Trade Policy and Global Sourcing:
A Rationale for Tariff Escalation

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Online Appendix (Not for Publication)

A Optimal Policies for a Small Open Economy: Derivations

A.1 First Best Policies for a Small Open Economy

A.1.1 Optimality Conditions of the Planner Problem

We begin by characterizing the solution to the program

$$\max \quad U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned} s.t. \quad & L^u_H + L^d_H = L_H \\ & \hat{A}^u_H\left(L^u_H\right) L^u_H = Q^u_{HH} + \tau^u Q^u_{HF} \\ & \hat{A}^d_H\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + \tau^d Q^d_{HF} \\ & P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}, \qquad \hat{A}^u_H = \bar{A}^u_H \left(L_H\right)^{\gamma^u},$$

with,

$$F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(L^d_i\right)^{\alpha}\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}.$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}, L^u_H, and L^d_H are as follows:

$$\begin{aligned} U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && \text{(A.1)} \\ U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} && \text{(A.2)} \\ \tau^d \mu_d &= \mu_{TB} \frac{\sigma-1}{\sigma} P^d_{HF} && \text{(A.3)} \\ \mu_u &= \mu_d\left(1+\gamma^d\right) \bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) && \text{(A.4)} \\ \mu_{TB} P^u_{FH} &= \mu_d\left(1+\gamma^d\right) \bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) && \text{(A.5)} \\ \tau^u \mu_u &= \mu_{TB} \frac{\theta-1}{\theta} P^u_{HF} && \text{(A.6)} \\ \mu_L &= \mu_u\left(1+\gamma^u\right) \bar{A}^u_H\left(L_H\right)^{\gamma^u} && \text{(A.7)} \\ \mu_L &= \mu_d\left(1+\gamma^d\right) \bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right), && \text{(A.8)} \end{aligned}$$

where we used μ_L, μ_u, and μ_d for the multiplier on the first, second and third feasibility constraints, respectively, and μ_{TB} for the multiplier in the trade balanced condition.

Dividing equation (A.1) by equation (A.2), and plugging in (A.3), we obtain:

$$\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{\frac{\sigma-1}{\sigma} P^d_{HF}/\tau^d}{P^d_{FH}},$$

which corresponds to the first optimality condition (9) in the main text.

Next, we divide equation (A.4) by equation (A.5), and plugging in (A.6), delivers

$$\frac{F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} = \frac{\frac{\theta-1}{\theta} P^u_{HF}/\tau^u}{P^u_{FH}},$$

which corresponds to the second optimality condition (10) in the main text.

Next, combining equation (A.4) with the ratio of equations (A.3) and (A.6) produces

$$\left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}} \frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d},$$

which corresponds to the third optimality condition (11) in the main text.

Finally, from dividing equation (A.7) by equation (A.8), and plugging in (A.4), we obtain

$$F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(1+\gamma^u\right) \bar{A}^u_H \left(L_H\right)^{\gamma^u} F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right),$$

which corresponds to equation (12) in the main text.

A.1.2 Optimality Conditions of the Decentralized Market Equilibrium

In this Appendix, we provide more details on the derivation of the equilibrium conditions (13)–(16).

Remember that Home consumers maximize

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}},$$

and face a price P^d_{HH} for domestic goods and a price $\left(1+t^d_H\right) P^d_{FH}$ for imports (remember that P^d_{FH} is defined inclusive of trade costs but exclusive of import tariffs). Exporters at Home must be indifferent between domestic sales or exports, which implies that $P^d_{HH} = \left(1-v^d_H\right) P^d_{HF}/\tau^d$. Equating the marginal rate of substitution to the relative price faced by consumers delivers

$$\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{1-v^d_H}{1+t^d_H} \frac{P^d_{HF}/\tau^d}{P^d_{FH}},$$

which corresponds to equation (13).

Home producers in turn maximize profits, which are given by

$$\pi^d_H = \frac{1}{1-s^d_H} P^d_{HH} \hat{A}^d_H F^d\left(\ell^d_H, q^u_{HH}, q^u_{FH}\right) - w\ell^d_H - P^u_{HH} q^u_{HH} - \left(1+t^u_H\right) P^u_{FH} q^u_{FH}.$$

Imposing $P^d_{HH} = \left(1-v^d_H\right) P^d_{HF}/\tau^d$ and $P^u_{HH} = \left(1-v^u_H\right) P^u_{HF}/\tau^u$, we can write this as

$$\pi^d_H = \frac{1}{1-s^d_H} \frac{\left(1-v^d_H\right) P^d_{HF}}{\tau^d} \hat{A}^d_H F^d\left(\ell^d_H, q^u_{HH}, q^u_{FH}\right) - w\ell^d_H - \frac{\left(1-v^u_H\right) P^u_{HF}}{\tau^u} q^u_{HH} - \left(1+t^u_H\right) P^u_{FH} q^u_{FH}.$$

The optimal mix of domestic and foreign inputs then ensures that the marginal rate of substitution between foreign and domestic inputs is equal to its relative price, or

$$\frac{F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} = \frac{1-v^u_H}{1+t^u_H}\frac{P^u_{HF}/\tau^u}{P^u_{FH}},$$

which corresponds to equation (14).

Next, note that equating the marginal rate of substitution between domestic inputs and labor delivers

$$\frac{F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} = \frac{\left(1-v^u_H\right)P^u_{HF}/\tau^u}{w}. \tag{A.9}$$

Now note that w also needs to correspond to the value of the marginal product of labor in the upstream sector, or

$$w = \frac{1}{1-s^u_H}\hat{A}^u_H\left(1-v^u_H\right)\frac{P^u_{HF}}{\tau^u}, \tag{A.10}$$

which, plugged into (A.9), delivers

$$F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \frac{1}{1-s^u_H}\hat{A}^u_H F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) \tag{A.11}$$

which corresponds to equation (16).

Finally, equating the value of the marginal product of labor w in both sectors implies

$$\frac{1}{1-s^u_H}\frac{\left(1-v^u_H\right)P^u_{HF}}{\tau^u}\hat{A}^u_H = w = \frac{1}{1-s^d_H}\frac{\left(1-v^d_H\right)P^d_{HF}}{\tau^d}\hat{A}^d_H F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right),$$

which, plugging (A.11), can be written as

$$\frac{\left(1-v^u_H\right)P^u_{HF}}{\tau^u} = \frac{1}{1-s^d_H}\frac{\left(1-v^d_H\right)P^d_{HF}}{\tau^d}\hat{A}^d_H F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right),$$

which corresponds to equation (15).

A.2 Second Best Policies for a Small Open Economy

In this Appendix, we characterize the second-best import tariffs when the government only has access to import tariffs upstream and downstream. The optimal import tariff are given by

$$\left(1+t^d_H\right) = \frac{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}\frac{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}/\tau^d}{P^d_{FH}},$$
$$\left(1+t^u_H\right) = \frac{F^d_{Q^u_{FH}}\left(L^d, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(L^d, Q^u_{HH}, Q^u_{FH}\right)}\frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}}P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}/\tau^u}{P^u_{FH}},$$

where the allocation is given by the solution of choosing L_H^u, L_H^d, Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, and Q_{HF}^u to

$$\begin{aligned}
\max \quad & U\left(Q_{HH}^d, Q_{FH}^d\right) \\
s.t. \quad & L_H^u + L_H^d = L_H \\
& \hat{A}_H^u\left(L_H^u\right) L_H^u = Q_{HH}^u + \tau^u Q_{HF}^u \\
& \hat{A}_H^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = Q_{HH}^d + \tau^d Q_{HF}^d \\
& P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u = Q_{HF}^d (Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} \\
& \hat{A}_H^d F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \frac{(Q_{HF}^u)^{-\frac{1}{\theta}} P_{FF}^u (Q_{FF}^u)^{\frac{1}{\theta}} / \tau^u}{(Q_{HF}^d)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} / \tau^d} \\
& F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right).
\end{aligned}$$

To prove Proposition 6 up to Propositions 8 we find useful to work with a modified version of this problem where we replace the last two equality constraints and the definition of the input tariff with:

$$\frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} / \tau^u}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} / \tau^d} = \bar{\kappa}_1 \hat{A}_H^d F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) \tag{A.12}$$

$$\frac{F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)} = \bar{\kappa}_2 \hat{A}^u, \tag{A.13}$$

$$\frac{F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)} \frac{P_{HF}^u / \tau^u}{P_{FH}^u} = \left(1 + t_H^u\right) \bar{\kappa}_1, \tag{A.14}$$

where $\bar{\kappa}_1$ and $\bar{\kappa}_2$ are two parameters satisfying $\bar{\kappa}_i \geq 1$.

This problem is useful because it allows us to think about the problem of a government choosing taxes when some but not all instruments are available, and eventually to derive parameter restrictions that ensure that the second-best import tariffs are escalated. For example, when $\bar{\kappa}_1 = \left(1 + \gamma^d\right) \frac{\theta}{\theta - 1} \frac{\sigma - 1}{\sigma}$ and $\bar{\kappa}_2 = 1 + \gamma^u$, the problem reduces to solving for the first-best allocation as we did in Appendix A.1. On the other hand, when $\bar{\kappa}_i = 1$ for all i the allocation is consistent with solving for second-best import tariffs. Intermediate values of $\bar{\kappa}_1$ correspond to fixing an arbitrary value for the export tax upstream equal to $1 - \nu_H^u = \frac{1}{\bar{\kappa}_1}$ (see equation (15)), while intermediate values of $\bar{\kappa}_2$ correspond to fixing an arbitrary value for the production subsidy upstream equal to $1 - s_H^u = \frac{1}{\bar{\kappa}_2}$ (see equation (16)).

A.2.1 Optimality Conditions of the Modified Planner Problem

The first order conditions associated with the choices of Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u, L_H^u, and L_H^d are as follows:

$$U_{Q^d_{HH}}(\cdot) = \mu_d \tag{A.15}$$

$$U_{Q^d_{FH}}(\cdot) = \mu_{TB} P^d_{FH} \tag{A.16}$$

$$\mu_d \tau^d = \frac{\sigma-1}{\sigma}\mu_{TB} P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d} \tag{A.17}$$

$$\begin{aligned}\mu_u = {} & \mu_d\left(1+\gamma^d\right) X^d_H \frac{F^d_{Q^u_{HH}}(\cdot)}{F^d(\cdot)} - \mu_{LC}\left[\frac{1}{\theta} + \left(\frac{\theta-1}{\theta}\right)\pi^u_{HH}\right]\frac{F^d_{L^d_H}(\cdot)}{Q^u_{HH}} \\ & + \mu_{SB}\frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d}\frac{\left[\left(1+\gamma^d\right)(1-\alpha) - \left(\frac{\theta-1}{\theta}\right)\right]\pi^u_{HH} - \frac{1}{\theta}}{Q^u_{HH}}\end{aligned} \tag{A.18}$$

$$\begin{aligned}\mu_{TB} P^u_{FH} = {} & \mu_d \frac{\left(1+\gamma^d\right) X^d_H F^d_{Q^u_{FH}}(\cdot)}{F^d(\cdot)} - \mu_{LC}\left(\frac{\theta-1}{\theta}\right)\frac{F^d_{L^d_H}(\cdot)\,\pi^u_{FH}}{Q^u_{FH}} \\ & + \mu_{SB}\frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d}\frac{\pi^u_{FH}}{Q^u_{FH}}\left[\left(1+\gamma^d\right)(1-\alpha) - \left(\frac{\theta-1}{\theta}\right)\right]\end{aligned} \tag{A.19}$$

$$\mu_u \tau^u = \mu_{TB}\frac{\theta-1}{\theta} P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d} \tag{A.20}$$

$$\mu_L = \mu_u\left(1+\gamma^u\right)\frac{X^u_H}{L^u_H} + \mu_{LC}\gamma^u \frac{F^d_{L^d_H}(\cdot)}{L^u_H} \tag{A.21}$$

$$\mu_L = \mu_d\left(1+\gamma^d\right) X^d_H \frac{F^d_{L^d_H}(\cdot)}{F^d(\cdot)} + \mu_{LC}\frac{F^d_{L^d_H}(\cdot)}{L^d_H} + \mu_{SB}\left(1+\gamma^d\right)\frac{\alpha}{L^d_H}\frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d}, \tag{A.22}$$

where as before we use μ_L, μ_u, and μ_d for the multipliers in each of the feasibility constraints, μ_{TB} for the trade balanced condition, while μ_{SB} is the multiplier on equation (A.12), and μ_{LC} is the multiplier on equation (A.13). The variable X^s_H represents total output in sector $s \in \{u, d\}$, and π^u_{HH} and π^u_{FH} are defined as:

$$\pi^u_{HH} \equiv \frac{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}}, \qquad \pi^u_{FH} = 1 - \pi^u_{HH}.$$

As a reminder the two added equilibrium constraints are:

$$\bar{\kappa}_1 X^d_H \frac{F^d_{Q^u_{HH}(\cdot)}}{F^d(\cdot)} = \frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d} \tag{A.23}$$

$$\bar{\kappa}_2 X^u_H \frac{F^u_{L^u_H}(\cdot)}{F^u(\cdot)} F^d_{Q^u_{HH}} = F^d_{L^d_H}(\cdot) \tag{A.24}$$

Manipulating the First-Order Conditions From equations (A.15), (A.16), and (A.17), we obtain:

$$\frac{U_{Q^d_{FH}}(\cdot)}{U_{Q^d_{HH}}(\cdot)} = \frac{P^d_{FH}}{P^d_{HF}}\left[\frac{\sigma}{\sigma-1}\tau^d + \frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}/\tau^u}{P^d_{HF}/\tau^d}\right].$$

Because in a competitive equilibrium with tariffs we have

$$\begin{aligned}\frac{U_{Q_{FH}^d}(\cdot)}{U_{Q_{HH}^d}(\cdot)} &= \left(1+t_H^d\right)\frac{P_{FH}^d}{P_{HF}^d/\tau^d} &\quad\text{(A.25)}\\ \frac{F_{Q_{FH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)} &= \bar{\kappa}_1\left(1+t_H^u\right)\frac{P_{FH}^u}{P_{HF}^u/\tau^u}. &\quad\text{(A.26)}\end{aligned}$$

we can establish that

$$1+t_H^d=\frac{\mu_{TB}P_{HF}^d/\tau^d}{\mu_d}=\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{\tau^d Q_{HF}^d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}. \quad\text{(A.27)}$$

Now combine equations (A.19), (A.23), (A.24), (A.26), and (A.27),

$$\frac{1}{\bar{\kappa}_1}\frac{1+t_H^d}{1+t_H^u}=\frac{1+\gamma^d}{\bar{\kappa}_1}+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left(1+\gamma^d\right)(1-\alpha)-\frac{\theta-1}{\theta}\right]-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{\bar{\kappa}_1\hat{A}_H^d L_H^d}. \quad\text{(A.28)}$$

We next plug equation (A.19) into equation (A.18) to obtain

$$\mu_{TB}P_{FH}^u\frac{\pi_{HH}^u}{Q_{HH}^u}\frac{Q_{FH}^u}{\pi_{FH}^u}=\mu_u+\mu_{LC}\frac{1}{\theta}\frac{F_{L_H^d}^d(\cdot)}{Q_{HH}^u}+\mu_{SB}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}\frac{1}{\theta}\frac{1}{Q_{HH}^u}.$$

Next, plugging μ_u from equation (A.20), and using what input tariff are in equilibrium we obtain

$$\mu_{TB}\frac{P_{HF}^u}{\tau^u}\left[\frac{1}{\bar{\kappa}_1}\frac{1}{1+t_H^u}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}\frac{1}{\theta}\left[\frac{1}{\tau^u Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right]+\mu_{LC}\frac{F_{L_H^d}^d(\cdot)}{Q_{HH}^u}\frac{1}{\theta}.$$

And, dividing by μ_d and plugging in (A.27), this delivers:

$$\frac{1}{\bar{\kappa}_1}\frac{1+t_H^d}{1+t_H^u}-\left(1+t_H^d\right)\frac{\theta-1}{\theta}=\frac{\mu_{SB}}{\mu^d}\frac{1}{\theta}\left(\frac{1}{\tau^u Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right)+\frac{1}{\theta}\frac{\mu_{LC}}{\mu^d}\frac{P_{HF}^d/\tau^d}{P_{HF}^u/\tau^u}\frac{F_{L_H^d}^d(\cdot)}{Q_{HH}^u}. \quad\text{(A.29)}$$

We finally seek to solve for μ_{LC} as a function of μ_{SB}. We begin with equation (A.21) and (A.22)

$$\frac{\mu_u}{\mu_d}\left(1+\gamma^u\right)\frac{X_H^u}{L_H^u}=\left(1+\gamma^d\right)X_H^d\frac{F_{L_H^d}^d(\cdot)}{F^d(\cdot)}+\frac{\mu_{LC}}{\mu_d}F_{L_H^d}^d(\cdot)\left[\frac{1}{L_H^d}-\frac{\gamma^u}{L_H^u}\right]+\frac{\mu_{SB}}{\mu_d}\left(1+\gamma^d\right)\frac{\alpha}{L^d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}.$$

Next, plug equation (A.18) and using (A.24), we obtain

$$\frac{\mu_{LC}}{\mu_d}=\frac{\left(1+\gamma^d\right)X_H^d\frac{F_{Q_{HH}^d}^d}{F^d(\cdot)}\left(1-\frac{\bar{\kappa}_2}{1+\gamma^u}\right)+\frac{\mu_{SB}}{\mu_d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left(1+\gamma^d\right)(1-\alpha)\left(1-\frac{\bar{\kappa}_2}{1+\gamma^u}\right)-\frac{\frac{\theta-1}{\theta}\pi_{HH}^u+\frac{1}{\theta}}{\pi_{HH}^u}\right]}{F_{L_H^d}^d(\cdot)\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\frac{\frac{1}{\theta}+\frac{\theta-1}{\theta}\pi_{HH}^u}{\pi_{HH}^u}+\frac{\bar{\kappa}_2}{1+\gamma^u}\frac{1-\alpha}{\alpha}\left(1-\gamma^u\frac{L_H^d}{L_H^u}\right)\right]}. \quad\text{(A.30)}$$

Recap of Key Equations The above derivations were lengthy and tedious, so it is useful to recap the four key equations that characterize the second-best optimal tariffs, which are equations (A.27), (A.28), (A.29), and (A.30):

$$
\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1}+\frac{1}{\sigma-1}\frac{\mu_{SB}}{\mu_d}\frac{1}{\tau^d Q_{HF}^d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d} \\
\frac{1}{\bar{\kappa}_1}\frac{1+t_H^d}{1+t_H^u} &= \frac{1+\gamma^d}{\bar{\kappa}_1}+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left(1+\gamma^d\right)(1-\alpha)-\frac{\theta-1}{\theta}\right] \\
&\quad -\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{\bar{\kappa}_1\hat{A}_H^d L_H^d} \\
\frac{1}{\bar{\kappa}_1}\frac{1+t_H^d}{1+t_H^u}-\left(1+t_H^d\right)\frac{\theta-1}{\theta} &= \frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left(\frac{1}{\tau^u Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right)+\frac{\mu_{LC}}{\mu_d}\frac{P_{HF}^d/\tau^d}{P_{HF}^u/\tau^u}\frac{F_{L_H^d}^d(\cdot)}{Q_{HH}^u}\frac{1}{\theta} \\
\frac{\mu_{LC}}{\mu_d} &= \frac{\left(1+\gamma^d\right)X_H^d\frac{F_{Q_{HH}^d}^d}{F^d(\cdot)}\left(1-\frac{\bar{\kappa}_2}{1+\gamma^u}\right)}{F_{L_H^d}^d(\cdot)\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\frac{\frac{1}{\theta}+\frac{\theta-1}{\theta}\pi_{HH}^u}{\pi_{HH}^u}+\frac{\bar{\kappa}_2}{1+\gamma^u}\frac{1-\alpha}{\alpha}\left(1-\gamma^u\frac{L_H^d}{L_H^u}\right)\right]} \\
&\quad +\frac{\frac{\mu_{SB}}{\mu_d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left(1+\gamma^d\right)(1-\alpha)\left(1-\frac{\bar{\kappa}_2}{1+\gamma^u}\right)-\frac{\frac{\theta-1}{\theta}\pi_{HH}^u+\frac{1}{\theta}}{\pi_{HH}^u}\right]}{F_{L_H^d}^d(\cdot)\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\frac{\frac{1}{\theta}+\frac{\theta-1}{\theta}\pi_{HH}^u}{\pi_{HH}^u}+\frac{\bar{\kappa}_2}{1+\gamma^u}\frac{1-\alpha}{\alpha}\left(1-\gamma^u\frac{L_H^d}{L_H^u}\right)\right]}.
\end{aligned}
$$

We can now explore various special cases of this system.

A.2.2 Proof Proposition 6 and Proposition 7

We start setting $\bar{\kappa}_i = 1$ for all i, the allocation corresponds to the second-best import tariff.

Second-Best Import Tariffs with No Labor Employed Downstream ($\alpha = 0$). Consider first the case in which $\alpha = 0$, so labor is only used in the upstream sector. As noted in the main text, the labor-market constraint and associated multiplier μ_{LC} become irrelevant, i.e., $\mu_{LC} = 0$, and the above system reduces to

$$
\begin{aligned}
1+t_H^d &= =\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d\tau^d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d} \\
\frac{1+t_H^d}{1+t_H^u} &= \left(1+\gamma^d\right)+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left(1+\gamma^d\right)-\left(\frac{\theta-1}{\theta}\right)\right].
\end{aligned}
$$

As long as Assumption 1 holds, i.e., $1+\gamma^d > \frac{(\theta-1)/\theta}{(\sigma-1)/\sigma}$, then $\mu_{SB} > 0$. We must thus have

$$
1+t_H^d > \frac{\sigma}{\sigma-1}, \qquad \frac{1+t_H^d}{1+t_H^u} > 1+\gamma^d.
$$

This proves Proposition 6 for the case $\alpha = 0$.

Second-Best Import Tariffs with No Scale Economies Upstream ($\gamma^u = 0$). We next study the case in which $\alpha > 0$ but $\gamma^u = 0$. In that case, equation (A.30) reduces to

$$
\frac{\mu_{LC}}{\mu_d} = -\frac{\mu_{SB}}{\mu_d}\hat{A}_H^d\frac{F_{Q_{HH}^u}^d(\cdot)}{F_{L_H^d}^d(\cdot)}\frac{\alpha\left[\left(\frac{\theta-1}{\theta}\right)\pi_{HH}^u+\frac{1}{\theta}\right]}{\alpha\left[\left(\frac{\theta-1}{\theta}\right)\pi_{HH}^u+\frac{1}{\theta}\right]+(1-\alpha)\pi_{HH}^u}.
$$

Then, using this information in equations (A.27) and (A.28), we get the following two relationships:

$$
\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{\tau^d Q_{HF}^d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d} && \text{(A.31)}\\
\frac{1+t_H^d}{1+t_H^u} &= \left(1+\gamma^d\right)+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\left(1+\gamma^d\right)-\frac{\pi_{HH}^u\left(\frac{\theta-1}{\theta}\right)}{\alpha\left[\left(\frac{\theta-1}{\theta}\right)\pi_{HH}^u+\frac{1}{\theta}\right]+(1-\alpha)\pi_{HH}^u}\right]. && \text{(A.32)}
\end{aligned}
$$

As long as Assumption 1 holds, i.e., $1+\gamma^d>\frac{(\theta-1)/\theta}{(\sigma-1)/\sigma}$, $\mu_{SB}>0$. Then, it follows that:

$$
1+t_H^d>\frac{\sigma}{\sigma-1}, \qquad \frac{1+t_H^d}{1+t_H^u}>1+\gamma^d.
$$

This proves Proposition 6 for the case $\gamma^u=0$.

Second-Best Import Tariffs with No Scale Economies in Either Sector. In this case, we only need to set $\gamma_d=0$ into equations (A.31) and (A.32) to get:

$$
\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{\tau^d Q_{HF}^d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}\\
\frac{1+t_H^d}{1+t_H^u} &= 1+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\frac{1}{\theta}\frac{\alpha+(1-\alpha)\pi_{HH}^u}{\alpha\left[\left(\frac{\theta-1}{\theta}\right)\pi_{HH}^u+\frac{1}{\theta}\right]+(1-\alpha)\pi_{HH}^u}\right].
\end{aligned}
$$

Now notice that when $\gamma^d=0$, Assumption 1 simplifies to $\frac{\sigma-1}{\sigma}\geq\frac{\theta-1}{\theta}$, or $\sigma\geq\theta$. Then, as long as Assumption 1 holds, we have $\mu_{SB}>0$ and tariffs are escalated. On the contrary, when $\sigma<\theta$, then $\mu_{SB}<0$, and tariffs are de-escalated. This concludes the proof of Proposition 7.

A.2.3 Proof of Proposition 8

To prove Proposition 8 we proceed in multiple steps. We start with two alternative second-best scenarios where we allow the planner to use either an upstream production subsidy or an upstream export tax on top of the two import tariffs. Although these two cases are not directly related to the main message in the paper, they are useful for deriving the restrictions in the parameter space that guarantee escalated tariffs as a the second-best policy. In this section, we further impose Assumption 2, i.e., $1<1+\gamma^u\leq\frac{\theta}{\theta-1}$.

Second-best with import tariffs and production subsidy. From our previous discussion at the beginning of Section A.2, this case corresponds to setting $\frac{1}{\bar{\kappa}_2}=1-s_H^u$, with s_H^u being the optimal production subsidy that the planner chooses when import tariffs and a production subsidy upstream are available. In this case, equation (A.13) trivially holds at the chosen allocation so we can drop it from the problem, i.e., $\mu_{LC}=0$. We also impose $\bar{\kappa}_1=1$,which implies zero export taxes. Our system equations (A.27), (A.28), (A.29), and (A.30), reduces to:

$$
\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{\tau^d Q_{HF}^d}\frac{P_{HF}^u/\tau^u}{P_{HF}^d/\tau^d}\\
\frac{1+t_H^d}{1+t_H^u} &= \left(1+\gamma^d\right)+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left(1+\gamma^d\right)(1-\alpha)-\left(\frac{\theta-1}{\theta}\right)\right]\\
\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right) &= \frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{\tau^u Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right]\\
\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\frac{\left(\frac{\theta-1}{\theta}\right)\pi_{HH}^u+\frac{1}{\theta}}{\pi_{HH}^u} &= \left(1+\gamma^d\right)\left[1-\frac{1}{\left(1-s_H^u\right)\left(1+\gamma^u\right)}\right]\left(1+\frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\frac{\mu_{SB}}{\mu_d}\right).
\end{aligned}
$$

Note then that under Assumption 1, $\mu_{SB}>0$. Therefore, we must have:

$$
1+t_H^d>\frac{\sigma}{\sigma-1}, \qquad \text{and} \qquad 1-s_H^u>\frac{1}{1+\gamma^u}.
$$

Furthermore, if

$$
\left(1+\gamma^d\right)(1-\alpha)\geq\frac{\theta-1}{\theta} \tag{A.33}
$$

tariffs are escalated with a level of escalation above its first-best level, i.e., $\frac{1+t_H^d}{1+t_H^u}\geq 1+\gamma^d$.

Second-best with import tariffs and an export tax. This case correspond to setting $\frac{1}{\bar{\kappa}_1}=1-\nu_H^u$, with ν_H^u being the optimal upstream export tax that the planner chooses when import tariffs and an upstream export tax are available. In this case, equation (A.12) trivially holds at the chosen allocation so we can drop it from the problem, i.e., $\mu_{SB}=0$. We also impose $\bar{\kappa}_2=1$,which implies zero production subsidies. Our system equations (A.27), (A.28), (A.29), and (A.30), reduces to:

$$
\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1} &\text{(A.34)}\\
\frac{1+t_H^d}{1+t_H^u} &= \left(1+\gamma^d\right)-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{\hat{A}_H^d}\frac{1}{L_H^d} &\text{(A.35)}\\
\frac{1+t_H^d}{1+t_H^u}\left(1-\nu_H^u\right)-\frac{\theta-1}{\theta}\left(1+t_H^d\right) &= \frac{\mu_{LC}}{\mu_d}\frac{1}{\theta}\frac{P_{HF}^d/\tau^d}{P_{HF}^u/\tau^u}\frac{F_{L_H^d}^d(\cdot)}{Q_{HH}^u} &\text{(A.36)}\\
\frac{\mu_{LC}}{\mu_d} &= \frac{\left(1+\gamma^d\right)X_H^d\frac{F_{Q_{HH}^u}^d(\cdot)}{F^d(\cdot)}\left[1-\frac{1}{(1+\gamma^u)}\right]}{F_{L_H^d}^d(\cdot)\frac{\pi_{HH}^u}{Q_{HH}^u}\left\{\frac{\left[\frac{1}{\theta}+\left(\frac{\theta-1}{\theta}\right)\pi_{HH}^u\right]}{\pi_{HH}^u}+\frac{1}{(1+\gamma^u)}\frac{1-\alpha}{\alpha}\left[1-\frac{\gamma^u L_H^d}{L_H^u}\right]\right\}} &\text{(A.37)}
\end{aligned}
$$

Next, use equation (A.37) to eliminate $\frac{\mu_{LC}}{\mu_d}$. Then, combine equations (A.34), (A.35) and (A.36) with equation (A.23), and solve for the upstream export tax to get:

$$
\frac{1}{\left(1-\nu_H^u\right)}=\left(1+\gamma^d\right)\frac{\sigma-1}{\sigma}\frac{\theta}{\theta-1}\frac{\frac{\theta-1}{\theta}-A}{\left(1+\gamma^u\right)\frac{\theta-1}{\theta}-A}, \tag{A.38}
$$

where $\bar{\omega}\equiv\frac{\frac{\theta-1}{\theta}\pi_{HH}^u}{\frac{1}{\theta}+\frac{\theta-1}{\theta}\pi_{HH}^u}$ satisfies $\bar{\omega}\in(0,1)$, and $A\equiv\frac{(1-\alpha)}{\alpha}L^d\left[\frac{\gamma^u}{L^u}-\frac{1}{L^d}\right]\bar{\omega}$ satisfies $A\leq\frac{\theta-1}{\theta}$.[1]

[1]The upper bound on A follows from the fact that with $\gamma_u\geq 0$ we have $\mu_{LC}\geq 0$.

More importantly, from equations (A.35) and (A.37), we get that the ratio of import tariffs is given by,

$$\frac{1+t_H^d}{1+t_H^u} = \left(1+\gamma^d\right)\left((1-\bar{\omega}) + \bar{\omega}\frac{\frac{\theta-1}{\theta} - A}{(1+\gamma^u)\frac{\theta-1}{\theta} - A}\right). \tag{A.39}$$

Note that the efficient level of escalation is multiplied by a convex combination of two numbers less or equal to one, so the degree of escalation is below its first-best level.

Now we are ready to show that if the following condition holds,

$$1+\gamma_u \leq \frac{\theta}{\theta-1}, \qquad \text{and} \qquad \left(1+\gamma^d\right)\left(1-\gamma_u\alpha\frac{\theta-1}{\theta}\right) \geq 1, \tag{A.40}$$

then tariffs are escalated. We split the proof in two parts. First, we will bound the equilibrium allocation of labor upstream. Then, we will use this information to bound the ratio of import tariffs.

1. Bounding the allocation of labor. At the first-best allocation we have:

$$\left(\frac{L_H^u}{L_H}\right)^* = \frac{\frac{(1-\alpha)}{\alpha}\gamma^u\bar{\omega}^*}{\left(\frac{\theta-1}{\theta}\right) + \frac{(1-\alpha)}{\alpha}\bar{\omega}^*(1+\gamma^u)} < \frac{(1-\alpha)\gamma^u}{\alpha+(1-\alpha)(1+\gamma^u)},$$

where the inequality comes from the fact that $\bar{\omega}^*$ is increasing in π_{HH}^u. We also know that the second-best allocation is inefficient because it allocates a lower amount of labor upstream, and so the equilibrium allocation of labor upstream satisfies:

$$\left(\frac{L_H^u}{L_H}\right) \leq \left(\frac{L_H^u}{L_H}\right)^* < \frac{(1-\alpha)\gamma^u}{\alpha+(1-\alpha)(1+\gamma^u)}. \tag{A.41}$$

To bound the allocation of labor upstream from below, we start from the labor market clearing condition for labor, and use (in order): (i) the optimal choice of labor downstream, (ii) feasibility in the downstream sector; (iii) the household budget constraint, (iv) the definition of the government revenue and trade balance; (v) feasibility upstream and the relationship between upstream prices; and (vi) the zero-profit condition upstream to get:

$$\begin{aligned}
w_H L_H &= w_H L_H^d + w_H L_H^u \\
&= \alpha P_{HH}^d X_H^d + w_H L_H^u \\
&= \alpha\left[P_{HH}^d Q_{HH}^d + P_{HH}^d Q_{HF}^d \tau^d\right] + w_H L_H^u \\
&= \alpha\left[w_H L_H + R_H - \left(1+t_H^d\right)P_{FH}^d Q_{FH}^d + P_{HF}^d Q_{HF}^d \tau^d\right] + w_H L_H^u \\
&= \alpha\left[w_H L_H + (1+t_H^u)P_{FH}^u Q_{FH}^u - (1-\nu_H^u)P_{HF}^u Q_{HF}^u\right] + w_H L_H^u \\
&= \alpha\left[w_H L_H + (1+t_H^u)P_{FH}^u Q_{FH}^u + P_{HH}^u Q_{HH}^u - P_{HH}^u X_H^u\right] + w_H L_H^u \\
&= \alpha\left[w_H L_H + (1+t_H^u)P_{FH}^u Q_{FH}^u + P_{HH}^u Q_{HH}^u - w_H L_H^u\right] + w_H L_H^u \\
&= \alpha\left[w_H L_H + P_{HH}^u Q_{HH}^u\left[\frac{\pi_{FH}^u}{\pi_{HH}^u} + 1\right] - w_H L_H^u\right] + w_H L_H^u \\
&= \alpha w_H L_H + \left[\alpha\frac{P_{HH}^u Q_{HH}^u}{P_{HH}^u X_H^u}\frac{1}{\pi_{HH}^u} + (1-\alpha)\right] w_H L_H^u.
\end{aligned}$$

Solving for L_H^u/L_H gives us:

$$\frac{L_H^u}{L_H} = \frac{(1-\alpha)}{\frac{P_{HH}^u Q_{HH}^u}{P_{HH}^u X_H^u}\frac{1}{\pi_{HH}^u}\alpha + (1-\alpha)}.$$

Therefore,

$$\frac{L^u}{L} > \frac{(1-\alpha)}{\frac{1}{\pi^u_{HH}}\alpha + (1-\alpha)}, \quad \forall \pi^u_{HH} \in (0,1)\,. \tag{A.42}$$

2. Bounding from below the ratio of import tariffs. Remember from equation (A.39) that the ratio of import tariffs is given by:

$$\frac{1+t^d_H}{1+t^u_H} = \left(1+\gamma^d\right)\left((1-\bar{\omega}) + \bar{\omega}\frac{\frac{\theta-1}{\theta} - A}{(1+\gamma^u)\frac{\theta-1}{\theta} - A}\right) = \left(1+\gamma^d\right)\left(1 - \frac{\theta-1}{\theta}\bar{\omega}\frac{\gamma^u}{(1+\gamma^u)\frac{\theta-1}{\theta} - A}\right).$$

Define then the following function[2]

$$f(x) \equiv \left(1+\gamma^d\right)\left(1 - \frac{x\frac{\theta-1}{\theta}\gamma^u}{\left[(1+\gamma^u)\left(\frac{\theta-1}{\theta}\right) + \frac{(1-\alpha)}{\alpha}\right]x + \frac{1}{\theta}(1+\gamma^u) - \gamma^u}\right).$$

which has the following properties: (i) the function $f(\cdot)$ has both a vertical and a horizontal asymptote at $x_{VA} = \frac{\gamma^u - (1+\gamma^u)\frac{1}{\theta}}{\left[(1+\gamma^u)\left(\frac{\theta-1}{\theta}\right) + \frac{(1-\alpha)}{\alpha}\right]}$ and $y_{HA} = \left(1+\gamma^d\right)\frac{\left[\left(\frac{\theta-1}{\theta}\right) + \frac{(1-\alpha)}{\alpha}\right]}{(1+\gamma^u)\left(\frac{\theta-1}{\theta}\right) + \frac{(1-\alpha)}{\alpha}}$, respectively; (ii) for $1+\gamma^u \leq \frac{\theta}{\theta-1}$, the function $f(\cdot)$ is decreasing in x for $x \in [0,1]$, and the vertical asymptote satisfies $x_{VA} < 0$.

Because the ratio of import tariffs is a an increasing function of L^u_H/L_H, we have that for all $\pi^u_{HH} \in [0,1]$ the following inequality holds:[3]

$$\frac{1+t^d_H}{1+t^u_H} \geq f\left(\pi^u_{HH}\right) \geq f(1) > 1,$$

where the last inequality follows from the restriction $\left(1+\gamma^d\right)\left(1 - \alpha\gamma^u\frac{\theta-1}{\theta}\right)$. Then, if restriction (A.40) holds, tariffs are escalated in this scenario as well.

Second-best with only import tariff featuring escalation. Finally, we set $\bar{\kappa}_1$ and $\bar{\kappa}_2$ to one, so both export taxes and the upstream production subsidies equal zero. The ratio of import tariffs is given in this case by equation (A.28):

$$\frac{1+t^d_H}{1+t^u_H} = \left(1+\gamma^d\right) + \frac{\mu_{SB}}{\mu_d}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\left(1+\gamma^d\right)(1-\alpha) - \left(\frac{\theta-1}{\theta}\right)\right] - \frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\left(\frac{\theta-1}{\theta}\right)\frac{1}{\hat{A}^d_H}\frac{1}{L^d_H}.$$

This expression conveys the necessary intuition to finish proving Proposition 8. The first term is the level of escalation that the planner would like to impose if the first-best allocation were feasible. However, the planner cannot use export taxes nor production subsidies so the ratio of tariffs is distorted from its first-best value.[4] The absent export tax upstream adds the first deviation from first-best captured by the second term. When export taxes are not available, and provided condition (A.33) holds, the planner would like to set more escalated tariffs. However, how much more escalation the planner is able to impose is constrained by the absence of an upstream production subsidy. This is captured by the last term in the previous equation.

In general, it is not possible to determine the size of these forces to conclude the resulting degree of

[2]This function comes from imposing our bound equation (A.41) into our formula for tariff escalation.

[3]The restriction $\gamma^u \leq \frac{1}{\theta-1}$ is important, otherwise the function $f(\cdot)$ has a vertical asymptote at $x \in (0,1)$. Therefore, at that point the inequality $\frac{1+t^d_H}{1+t^u_H} \geq f(x_{VA} - \epsilon)$ breaks for ϵ very small.

[4]Remember that under Assumption 1, and provided that γ_u>0, we have that μ_{SB} and μ_{LC} are non-negative.

escalation. However, we have shown that if condition (A.40) holds, the second term is not strong enough to push the ratio of tariff below one. Therefore, when both condition (A.33) and (A.40) hold, the ratio of import tariffs must be above one. In other words, whenever

$$\left(1+\gamma^{d}\right) \times \min \left\{1-\alpha \gamma^{u} \frac{\theta-1}{\theta},(1-\alpha) \frac{\theta}{\theta-1}, \frac{\sigma-1}{\sigma} \frac{\theta}{\theta-1}\right\} \geq 1,$$

we have,

$$\frac{1+t_{H}^{d}}{1+t_{H}^{u}}>1.$$

This completes the proof of Proposition 8.

B Inspecting the Mechanism : Derivations

B.1 Closed-Economy Model

In this Appendix, we provide more details on the closed-economy version of our model.

B.1.1 Social Planner Problem

The social planner maximizes the Household utility, choosing L^u and L^d subject to feasibility, or:

$$\begin{aligned} \max_{L^u,L^d} \quad & U = Q^d \\ s.t. \quad & L = L^u + L^d \\ & Q^u = \hat{A}^u \left(L^u\right) L^u \\ & Q^d = \hat{A}^d \left(F^d \left(L^d, Q^u\right)\right) F^d \left(L^d, Q^u\right). \end{aligned}$$

Clearly, this reduces to

$$\begin{aligned} \max_{L^u,L^d} \quad & U = \bar{A}^d \left(F^d \left(L^d, \bar{A}^u \left(L^u\right)^{1+\gamma^u}\right)\right)^{1+\gamma^d} \\ s.t. \quad & L = L^u + L^d, \end{aligned}$$

which gives rise to the unique optimality condition

$$F^d_{L^d} \left(L^d, Q^u\right) = \left(1+\gamma^u\right) \bar{A}^u \left(L^u\right)^{\gamma^u} F^d_{Q^u} \left(L^d, Q^u\right). \tag{B.1}$$

Given the Cobb-Douglas functional form in (2) and the fact that $Q^u = \bar{A}^u \left(L\right)^{1+\gamma^u}$, this condition reduces to

$$\frac{\alpha}{L^d} = \left(1+\gamma^u\right) \frac{1-\alpha}{L^u},$$

which imposing labor market clearing implies that the socially optimal share of labor allocated to the upstream sector is given by

$$\left(\frac{L^u}{L}\right)^* = \frac{\left(1+\gamma^u\right)\left(1-\alpha\right)}{\alpha + \left(1+\gamma^u\right)\left(1-\alpha\right)}. \tag{B.2}$$

B.1.2 Decentralized Equilibrium

We next compare this social optimum with the decentralized equilibrium of the closed economy. Assume the government sets production subsidies on downstream (s^d) and upstream (s^u) production. Final-good producers maximize profits taking $\hat{A}^u_H$ and $\hat{A}^d_H$ as given, or

$$\max \quad \frac{1}{1-s^d} P^d \hat{A}^d F^d \left(\ell^d, q^u\right) - w\ell^d - P^u q^u,$$

which delivers the first order conditions:

$$\begin{aligned} \frac{1}{1-s^d} P^d \hat{A}^d F^d_{\ell^d} \left(\ell^d, q^u\right) &= w \\ \frac{1}{1-s^d} P^d \hat{A}^d F^d_{q^u} \left(\ell^d, q^u\right) &= P^u. \end{aligned}$$

Upstream producers only hire labor, and their zero-profit conditions imposes

$$P^u = (1 - s^u) \frac{w}{\hat{A}^u}.$$

Combining these conditions, we find that

$$F^d_{\ell^d}\left(\ell^d, q^u\right) = \frac{1}{1-s^u} \hat{A}^u F^d_{q^u}\left(\ell^d, q^u\right).$$

Imposing symmetric behavior across all agents then implies

$$F^d_{L^d}\left(L^d, Q^u\right) = \frac{1}{1-s^u} \bar{A}^u \left(L^u\right)^{\gamma^u} F^d_{Q^u}\left(L^d, Q^u\right), \tag{B.3}$$

which is the decentralized equilibrium analogue of equation (B.1).

Given the Cobb-Douglas functional form in (2) and the fact that $Q^u = \bar{A}^u \left(L\right)^{1+\gamma^u}$, this condition implies that

$$\frac{\alpha}{L^d} = \frac{1}{1-s^u} \frac{1-\alpha}{L^u},$$

which imposing labor market clearing implies that the socially optimal share of labor allocated to the upstream sector is given by

$$\frac{L^u}{L} = \frac{1-\alpha}{\alpha\left(1-s^u\right)+1-\alpha}. \tag{B.4}$$

Comparing equations (B.2)-(B.4) to the corresponding ones in the market equilibrium, we conclude that:

Proposition B.1. In the decentralized equilibrium with no production subsidies, there is too little labor allocated upstream unless $\alpha = 1$ (so the upstream sector is shut down), $\alpha = 0$ (so the downstream sector does not use labor directly in production), or $\gamma^u = 0$ (so the upstream sector features constant returns to scale).

Notice also that the ratio of the socially efficient allocation of labor upstream to its decentralized equilibrium allocation with $s^u = 0$ is given by

$$\frac{\left(L^u\right)^*}{L^u} = \frac{1+\gamma^u}{\alpha + \left(1+\gamma^u\right)\left(1-\alpha\right)},$$

which is higher, the higher is α. Thus, the decentralized market misallocation of upstream labor is increasing in the final-good sector's labor share α.

B.1.3 Optimal Policy

A simple comparison of equations (B.2) and (B.4) also reveals that:

Proposition B.2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $\left(s^u\right)^* = \gamma^u / \left(1+\gamma^u\right)$. Downstream production subsidies have no impact on the decentralized equilibrium.

In sum, we have shown that a vertical model with external economies of scale features inefficient entry upstream, and that this market failure can be addressed with an upstream production subsidy. Conversely, downstream production subsidies are not helpful in addressing this market failure.

Notice finally that because the social planner seeks to maximize Q^d, the optimal upstream subsidy increases downstream output even though it pulls labor away from the downstream sector. This is due to the increasing returns to scale upstream. By increasing the size of the upstream sector, the optimal subsidy also raises its efficiency, which provides the downstream sector with more inputs such that it also grows.

B.1.4 Extensions

We finally briefly develop two extensions of our closed-economy model, both featuring a more complex input sector.

Roundabout Production Upstream We first allow the upstream sector to use not only labor in production, but also the same bundle of inputs Q^u used in the final-good sector. More specifically, we now assume

$$\begin{aligned} x^u &= \bar{A}^u (\ell^u)^\beta (q^u)^{1-\beta} \left((L^u)^\beta (Q^u)^{1-\beta} \right)^{\gamma^u} \\ x^d &= \bar{A}^d \left(\ell^d\right)^\alpha (q^u)^{1-\alpha} \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{\gamma^d}, \end{aligned}$$

where β governs the labor intensity of production upstream. It is clear from the second of these expressions that firms in the downstream sector will spend a fraction of their costs on the upstream sector, or

$$P^u q^u = (1-\alpha) P^d x^d.$$

Noting that, due to symmetry, $x^u = Q^u$ and $x^d = Q^d$, and that the decentralized market prices for the downstream sector is given by

$$P^d = \frac{1}{\bar{A}^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$ and $Q^d = \bar{A}^d \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{1+\gamma^d}$ we thus obtain

$$\frac{1}{\bar{A}^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u Q^u}{1-\alpha}\right)^{1-\alpha} \bar{A}^d \left(L^d\right)^\alpha = wL$$

or

$$\left(\frac{w}{\alpha}\right)^\alpha (wL)^{1-\alpha} \left(L^d\right)^\alpha = wL,$$

from which it is immediate that

$$L^u = (1-\alpha) L, \quad \text{and} \quad L^d = \alpha L,$$

just as in our baseline model

We next consider the planner problem,

$$\begin{aligned} \max_{L^u, L^d} \quad & Q^d = \bar{A}^d \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{1+\gamma^d} \\ s.t. \quad & Q^u = \bar{A}^u \left((L^u)^\beta (Q^u)^{1-\beta} \right)^{1+\gamma^u} \\ & L^u + L^d = L. \end{aligned}$$

Noting that the second constraint can be written as

$$Q^u = \tilde{A}^u (L^u)^{1+\tilde{\gamma}^u},$$

where

$$\begin{aligned} \tilde{A}^u &= \left(\bar{A}^u\right)^{\frac{1}{1-(1-\beta)(1+\gamma^u)}} \\ \tilde{\gamma}^u &= \frac{\gamma^u}{1-(1-\beta)(1+\gamma^u)}, \end{aligned}$$

it then becomes clear that this program is identical to the one in our baseline model, except for the fact that the scale elasticity upstream is now not given by γ^u, but by $\tilde{\gamma}^u > \gamma^u$ (the program also features a rescaled upstream productivity, but that is immaterial). Note that the gap between $\tilde{\gamma}^u$ and γ^u is decreasing in β.

Analogously to equation (B.2), the socially optimal allocation of labor is given by

$$(L^u)^* = \frac{1+\tilde{\gamma}^u}{\tilde{\gamma}^u(1-\alpha)+1}(1-\alpha)L, \quad \text{and} \quad \left(L^d\right)^* = \frac{1}{\tilde{\gamma}^u(1-\alpha)+1}\alpha L.$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$, just as in our baseline model, but the inefficiency is now decreasing in β. Finally, one can also verify that an upstream subsidy equal to $(s^u)^* = \tilde{\gamma}^u / (1+\tilde{\gamma}^u)$ is sufficient to restore efficiency. Because $\tilde{\gamma}^u > \gamma^u$, this subsidy is now higher than in our baseline model, and it is decreasing in β.

Multi-Stage Production We next develop a multi-stage extension of the model. We begin with a simple three-stage production process with a downstream sector, a midstream sector, and an upstream sector. The technologies are given by

$$\begin{aligned} x^u &= \bar{A}^u (\ell^u)(L^u)^{\gamma^u} \\ x^m &= \bar{A}^m (\ell^m)^{\beta} (q^u)^{1-\beta} \left((L^m)^{\beta}(Q^u)^{1-\beta}\right)^{\gamma^m} \\ x^d &= \bar{A}^d \left(\ell^d\right)^{\alpha} (q^m)^{1-\alpha} \left(\left(L^d\right)^{\alpha}(Q^m)^{1-\alpha}\right)^{\gamma^d}, \end{aligned}$$

Using the fact that, in a decentralized equilibrium, we have

$$\begin{aligned} P^d Q^d &= wL; \\ Q^d &= \bar{A}^d \left(\left(L^d\right)^{\alpha}(Q^m)^{1-\alpha}\right)^{1+\gamma^d}; \\ P^d &= \frac{1}{\bar{A}^d}\left(\frac{w}{\alpha}\right)^{\alpha}\left(\frac{P^m}{1-\alpha}\right)^{1-\alpha}\left(\left(L^d\right)^{\alpha}(Q^m)^{1-\alpha}\right)^{-\gamma^d}; \\ P^m Q^m &= (1-\alpha) P^d Q^d, \end{aligned}$$

we immediately obtain

$$L^d = \alpha L.$$

Next, because

$$\begin{aligned}
P^m Q^m &= (1-\alpha)\, wL; \\
Q^m &= \bar{A}^m \left((L^m)^{\alpha} (Q^u)^{1-\alpha} \right)^{1+\gamma^m}; \\
P^m &= \frac{1}{\bar{A}^m} \left(\frac{w}{\beta} \right)^{\beta} \left(\frac{P^u}{1-\beta} \right)^{1-\beta} \left((L^m)^{\beta} (Q^u)^{1-\beta} \right)^{-\gamma^m}; \\
P^u Q^u &= (1-\beta)\, P^m Q^m,
\end{aligned}$$

we obtain

$$L^m = \beta (1-\alpha) L, \quad \text{and} \quad L^u = (1-\beta)(1-\alpha) L.$$

Now consider the planner problem

$$\begin{aligned}
\max_{L^u, L^m, L^d} \quad & Q^d = \bar{A}^d \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{1+\gamma^d} \\
s.t. \quad & Q^m = \bar{A}^m \left((L^m)^{\beta} (Q^u)^{1-\beta} \right)^{1+\gamma^u} \\
& Q^u = \bar{A}^u (L^u)^{1+\gamma^u} \\
& L^u + L^m + L^d = L.
\end{aligned}$$

which delivers

$$\begin{aligned}
(L^u)^* &= \frac{(1+\gamma^u)(1+\gamma^m)}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} (1-\beta)(1-\alpha) L \\
(L^m)^* &= \frac{1+\gamma^m}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \beta (1-\alpha) L \\
\left(L^d\right)^* &= \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \alpha L.
\end{aligned}$$

Notice that the gap between the socially optimal and the market allocation of labor is higher the more upstream the stage. Does that mean that subsidies are higher, the more upstream a sector? To answer this question, consider the following key conditions which identify a market equilibrium with subsidies:

$$\begin{aligned}
P^d Q^d &= wL - s^m P^m Q^m - s^u P^u Q^u \\
Q^d &= \bar{A}^d \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{1+\gamma^d}; \\
P^d &= \frac{1}{\bar{A}^d} \left(\frac{w}{\alpha} \right)^{\alpha} \left(\frac{(1-s^m) P^m}{1-\alpha} \right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{-\gamma^d}; \\
(1-s^m) P^m Q^m &= (1-\alpha) P^d Q^d \\
Q^m &= \bar{A}^m \left((L^m)^{\alpha} (Q^u)^{1-\alpha} \right)^{1+\gamma^m} \\
P^m &= \frac{1}{\bar{A}^m} \left(\frac{w}{\beta} \right)^{\beta} \left(\frac{(1-s^u) P^u}{1-\beta} \right)^{1-\beta} \left((L^m)^{\beta} (Q^u)^{1-\beta} \right)^{-\gamma^m} \\
(1-s^u) P^u Q^u &= (1-\beta) P^m Q^m
\end{aligned}$$

Note that

$$P^d Q^d = wL - \frac{s^m}{1-s^m} (1-\alpha) P^d Q^d - \frac{s^u}{1-s^u} \frac{(1-\beta)}{1-s^m} (1-\alpha) P^d Q^d$$

or

$$P^d Q^d = \frac{wL}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}.$$

Next

$$\begin{aligned} P^d Q^d &= \left(\frac{wL^d}{\alpha}\right)^{\alpha}\left(\frac{(1-s^m)\,P^m Q^m}{1-\alpha}\right)^{1-\alpha} \\ &= \left(\frac{wL^d}{\alpha}\right)^{\alpha}\left(P^d Q^d\right)^{1-\alpha}, \end{aligned}$$

so

$$\frac{L^d}{L} = \frac{\alpha}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}.$$

Next,

$$\begin{aligned} P^m Q^m &= \left(\frac{wL^m}{\beta}\right)^{\beta}\left(\frac{(1-s^u)\,P^u Q^u}{1-\beta}\right)^{1-\beta} \\ &= \left(\frac{wL^m}{\beta}\right)^{\beta}\left(P^m Q^m\right)^{1-\beta}, \end{aligned}$$

so

$$P^m Q^m = \frac{(1-\alpha)}{(1-s^m)} P^d Q^d = \frac{wL^m}{\beta}$$

or

$$\frac{L^m}{L} = \frac{\beta\frac{(1-\alpha)}{(1-s^m)}}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}.$$

We thus have that the subsidies s^m and s^u need to satisfy

$$\frac{\alpha}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)} = \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))}\alpha$$

and

$$\frac{\beta\frac{(1-\alpha)}{(1-s^m)}}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)} = \frac{1+\gamma^m}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))}\beta(1-\alpha),$$

which delivers

$$s^m = \frac{\gamma^m}{1+\gamma^m}; \quad s^u = \frac{\gamma^u}{1+\gamma^u}.$$

As is clear from this expression, subsidies in all sectors producing inputs are positive, but notice that subsidies are higher upstream relative to midstream only if $\gamma^u > \gamma^m$, i.e., only if the scale elasticity is higher upstream than midstream. This contrasts with the results of Liu (2019), who finds that optimal subsidies should necessarily be higher, the more upstream the sector. The reason is that, unlike in Liu's work, we solve for first-best subsidy policy: when the government can only set subsidies in one sector, the size of the subsidy would be higher, the more upstream the sector, because as we have seen above, the gap between the social optimal and market allocation of labor is highest in the upstream sector.

B.2 Equilibrium of Isomorphic Economy with Internal Scale Economies

In this Appendix, we outline the equilibrium conditions corresponding to the two-country model featuring internal scale economies, product differentiation and monopolistic competition outlined in Section 4.2. We will then work with these equations in Appendix to demonstrate the isomorphism with the model with external economies of scale developed in the main text.

B.2.1 Environment

Consider a world economy with two countries (Home and Foreign), indexed by i or j (and sometimes by H and F), each populated by L_i consumers/workers. The representative consumer in each country i values the consumption of differentiated varieties of manufacturing goods according to the utility function

$$U_i = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \quad i \in \{H,F\}, \tag{B.5}$$

where M_j^d is the endogenous measure of firms in country j, $q_{ji}^d(\omega)$ is the quantity consumed of variety ω from country j in country i, and σ is the elasticity of substitution across varieties. Individuals supply one unit of labor inelastically, with L_i denoting the total labor force. There are no other factors of production, so labor should be interpreted as representing "equipped" labor.

Labor is used for the production of intermediate inputs (the upstream sector) and (possibly) in producing final goods (the downstream sector). More specifically, we represent technologies in the upstream and downstream sectors with

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H,F\}, \tag{B.6}$$

and

$$f_i^d + x_i^d(\omega) = A_i^d (\ell_i^d(\omega))^{\alpha} Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H,F\}. \tag{B.7}$$

respectively. In these expressions, f_i^s denotes the fixed output requirements for entry in sector $s \in \{D, U\}$ in country i, $x_i^s(\omega)$ is the output produced for sale by variety ω in sector $s \in \{D, U\}$ in country i, A_i^s is a sector- and country-specific technology parameter, and $Q_i^u(\omega)$ is a composite of all intermediate goods available in country i, which is in turn given by

$$Q_i^u(\omega) = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right) \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H,F\}, \tag{B.8}$$

where $q_{ji}^u(\varpi)$ is the quantity consumed of input variety ϖ from country j in country i. In words, the upstream sector uses only labor, and technology features increasing returns to scale due to the presence of a fixed overhead cost. The downstream sector combines labor with a continuum of intermediate inputs (domestic and foreign), and technology again exhibits increasing returns stemming from a fixed overhead cost. Notice that $\theta > 1$ governs the degree of substitutability across inputs, while $\alpha \in [0,1]$ corresponds to the downstream labor (or value-added) intensity in production.[5]

[5]Note that we specify the fixed costs of production in terms of output rather than labor. This assumption is immaterial for our main results, and it avoids introducing additional sources of inefficiency into our framework (see also Costinot and Rodríguez-Clare, 2014, footnote 20).

There is an endogenous measure, M_i^d, of manufacturing firms in the downstream sector in country i, each producing a single final-good variety. Analogously, there is an endogenous measure, M_i^u, of manufacturing firms in the upstream sector in country i, each producing a single intermediate-input variety. All entrants have access to the same technologies in (B.6), (B.7) and (B.8). Market structure in both sectors is characterized by monopolistic competition and free entry.

Trade is costly due to the presence of both iceberg trade costs and import tariffs. We denote the symmetric iceberg trade costs that apply to final goods and inputs by $\tau^d > 1$ and $\tau^u > 1$, respectively, and we denote the tariffs set by country i on imports of final goods and intermediate inputs by t_i^d and t_i^u, respectively. We also consider additional instruments, namely domestic production subsidies (s_i^d and s_i^u) and export taxes (v_i^d and v_i^u) in both sectors.[6]

Given symmetry across firms, the tariff revenue collected by the government is rebated to households via lump-sum transfers in an amount

$$R_i = \frac{t_i^d}{1+t_i^d} M_j^d p_{ji}^d q_{ji}^d + \frac{t_i^u}{1+t_i^u} M_i^d M_j^u p_{ji}^u q_{ji}^u + \frac{v_i^d}{1-v_i^d} M_i^d \tilde{p}_{ij}^d q_{ij}^d + \frac{v_i^u}{1-v_i^u} M_j^d M_i^u \tilde{p}_{ij}^u q_{ij}^u, \tag{B.9}$$

where p_{ji}^d and p_{ji}^u are the prices paid by consumers in i for final goods and by firms in i for inputs, and where $\tilde{p}_{ij}^d$ and $\tilde{p}_{ij}^u$ are the prices collected by producers in i when selling final goods and inputs in country j. When the government also levies production subsidies, this government balance condition needs to be modified in a straightforward manner.

B.2.2 Firm Behavior

Import tariffs on the downstream sector create a wedge between consumer prices in country i and producer prices in country j. More specifically, given CES preferences, consumer prices in i for goods originating in j are given by:

$$p_{ji}^d = \left(1+t_i^d\right) \frac{\sigma}{\sigma-1} \tau^d \frac{1}{A_j^d} \left(\frac{w_j}{\alpha}\right)^{\alpha} \left(\frac{P_j^u}{1-\alpha}\right)^{1-\alpha} = \frac{1+t_i^d}{1-v_j^d} \tilde{p}_{ji}^d. \tag{B.10}$$

Similarly, import tariffs on the upstream sector create a wedge between the price paid by final-good producers in i for inputs from j, and the producer price for those inputs obtained by suppliers in country j. In particular, we have

$$p_{ji}^u = (1+t_i^u) \frac{\theta}{\theta-1} \tau^u \frac{w_j}{A_j^u} = \frac{1+t_i^u}{1-v_j^u} \tilde{p}_{ji}^u. \tag{B.11}$$

In equation (B.10), the price index P_i^u is given by

$$P_i^u = \left[\sum_{j \in \{H,F\}} \left(P_{ji}^u\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}, \tag{B.12}$$

with

$$P_{ji}^u = \left[\int_0^{M_j^u} \left(p_{ji}^u(\varpi)\right)^{1-\theta} d\varpi \right]^{\frac{1}{1-\theta}}. \tag{B.13}$$

When setting $j = i$, the above pricing equations also characterize domestic prices in country j after setting $t_i^d = t_i^u = v_i^d = v_i^u = 0$ and $\tau^d = \tau^u = 1$. Note that $p_{ii}^d = \tilde{p}_{ii}^d$ and $p_{ii}^u = \tilde{p}_{ii}^u$.

[6] Implicit in the equations derived above is the fact that because trade costs are all ad-valorem and preferences are CES, all firms in all sectors find it profitable to sell in both markets.

Next, utility maximization implies that when consuming country j varieties, consumers in i allocate to each variety ω a share of spending equal to

$$\frac{p_{ji}^{d}(\omega)\, q_{ji}^{d}(\omega)}{P_{ji}^{d} Q_{ji}^{d}} = \left(\frac{p_{ji}^{d}(\omega)}{P_{ji}^{d}}\right)^{1-\sigma}, \tag{B.14}$$

of their total spending on country j varieties, where

$$P_{ji}^{d} = \left[\int_{0}^{M_{j}^{d}} \left(p_{ji}^{d}(\omega)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}. \tag{B.15}$$

Consumers' (aggregate) spending on Home and Foreign varieties is in turn determined by

$$P_{ji}^{d} Q_{ji}^{d} = \left(\frac{P_{ji}^{d}}{P_{i}^{d}}\right)^{1-\sigma} (w_i L_i + R_i), \tag{B.16}$$

where P_i^d is the aggregate consumer price index in i

$$P_{i}^{d} = \left[\sum_{j\in\{H,F\}} \left(P_{ji}^{d}\right)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}, \tag{B.17}$$

and R_i is tariff revenue, which we have defined in equation (B.9).

We now turn to profit maximization by downstream producers in country i. First note that free entry implies that firm revenue (net of tariffs) will equal total costs, and that a share of those costs will go to pay labor. As a result, labor compensation by each final-good producer in i is given by

$$w_i \ell_i^d = \alpha \left(\tilde{p}_{ii}^{d} q_{ii}^{d} + \frac{1-v_i^d}{1+t_j^d} \tilde{p}_{ij}^{d} q_{ij}^{d}\right). \tag{B.18}$$

Next, when purchasing inputs from upstream producers in country j, final-good producers in country i, will demand an amount of each variety ϖ from country j equal to

$$q_{ji}^{u}(\varpi) = Q_{ji}^{u}(\omega) \left(\frac{p_{ji}^{u}}{P_{ji}^{u}}\right)^{-\theta},$$

while aggregate spending on all country j's input varieties is given by

$$P_{ji}^{u} Q_{ji}^{u} = (1-\alpha) \left(\tilde{p}_{ii}^{d} q_{ii}^{d} + \frac{1-v_i^d}{1+t_j^d} \tilde{p}_{ij}^{d} q_{ij}^{d}\right) \left(\frac{P_{ji}^{u}}{P_{i}^{u}}\right)^{1-\theta} M_i^d. \tag{B.19}$$

Aggregate spending on Home and Foreign intermediate inputs in country i is then given by

$$P_{i}^{u} Q_{i}^{u} = (1-\alpha) \left(\tilde{p}_{ii}^{d} q_{ii}^{d} + \frac{1-v_i^d}{1+t_j^d} \tilde{p}_{ij}^{d} q_{ij}^{d}\right) M_i^d. \tag{B.20}$$

Our final set of equilibrium conditions impose market clearing. First, labor-market clearing in both countries implies that

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u, \tag{B.21}$$

where ℓ_i^d is given in (B.18), and $\ell_i^u = (f_i^u + x_i^u)/A_i^u$.[7] Second, goods-market clearing imposes

$$q_{ii}^d + \tau^d q_{ij}^d = x_i^d \tag{B.22}$$

and

$$M_i^d q_{ii}^u + M_j^d \tau^u q_{ij}^u = x_i^u. \tag{B.23}$$

Note that free entry upstream and downstream implies that firm revenue is equal to total costs, which delivers

$$x_i^d = (\sigma - 1) f_i^d; \qquad x_i^u = (\theta - 1) f_i^u \tag{B.24}$$

for $i = \{H, F\}$. Firm-level production levels are thus independent of tariff choices, and the only way in which tariffs can affect the allocation of labor across sectors is by changing the measure of firms in each of the two sectors. As a result, optimal trade policies will seek to achieve a social-welfare maximizing allocation of labor across sectors, with no concern for the allocation of labor within sectors (across fixed costs of entry versus marginal costs of production).

B.2.3 Isomorphism of Preferences, Technology and Resource Constraints

We next show that equilibrium conditions of the decentralized equilibrium of this two-country model featuring internal scale economies, product differentiation and monopolistic competition can be reduced to a set of equations identical to equations (5) through (16) applying to the competitive model with external economies of scale developed in the main text.

Preferences We begin by noting that given symmetry in final-good production, we can express preferences as

$$\begin{aligned} U_i &= \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[M_i^d (q_{ii}^d)^{\frac{\sigma-1}{\sigma}} + M_j^d (q_{ji}^d)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left((Q_{ii}^d)^{\frac{\sigma-1}{\sigma}} + (Q_{ji}^d)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where

$$Q_{ii}^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} q_{ii}^d; \quad Q_{ji}^d \equiv \left(M_j^d\right)^{\frac{\sigma}{\sigma-1}} q_{ji}^d. \tag{B.25}$$

Starting from (B.5), we have thus derived (1), which are preferences in the isomorphic economy with two final goods (a Home one and a Foreign one) and external economies of scale.

[7]Naturally, equilibrium also requires trade balance, but this is ensured by the other equilibrium conditions outlined in this section.

Labor-Market Clearing Next, remember that ℓ_i^d and ℓ_i^u are the firm-level amounts of labor used downstream and upstream to cover fixed and variable costs. Hence, defining

$$L_i^d \equiv M_i^d \ell_i^d; \quad L_i^u \equiv M_i^u \ell_i^u, \tag{B.26}$$

we have that equation (B.21) in the monopolistic competition model implies equation (5) in the external economies model, or

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u = L_i^u + L_i^d.$$

Upstream Market Clearing and Upstream Endogenous Productivity Next let us define

$$Q_{ii}^u \equiv M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u; \quad Q_{ij}^u \equiv M_j^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ij}^u. \tag{B.27}$$

Given these definitions in (B.27), and given the definition of the input aggregate $Q_i^u(\omega)$ in the monopolistic competition model, that is

$$Q_i^u(\omega) = \left[\sum_{j\in\{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi\right)\right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H,F\},$$

we have that the total usage of inputs by firms in country i is given by

$$\begin{aligned} Q_i^u &= M_i^d Q_i^u(\omega) = \left[M_i^u \left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + M_j^u \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(M_i^d \left(M_j^u\right)^{\frac{\theta}{\theta-1}} \left(q_{ji}^u\right)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \end{aligned} \tag{B.28}$$

and thus is analogous to a CES aggregator of only two inputs: a Home and a Foreign one, as defined in equation (B.27). These inputs are either produced domestically or are imported.

Now consider the domestic production of those inputs. Let us start from the definition of upstream technology in the monopolistic competition model, that is

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H,F\}.$$

Imposing symmetry and firm-level output in equation (B.24) – i.e., $x_i^u = (\theta-1) f_i^u$ –, and invoking the definition of L_i^u in (B.26), we have

$$X_i^u \equiv \left(M_i^u\right)^{\frac{\theta}{\theta-1}} x_i^u = \left(\frac{A_i^u}{\theta f_i^u}\right)^{\frac{\theta}{\theta-1}} (\theta-1) f_i^u \left(L_i^u\right)^{\frac{\theta}{\theta-1}} \tag{B.29}$$

or

$$X_i^u = \hat{A}_i^u F_i^u\left(\ell_i^u\right) = \bar{A}_i^u \left(L_i^u\right)^{1+\gamma^u},$$

where

$$\bar{A}_i^u \equiv \left(\frac{A_i^u}{\theta f_i^u}\right)^{\frac{\theta}{\theta-1}} (\theta-1) f_i^u,$$

and

$$\gamma^u \equiv 1/(\theta - 1).$$

Because this domestic production X_i^u is sold domestically or exported, we have

$$\bar{A}_i^u (L_i^u)^{1+\gamma^u} = Q_{ii}^u + \tau^u Q_{ij}^u,$$

which corresponds exactly to equation (6) in the external economies model.

Downstream Market Clearing and Downstream Endogenous Productivity We can proceed analogously for final-good production. We begin with the definition of technology in the downstream sector in the monopolistic competition model:

$$f_i^d + x_i^d(\omega) = A_i^d(\ell_i^d(\omega))^\alpha Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H, F\}.$$

Imposing symmetry and (B.24), we obtain

$$M_i^d = \frac{A_i^d}{\sigma f_i^d}(M_i^d \ell_i^d(\omega))^\alpha \left(M_i^d Q_i^u\right)^{1-\alpha}$$

or

$$X_i^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^\alpha \left(\left(L_i^d\right)^\alpha \left((Q_{ii}^u)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}}. \tag{B.30}$$

where

$$\bar{A}_i^d \equiv \left(\frac{A_i^d}{\sigma f_i^d}\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f_i^d.$$

This aggregate output X_i^d is sold domestically or exported, and thus

$$\bar{A}_i^d \left(\left(L_i^d\right)^\alpha \left((Q_{ii}^u)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d} = Q_{ii}^d + \tau^d Q_{ij}^d,$$

where

$$\gamma^d \equiv 1/(\sigma - 1).$$

In sum, starting from the monopolistic competition model, we have derived equation (6) in the external economies model.

Trade Balance Consider next the trade balance condition. Starting from the monopolistic competition economy, we have

$$\frac{p_{ji}^d}{1+t_i^d} M_j^d q_{ji}^d + \frac{p_{ji}^u}{1+t_i^u} M_i^d M_j^u q_{ji}^u = \frac{\tilde{p}_{ij}^d}{1-v_i^d} M_i^d q_{ij}^d + \frac{\tilde{p}_{ij}^u}{1-v_i^u} M_j^d M_i^u q_{ij}^u, \tag{B.31}$$

which equates the import revenue in i paid to exporters in j with export revenue collected from j by producers in i.

Now from equations (B.14) and (B.15), notice that we have

$$\frac{p_{ji}^{d}(\omega)\, q_{ji}^{d}(\omega)}{P_{ji}^{d} Q_{ji}^{d}} = \left(\frac{p_{ji}^{d}(\omega)}{P_{ji}^{d}}\right)^{1-\sigma},$$

and

$$P_{ji}^{d} = \left[\int_{0}^{M_j^d} \left(p_{ji}^{d}(\omega)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

so given symmetry, we have

$$P_{ji}^{d} = \left(M_j^d\right)^{\frac{1}{1-\sigma}} p_{ji}^{d} \tag{B.32}$$

and

$$P_{ji}^{d} Q_{ji}^{d} = \left(M_j^d\right)^{\frac{-1}{\sigma-1}} p_{ji}^{d} \times \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} q_{ji}^{d} = M_j^d p_{ji}^{d} q_{ji}^{d}.$$

Similarly, for inputs

$$P_{ji}^{u} Q_{ji}^{u} = \left(M_j^u\right)^{\frac{1}{1-\theta}} p_{ji}^{u} \times M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ji}^{u} = p_{ji}^{u} M_i^d M_j^u q_{ji}^{u}.$$

This implies that we can write total imports in the trade balance condition (B.31) as

$$\frac{P_{ji}^{d}}{1+t_i^d} Q_{ji}^{d} + \frac{P_{ji}^{u}}{1+t_i^u} Q_{ji}^{u} = \bar{P}_{ji}^{d} Q_{ji}^{d} + \bar{P}_{ji}^{u} Q_{ji}^{u},$$

which corresponds to the left-hand-side of the trade balance condition (8) for the economy with external economies of scale after noting that $\bar{P}_{ji}^{d}$ and $\bar{P}_{ji}^{u}$ are the prices collected by country j (or Foreign) exporters (not those paid by domestic or country i consumers).

Now consider revenue from exporting final goods. Notice that, regardless of whether the Foreign government imposes import tariffs or not, we have that export revenue is

$$\frac{\tilde{p}_{ij}^{d}}{1-v_i^d} M_i^d q_{ij}^{d} + \frac{\tilde{p}_{ij}^{u}}{1-v_i^u} M_j^d M_i^u q_{ij}^{u}$$

Prices paid by country j are $\tilde{p}_{ij}^{d} / \left(1-v_i^d\right)$ and $\tilde{p}_{ij}^{u} / \left(1-v_i^u\right)$, so following analogous steps, the right-hand-side of (8) becomes

$$\frac{\tilde{P}_{ij}^{d}}{1-v_i^d} Q_{ij}^{d} + \frac{\tilde{P}_{ij}^{u}}{1-v_i^u} Q_{ij}^{u} = \bar{P}_{ij}^{d} Q_{ij}^{d} + \bar{P}_{ij}^{u} Q_{ij}^{u},$$

where $\bar{P}_{ij}^{d}$ and $\bar{P}_{ij}^{u}$ are the prices paid by country j (or Foreign) importers (not those collected by country i exporters).

Note: In the main text, we denote $\bar{P}_{ji}^{d}$, $\bar{P}_{ji}^{u}$, $\bar{P}_{ij}^{d}$, and $\bar{P}_{ij}^{u}$ as simply P_{ji}^{d}, P_{ji}^{u}, P_{ij}^{d}, and P_{ij}^{u}. We do so not to clutter the notation, but these are distinct from the price indices applying to the monopolistic competition model, which are always built based on prices paid by consumers, regardless of their country.

B.2.4 Isomorphism of Social Planner Problem

We have shown that we can reduce the planner problem of the Krugman model with internal economies of scale to

$$\max \quad U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned} s.t. \quad & L^u_H + L^d_H = L_H \\ & \hat{A}^u_H\left(L^u_H\right) L^u_H = Q^u_{HH} + \tau^u Q^u_{HF} \\ & \hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right) F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + \tau^d Q^d_{HF} \\ & P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}, \end{aligned}$$

where

$$\begin{aligned} Q^d_{ii} &\equiv \left(M^d_i\right)^{\frac{\sigma}{\sigma-1}} q^d_{ii}; \quad Q^d_{ji} \equiv \left(M^d_j\right)^{\frac{\sigma}{\sigma-1}} q^d_{ji}; \\ L^d_i &\equiv M^d_i \ell^d_i; \quad L^u_i \equiv M^u_i \ell^u_i; \\ Q^u_{ii} &\equiv M^d_i \left(M^u_i\right)^{\frac{\theta}{\theta-1}} q^u_{ii}; \quad Q^u_{ij} \equiv M^d_j \left(M^u_i\right)^{\frac{\theta}{\theta-1}} q^u_{ij}; \\ \hat{A}^u_H\left(L^u_H\right) &= \left(\frac{A^u_i}{\theta f^u_i}\right)^{\frac{\theta}{\theta-1}} (\theta-1) f^u_i \left(L^u_H\right)^{\frac{1}{\theta-1}}; \\ \hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right) &= \left(\frac{A^d_i}{\sigma f^d_i}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1) f^d_i \left[\left(L^d_i\right)^{\alpha}\left(\left(L^d_i\right)^{\alpha}\left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{1}{\sigma-1}}. \end{aligned}$$

It is then clear that this planner problem is identical to the one developed in Section 3.1 of the main text, so it naturally leads to the same exact optimality conditions (9)–(12) There are only two subtle aspects of the isomorphism. First, the productivity terms $\bar{A}^d_i$ and $\bar{A}^u_i$ in equations (3) and (4) are a function of the primitive parameters of the 'Krugman' model (including final-good and input elasticities of substitution and the levels of fixed costs). Second, the scale elasticities γ^d and γ^u are no longer free parameters and are in fact given by $\gamma^d = 1/(\sigma-1)$ and $\gamma^u = 1/(\theta-1)$, respectively.

B.2.5 Isomorphism of Decentralized Market Equilibrium

We have shown above that the four 'resource' constraints (5) through (8) of our baseline external economies economy can be derived from an isomorphic model with monopolistic competition and internal economies of scale. We next turn to an analogous derivation for the optimality conditions (13) through (16).

Optimality in Final-Good Consumption Let us begin with the first one, equating the marginal rate of substitution in final-good consumption to relative prices. Given equation (B.16) in the model with monopolistic competition, we have

$$\frac{Q^d_{ii}}{Q^d_{ji}} = \left(\frac{P^d_{ii}}{P^d_{ji}}\right)^{-\sigma},$$

where Q^d_{ji}, Q^d_{ji}, P^d_{ji} and P^d_{ji} are defined in (B.27) and (B.32). Thus, we have

$$\left(\frac{Q^d_{ii}}{Q^d_{ji}}\right)^{-\frac{1}{\sigma}} = \frac{P^d_{ii}}{P^d_{ji}} = \frac{\left(1-v^d_i\right)\bar{P}^d_{ij}}{\left(1+t^d_i\right)\bar{P}^d_{ji}},$$

where $P^d_{ii} = \left(1-v^d_i\right)\bar{P}^d_{ij}$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}^d_{ij}$ is the price paid by consumers in j for final goods from j). We have thus derived an equation that corresponds to (13) in the external economies of scale model.

Optimality in Input Consumption We next equate the marginal rate of substitution in input consumption to relative prices. In particular, from equation (B.19) in the model with monopolistic competition, we have

$$\frac{Q^u_{ii}}{Q^u_{ji}} = \left(\frac{P^u_{ii}}{P^u_{ji}}\right)^{-\theta},$$

where Q^u_{ji}, Q^u_{ji}, P^u_{ji} and P^u_{ji} are defined in (B.27) and (B.32). Thus, we have

$$\left(\frac{Q^u_{ii}}{Q^u_{ji}}\right)^{-\frac{1}{\theta}} = \frac{P^u_{ii}}{P^u_{ji}} = \frac{\left(1-v^u_i\right)\bar{P}^u_{ij}}{\left(1+t^u_i\right)\bar{P}^u_{ji}},$$

where $P^u_{ii} = \left(1-v^d_i\right)\bar{P}^u_{ij}$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}^u_{ij}$ is the price paid by consumers in j for final goods from j). We have thus an equation that corresponds to (14) in the external economies of scale model.

Optimal Domestic Input Allocation We next move to the third optimality condition (15), which equates the benefits of exporting domestic intermediate inputs with the benefits of using those additional domestic inputs to produce an additional amount of the final good that is in turn exported.

We begin with equation (B.20), and note that aggregate input use in country i in the monopolistic competition model is given by

$$P^u_i Q^u_i = (1-\alpha)\left(\tilde{p}^d_{ii}q^d_{ii} + \frac{1-v^d_i}{1+t^d_j}\tilde{p}^d_{ij}q^d_{ij}\right)M^d_i. \tag{B.33}$$

To reiterate this, note from (B.11) that $\frac{1-v^d_i}{1+t^d_j}\tilde{p}^d_{ij} = \tau^d p^d_{ii}$, and plugging in equation (B.22), we obtain

$$P^u_i Q^u_i = (1-\alpha)\, p^d_{ii}\left(q^d_{ii} + \tau^d q^d_{ij}\right)M^d_i = (1-\alpha)\, p^d_{ii} x^d_i M^d_i. \tag{B.34}$$

Next, invoke equation (B.19) applied to $P^u_{ii}Q^u_{ii}$ to obtain (after plugging in (B.33)):

$$P^u_i Q^u_i = P^u_{ii}Q^u_{ii}\left(\frac{P^u_{ii}}{P^u_i}\right)^{\theta-1}. \tag{B.35}$$

Combining (B.34) and (B.35), we obtain:

$$(1-\alpha)\, p^d_{ii} x^d_i M^d_i = P^u_{ii}Q^u_{ii}\left(\frac{P^u_{ii}}{P^u_i}\right)^{\theta-1},$$

which we decompose as

$$(1-\alpha)\times\left(M_i^d\right)^{\frac{-1}{\sigma-1}}p_{ii}^d\times\left(M_i^d\right)^{\frac{\sigma}{\sigma-1}}x_i^d=P_{ii}^uQ_{ii}^u\left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1},\tag{B.36}$$

Now remember from equation (B.30) derived above that

$$\left(M_i^d\right)^{\frac{\sigma}{\sigma-1}}x_i^d=\bar{A}_i^d\left[\left(L_i^d\right)^{\alpha}\left(\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}},$$

and also from (B.32) that $\left(M_i^d\right)^{\frac{-1}{\sigma-1}}p_{ii}^d=P_{ii}^d$, so we can write (B.36) as

$$P_{ii}^d\left(1-\alpha\right)\hat{A}_i^d\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{1}{Q_{ii}^u}=P_{ii}^u\left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}.$$

Now invoke (B.19)

$$\frac{Q_{ii}^u}{Q_i^u}=\left(\frac{P_{ii}^u}{P_i^u}\right)^{-\theta},$$

to obtain

$$P_{ii}^d\left(1-\alpha\right)\hat{A}_i^d\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{1}{Q_{ii}^u}=P_{ii}^u\left(\frac{Q_{ii}^u}{Q_i^u}\right)^{-\frac{\theta-1}{\theta}},$$

which given the definition of Q_i^u in (B.10) delivers

$$P_{ii}^d\left(1-\alpha\right)\hat{A}_i^d\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{1}{Q_{ii}^u}\frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}}=P_{ii}^u.$$

The final step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^d=\left(1-v_i^d\right)\bar{P}_{ij}^d$ and $P_{ii}^u=\left(1-v_i^u\right)\bar{P}_{ij}^u$, where remember that $\bar{P}_{ij}^d$ and $\bar{P}_{ij}^u$ are the prices paid by country j residents. In sum, we have derived

$$\left(1-\alpha\right)\hat{A}_i^d\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{1}{Q_{ii}^u}\frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}}=\frac{\left(1-v_i^u\right)}{\left(1-v_i^d\right)}\frac{\bar{P}_{ij}^u}{\bar{P}_{ij}^d},$$

which corresponds to equation (15) in the external economies of scale model

Optimal Labor Market Allocation We finally tackle the fourth optimality condition, associated with the optimal allocation of labor across sectors. We begin with the firm-level monopolistic competition model, equating the wage paid in both sectors. Because of free entry, total revenue upstream must equal total wage payments, while in the downstream sector, wage payments are a share α of total revenue, as indicated in equation (B.18), or

$$\frac{\alpha\left(\tilde{p}_{ii}^dq_{ii}^d+\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^dq_{ij}^d\right)}{\ell_i^d}=\frac{\tilde{p}_{ii}^uM_i^dq_{ii}^u+\frac{1-v_i^u}{1+t_j^u}\tilde{p}_{ij}^uM_j^dq_{ij}^u}{\ell_i^u}.$$

Now noting that from (B.11), we have $\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d = \tau^d p_{ii}^d$ (and analogously $\frac{1-v_i^u}{1+t_j^u}\tilde{p}_{ij}^u = \tau^u p_{ii}^u$), and using equations (B.22) and (B.23), we have

$$\frac{\alpha p_{ii}^d x_i^d}{\ell_i^d} = \frac{p_{ii}^u x_i^u}{\ell_i^u}. \tag{B.37}$$

Next, invoke the price index definitions – see equation (B.32) – as well as the definitions $L_i^d = M_i^d \ell_i^d$ and $L_i^u = M_i^u \ell_i^u$, to write (B.37) as

$$\alpha P_{ii}^d \frac{x_i^d \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}}}{L_i^d} = P_{ii}^u \frac{\left(M_i^u\right)^{\frac{\theta}{\theta-1}} x_i^u}{L_i^u}.$$

Next, plugging (B.29) and (B.30), delivers

$$\frac{\alpha P_{ii}^d}{L_i^d} \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha} = P_{ii}^u \hat{A}_i^u,$$

where remember that $\hat{A}_i^d$ and $\hat{A}_i^u$ are defined in equations (3) and (4) in the main text.

The next step is to note, as we did above, that indifference between selling domestically and exporting delivers $P_{ii}^d = \left(1 - v_i^d\right)\bar{P}_{ij}^d$ and $P_{ii}^u = \left(1 - v_i^u\right)\bar{P}_{ij}^u$, where remember that $\bar{P}_{ij}^d$ and $\bar{P}_{ij}^u$ are the prices paid by country j residents, so we have

$$\frac{\alpha}{L_i^d} \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha} = \hat{A}_i^u \frac{\left(1 - v_i^u\right)}{\left(1 - v_i^d\right)} \frac{\bar{P}_{ij}^u}{\bar{P}_{ij}^d}.$$

The final step is to plug optimality condition (B.2.5) and cancel terms to obtain

$$\frac{\alpha}{L_i^d} = (1-\alpha)\hat{A}_i^u \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}},$$

which corresponds to the last optimality condition (16) in the model with external economies of scale.

This completes the proof of the isomorphism claimed in Section 4.2 of the main text

B.3 General Functional Forms

As demonstrated in the derivations in the above Appendix A.1, we have made no use of the properties of the functions $U\left(Q_{HH}^d, Q_{FH}^d\right)$ and $F^d\left(Q_{HH}^u, Q_{FH}^u\right)$. In particular, we could assume that

$$U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma_H-1}{\sigma_H}} + \left(Q_{FH}^d\right)^{\frac{\sigma_H-1}{\sigma_H}}\right)^{\frac{\sigma_H}{\sigma_H-1}}$$

and that

$$F^d\left(Q_{HH}^u, Q_{FH}^u\right) = \left(\left(Q_{HH}^u\right)^{\frac{\theta_H-1}{\theta_H}} + \left(Q_{FH}^u\right)^{\frac{\theta_H-1}{\theta_H}}\right)^{\frac{\theta_H}{\theta_H-1}},$$

with potentially $\sigma_H \neq \sigma$ and $\theta_H \neq \theta$. It is clear from the derivations in Section B.3 that the first-best trade policies will continue to be shaped by σ and θ, and not by σ_H or θ_H.

The only significant difference in this case is that if we want to invoke our isomorphism to claim that these policies also implement the first-best in the Krugman vertical economy with internal economies of scale, then we necessarily need to impose $\gamma^d = 1/\left(\sigma_H - 1\right)$, and thus the level of the tariff escalation is closely related to the degree of differentiation in the Home final-good sector. This is not particularly surprising, since love-for-variety effects will be more powerful, the lower the degree of substitutability across final goods.

C Quantitative Analysis

C.1 Numerical Simulations of Second-Best Tariff Escalation

In this Appendix, we describe how we solve numerically for second-best trade policies in the small open economy (SOE), and report the results of solving this problem for various parameter values. We perform an extensive grid search over various values of the key parameters: (i) the two scale elasticities (γ_d and γ_u); (ii) the two demand elasticities (σ and θ); and (iii) the downstream labor share (α). Table C.1 provides the list of values we consider for each parameter.

Table C.1: List of Parameters in Grid Search Exercise

Parameter	List of values
Downstream Scale Elasticity, γ^d	$\{0.04, 0.09, 0.13, 0.17, 0.21, 0.26\}$
Upstream Scale Elasticity, γ^u	$\{0.04, 0.09, 0.13, 0.17, 0.21, 0.26\}$
Downstream Demand Elasticity, σ	$\{2.5, 3.5, 4.5, 5, 5.5, 6.5, 7.5\}$
Upstream Demand Elasticity, θ	$\{2.5, 3.5, 4.5, 5, 5.5, 6.5, 7.5\}$
Downstream Labor share, α	$\{0, 0.1, 0.2, \ldots, 0.9\}$

Notes: Each row presents the list of values considered for each parameter in the grid search exercise. Iceberg costs and the two productivities Iceberg costs, and the two productivity levels are fixed at their calibrated values. Table 1 in Section 6.1.

C.1.1 Solving for optimal import tariff

Solving for optimal tariffs in this second-best setting requires providing values for import prices and export demand shifters, both of which are exogenously given in the SOE, and for productivities in both sectors. To recover the value of these parameters, we use that the large open economy (LOE) approximates the SOE when Home's population is low relative to the rest of the world. In this case, import prices and export demand shifters in the SOE can be constructed from the equilibrium values of the LOE as follows,

$$\begin{aligned}
P^d_{soe,FH} &= \left(M^d_F\right)^{\frac{1}{1-\sigma}} p^d_{FH} \\
P^u_{soe,FH} &= \left(M^u_F\right)^{\frac{1}{1-\theta}} p^u_{FH} \\
P^d_{soe,HF} &= P^d_{HF} C^{\frac{1}{\sigma}}_{HF} \\
P^u_{soe,HF} &= P^u_{HF} Q^{\frac{1}{\theta}}_{HF}.
\end{aligned}$$

Similarly, productivity levels in the two sectors can be constructed using:

$$A^u_{soe} = \left(\frac{A^u_H}{f^\theta_H}\right)^{\frac{\theta}{\theta-1}} (\theta-1)\, f^u_H, \qquad A^d_{soe} = \left(\frac{A^d_H}{f^d_H \sigma}\right)^{\frac{\sigma}{\sigma-1}} (\sigma-1)\, f^d_H.$$

Given these values, it is straightforward to compute numerically the second-best import tariff. In practice, however, we solve for the optimal import tariff in the LOE with $L_H/L_F = 0.01$ for each combination of the parameters in Table C.1, keeping iceberg costs and productivity levels at those estimated in Section 6.1. We find this method to be more numerically stable than the corresponding one in the SOE. Table C.2 shows that using this method to compute the optimal taxes in SOE works well, at least for the calibrated values of

Section 6.1.

Table C.2: Optimal taxes in the large and small open-economy

	$1+t^d$	$1+t^u$	$1-v^u$	$1-s^u$
LOE - First-Best	1.25	1.07	0.86	0.85
SOE - First-Best	1.25	1.07	0.86	0.85
LOE - Second-Best	1.30	1.20		
SOE - Second-Best	1.31	1.20		

Notes: Table compares the first- and second-best policy in the LOE and SOE. We compute the optimal tariffs using the estimated parameters in section 6.1, but we imposed $L_H/L_F = 0.01$.

C.1.2 Grid over γ_d, γ_u and α

We solve for optimal trade policy for values of γ_d and γ_u ranging from 0.04 to 0.26, and for values of α ranging from 0 to 0.9, as described in Table C.1. We fix the values A_d, A_u, τ_d and τ_u to the values we estimate in Section 6.1. Overall, we explore $6 \times 6 \times 10 = 360$ configurations of parameters. We can successfully solve the model for all of them.

Statistics tariff ratio wedge

The next three tables report statistics related to the tariff escalation wedge, the mean, the median, the standard deviation, the minimum and the maximum. On average, downstream tariffs are 9 percent higher than upstream tariffs, and the medians show a similar divergence. Some cases feature tariff escalation levels as high as 1.29, while tariff de-escalation remains modest even in the most extreme cases (minimum of 0.82). The third and fourth column of Table C.6 recalculates the statistic but looking at the cases for which the tariff ratio is above and below 1, respectively. It is interesting to note that when our model predicts tariff de-escalation, it does so with fairly moderate levels (median of 0.96).

Table C.3: Statistics of the tariff escalation wedge: all cases

	All cases	With escalation	With de-escalation
Mean	1.09	1.12	0.95
Median	1.09	1.11	0.96
Standard Deviation	0.09	0.08	0.04
Minimum value	0.82	1.00	0.82
Maximum value	1.29	1.29	1.00
N	360	302	48

Notes: Table reports statistics for the tariff escalation wedge for optimal import tariffs computed in the grid search exercise.

C.1.3 Tariff escalation and parameter space

We now present results about the parameter combinations that generate de-escalated tariffs. Table C.4 reports the fraction of cases for which tariffs are de-escalated for a given parameter, for each possible value that this parameter can take. For example, we have 60 combinations with $\gamma^d = 0.21$, with 2 percent of these cases featuring de-escalated tariffs.

Table C.4: Tariff De-escalation across the parameter space, I

	Values Downstream Scale Elasticity (γ^d)					
	0.04	0.09	0.13	0.17	0.21	0.26
Share of cases w. De-escalation:	0.48	0.28	0.13	0.05	0.02	0.00

	Values Upstream Scale Elasticity (γ^u)					
	0.04	0.09	0.13	0.17	0.21	0.26
Share of cases w. De-escalation:	0.00	0.07	0.12	0.18	0.27	0.33

	Values Downstream Labor Share (α)									
	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Share of cases w. De-escalation:	0.00	0.00	0.00	0.06	0.11	0.17	0.25	0.28	0.33	0.42

Notes: Table reports the share of cases in each cell for which optimal tariffs are de-escalated, i.e $\left(1+t_H^d\right)/\left(1+t_H^u\right) < 1$.

Table C.5 presents the fraction of cases with de-escalated tariffs for the ten values of the downstream labor share (α) split into cases with $\gamma^d > \gamma^u$, $\gamma^d < \gamma^u$, and $\gamma^d = \gamma^u$. In line with the intuition in the draft, optimal tariffs are more likely to be de-escalated when the scale elasticity upstream is larger than the scale elasticity downstream, and when the downstream labor share is high.

Table C.5: Tariff De-escalation across the parameter space, II

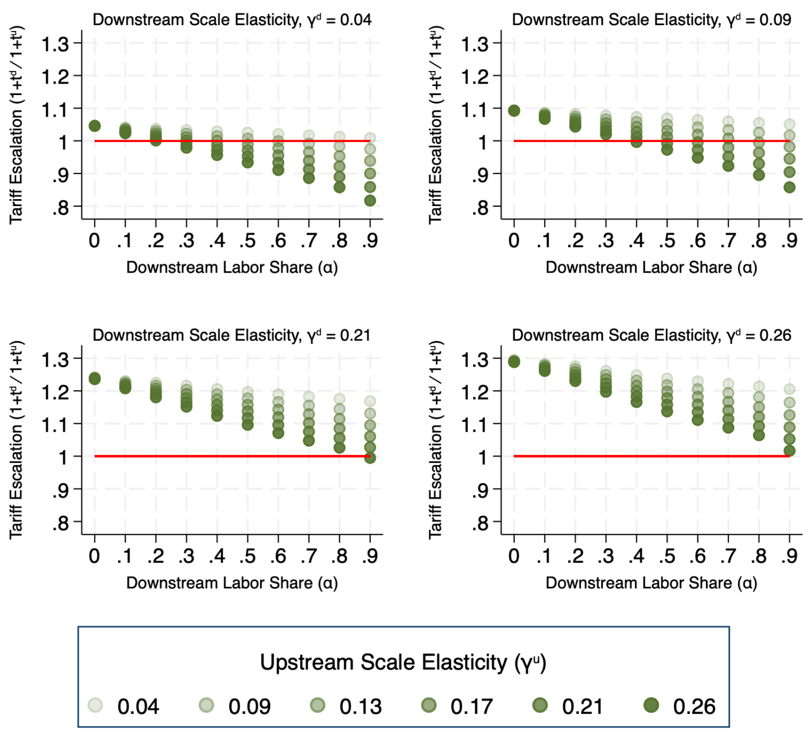
	Values of α									
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$\gamma^d > \gamma^u$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\gamma^d < \gamma^u$	0.00	0.00	0.00	0.13	0.27	0.40	0.60	0.67	0.80	1.00
$\gamma^d = \gamma^u$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Table presents the share of cases in each column for which the optimal tariff are de-escalated.

C.1.4 Second-best tariff escalation and the labor share

We replicate Figure 2 for different values of the demand elasticity downstream in Figure C.1. Furthermore, we plot tariff escalation as a function of the returns to scale downstream relative to those upstream in Figure C.2.

Figure C.1: Second-Best Tariff Escalation and the Labor Share



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the downstream labor share (α) and upstream scale elasticity (γ^u) for different values of the downstream scale elasticity (γ^d). The two trade elasticities (σ and θ) are both fixed at 5.

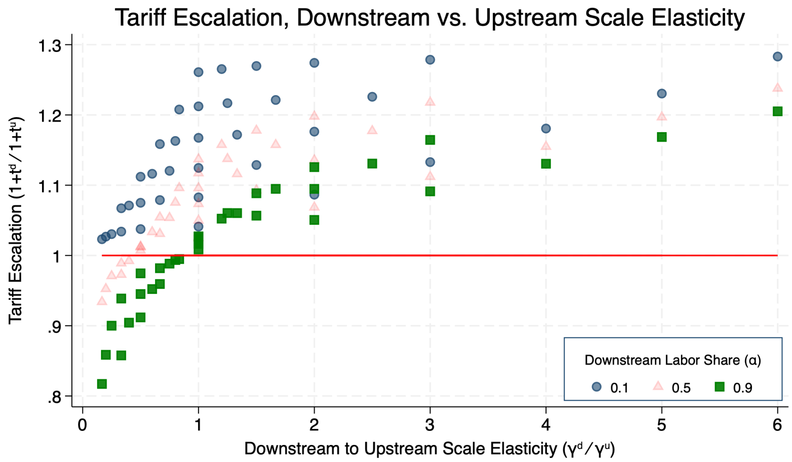
C.1.5 Grid over σ, θ and α

We solve for optimal trade policy for values of σ and θ ranging from 2.5 to 7.5, and for values of α ranging from 0 to 0.9, as described in Table C.1. We fix the values A_d, A_u, τ_d and τ_u to the values we estimate in Section 6.1. Overall, we explore $7 \times 7 \times 10 = 490$ configurations of parameters. We can successfully solve the model for 489 of these 490 cases.

Statistics tariff ratio wedge

The next three tables report statistics related to the tariff escalation wedge, the mean, the median, the standard deviation, the minimum and the maximum. The second column of Table C.6 provides values of these statistics for the 489 cases for which we have a solution. On average, downstream tariffs are 11 percent

Figure C.2: Second-Best Tariff Escalation and Relative Scale Economies



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the relative scale elasticity in downstream versus upstream production (γ^d/γ^u) and the downstream labor share (α). The trade elasticities (σ and θ) are both fixed at 5.

higher than upstream tariffs, and the medians show a similar divergence. Some cases feature tariff escalation levels as high as 1.34, while tariff de-escalation remains modest even in the most extreme cases (minimum of 0.98). The third and fourth column of Table C.6 recalculates the statistic but looking at the cases for which the tariff ratio is above and below 1, respectively. It is interesting to note that when our model predicts tariff de-escalation, it does so with fairly moderate levels (median of 0.99).

Table C.6: Statistics of the tariff escalation wedge: all cases

	All cases	With escalation	With de-escalation
Mean	1.11	1.11	0.98
Median	1.11	1.11	0.99
Standard Deviation	0.06	0.06	0.01
Minimum value	0.98	1.01	0.98
Maximum value	1.34	1.34	0.99
N	489	484	5

Notes: Table reports statistics for the tariff escalation wedge for optimal import tariffs computed in the grid search exercise.

C.1.6 Tariff escalation and parameter space

We now present results about the parameter combinations that generate de-escalated tariffs. Table C.7 reports the fraction of cases for which tariffs are de-escalated for a given parameter, for each possible value that this parameter can take. For example, we have 70 combinations with $\sigma = 4.5$, with 1 percent of these cases featuring de-escalated tariffs.

Table C.8 presents the fraction of cases with de-escalated tariffs for the ten values of the downstream labor share (α) split into cases with $\sigma > \theta$, $\sigma < \theta$, and $\sigma = \theta$. In line with the intuition in the draft, optimal

Table C.7: Tariff De-escalation across the parameter space, I

	Values Downstream Demand Elasticity (σ)						
	2.5	3.5	4.5	5	5.5	6.5	7.5
Share of cases w. De-escalation:	0.00	0.06	0.01	0.00	0.00	0.00	0.00

	Values Upstream Demand Elasticity (θ)						
	2.5	3.5	4.5	5	5.5	6.5	7.5
Share of cases w. De-escalation:	0.00	0.00	0.00	0.01	0.01	0.01	0.03

	Values Downstream Labor Share (α)									
	0	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Share of cases w. De-escalation:	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10

Notes: Table reports the share of cases in each cell for which optimal tariffs are de-escalated, i.e $\left(1+t_H^d\right)/\left(1+t_H^u\right) < 1$.

tariffs are more likely to be de-escalated when the demand elasticity upstream is larger than the elasticity downstream, and when the downstream labor share is high.

Table C.8: Tariff De-escalation across the parameter space, II

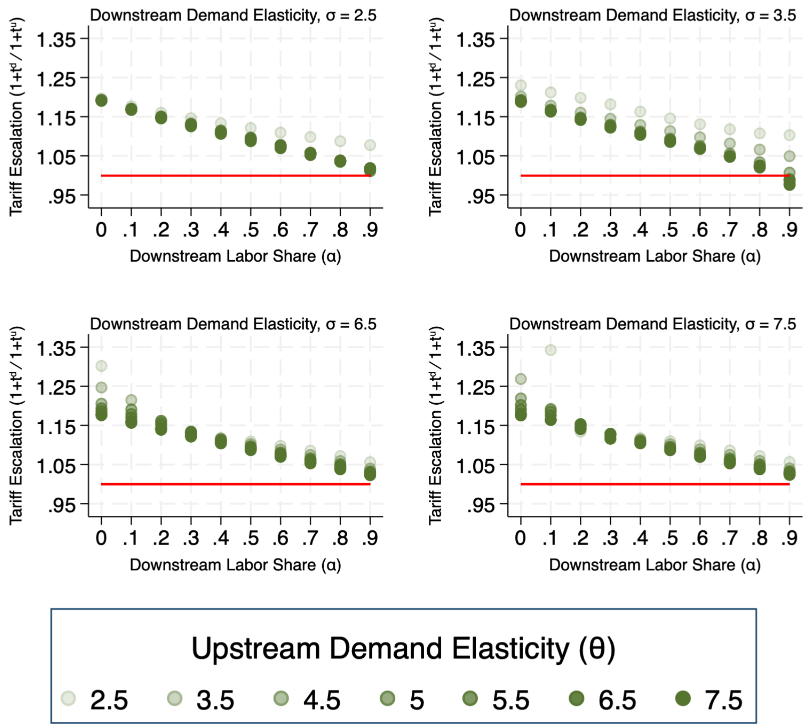
	Values of α									
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$\sigma > \theta$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma < \theta$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.24
$\sigma = \theta$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Notes: Table presents the share of cases in each column for which the optimal tariff are de-escalated.

C.1.7 Second-best tariff escalation and the labor share

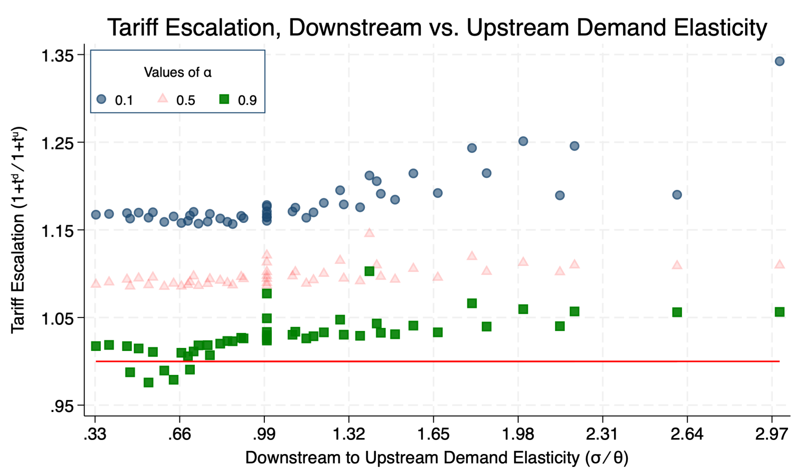
We replicate Figure 3 for different values of the demand elasticity downstream in Figure C.3. Furthermore, we plot tariff escalation as a function of the downstream elasticity downstream relative to the upstream elasticity in Figure C.4.

Figure C.3: Second-Best Tariff Escalation and the Labor Share



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the downstream labor share (α) and upstream demand elasticity (θ) for different values of the downstream demand elasticity (σ). The two scale elasticities (γ^d and γ^u) are both fixed at 0.17.

Figure C.4: Second-Best Tariff Escalation and Relative Scale Economies



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the relative demand elasticity in downstream versus upstream production (σ/θ) and the downstream labor share (α). The scale elasticities (γ^d and γ^u) are both fixed at 0.17.

C.2 Robustness for Calibrated Parameters

Table C.9 reports the estimated values for the two iceberg costs and the two productivity levels for the different combinations of the scale elasticities (γ^d and γ^u), demand elasticities (σ and θ), and the downstream labor share (α) over which we explored robustness of optimal policies featuring escalated tariffs.

Table C.9: Calibrated Parameters - Robustness

σ	θ	γ^d	γ^u	α	τ^u	τ^d	A^d_{RoW}	A^u_{RoW}
5	5	0.170	0.170	0.55	2.031	2.368	0.348	0.149
5	5	0.170	0.128	0.55	2.031	2.368	0.348	0.156
5	5	0.170	0.128	0.25	2.014	2.349	0.464	0.156
5	5	0.170	0.128	0.75	2.045	2.367	0.253	0.153
5	5	0.170	0.128	0.90	2.020	2.321	0.181	0.150
5	5	0.128	0.170	0.55	2.031	2.368	0.364	0.149
5	5	0.128	0.170	0.25	2.014	2.349	0.481	0.148
5	5	0.128	0.170	0.75	2.045	2.367	0.265	0.147
5	5	0.128	0.170	0.90	2.020	2.321	0.189	0.144
5	5	0.170	0.170	0.55	2.030	2.368	0.348	0.149
6.44	4.43	0.170	0.170	0.55	2.292	1.854	0.330	0.152
3.59	2.45	0.170	0.170	0.55	7.310	3.916	0.353	0.178
4	2.5	0.170	0.170	0.55	6.837	3.222	0.342	0.176
4	5.5	0.170	0.170	0.55	1.873	3.222	0.371	0.148
5	5	0.170	0.170	0	2.044	3.025	0.495	0.158
5	5	0.170	0.170	0.25	2.014	2.349	0.464	0.148
5	5	0.170	0.170	0.75	2.045	2.367	0.253	0.147
5	5	0.170	0.170	0.90	2.020	2.321	0.181	0.144

Notes: This table reports the re-calibrated parameters used in our robustness exercise in Table 2 and in Table 3.

D Data Appendix

D.1 Data Construction for Figure 4

US Tariff Data.

- We use US import tariff data at the 8-digit level from the US Harmonized Tariff Schedule (HTS) available at https://dataweb.usitc.gov/tariff/annual. We use the most-favored-nation (MFN) ad valorem tariff rate whenever possible. In approximately 25% of the cases, the MFN ad valorem rate is available and instead a "specific" tariff rate is applied such as "68 cents/head", "1 cents/kg", "0.9 cents each" etc. In these cases we perform an imputation by calculating an ad valorem equivalent tariff rate using unit values obtained from the US Census Bureau.

- In a next step we use the imputed ad valorem tariff rate to calculate applied MFN ad valorem tariff rates for all goods, taking trade agreements between the US and other countries into account. That is, we calculate the applied MFN ad valorem tariff rate as an import weighted average of the MFN ad-valorem rate and the tariff rate that is paid by countries that are members of a trade agreement.[8] US import data for the year 2015 come from the US Census Bureau.

- Data on tariffs imposed in February and March 2018 on almost all countries (washers; solar panels; iron and steel; aluminum) come from Fajgelbaum et al. (2020) and all subsequent tariffs imposed on imports from China throughout 2018 and 2019 from Chad Bown (available here).

ROW Tariff Data.

- We use tariff data for 115 countries plus the European Union at the 6-digit HS code level from the WTO Tariff Download Facility available at http://tariffdata.wto.org/default.aspx. We use the most-favored-nation (MFN) ad valorem tariff rate which constitutes the simple average duty of all products within a 6-digit HS code classification.

- We use data on retaliatory tariffs imposed by China throughout 2018 and 2019 from Chad Bown (available here). Data on retaliatory tariffs imposed by the European Union, Canada, Mexico, India and Turkey stem from Li (2018). Using data on these tariff waves we adjust the MFN applied tariff rates taking 2015 US export value weighted averages with US export data coming from the US Census Bureau.

Intermediate and Final Goods Classification

- We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC), rev. 5. The cross-walk between HTS codes (specifically HS 2012) and end-use categories is available here. We classify goods as intermediate when BEC indicates their final use is as an intermediate or capital good (third digit 1 or 2, respectively); we classify goods as final goods when their BEC end use is consumption (third digit 3). There are a handful of HTS codes corresponding with more than one BEC end use; in these cases, we classify them as intermediate if they are hybrid intermediate-capital,

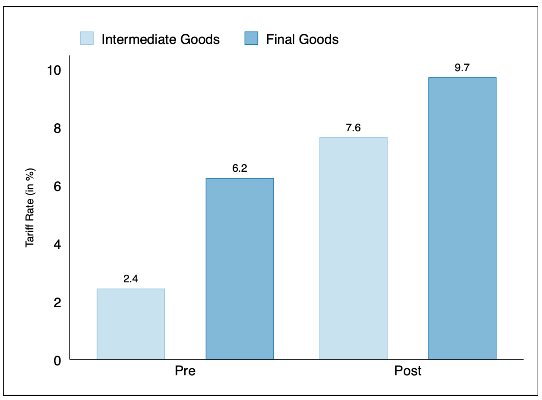
[8] We account for the following trade agreements: Generalized System of Preferences (GSP, 41 countries), The Agreement on Trade in Civil Aircraft (32 countries), NAFTA (3 countries), Caribbean Basin Initiative (CBI, 17 countries), African Growth and Opportunity Act (AGOA, 40 countries), Caribbean Basin Trade Partnership Act (CBTPA, 8 countries), Dominican Republic-Central America FTA (6 countries) and the Agreement on Trade in Pharmaceutical Products (7 countries).

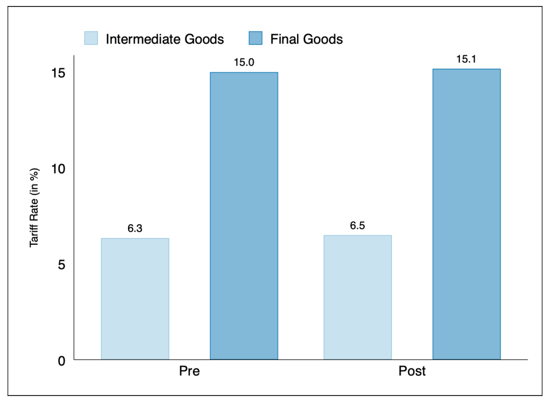
and exclude those with any other combination (e.g. mixed use as an intermediate and consumption good).

Tariff Escalation Unweighted

- As alternative to Figure 4 which shows trade-weighted tariff rates, Figure D.1 displays an unweighted version of the tariff increase on intermediate and final goods by the ROW on imports from the US throughout the trade war and vice versa.

Figure D.1: Comparison of ROW and US Input & Final-Good Tariffs (Unweighted)





Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC. Goods are classified as intermediate or final according to the Broad Economic Categories rev. 5 (BEC5) code corresponding to their HS code. BEC5 codes with a third digit of 1 or 2 (intermediate and capital final use in BEC5, respectively) are classified as intermediate goods and BEC5 codes with a third digit of 3 are classified as final consumption goods. When the HS to BEC5 correspondence implies mixed use as intermediate and consumption or capital and consumption, we assign no classification and omit from consideration.

D.2 Elasticity Estimation

Below we explain the estimation of the elasticities of substitution in the upstream and downstream sectors using two different approaches: the trade elasticity approach, and the sectoral markup approach. We present results on optimal taxes for the two approaches and demonstrate how they differ.

Trade Elasticity Approach. We estimate elasticities in the upstream and downstream sectors by measuring the response of imports in the upstream and downstream sectors to changes in import tariffs. More specifically, we calculate the changes in US import values in both sectors during the US-China trade war (January 2018 to December 2019) that raised US import tariffs on upstream goods by 4.5 percentage points and downstream goods by 3.6 percentage points. We obtain data on import values at the country-HTS10-month level from the US Census Bureau's Application Programming Interface (API). Data on US import tariffs are constructed as described in Section D.1.

We regress 12-month log changes in import values on 12-month log changes in tariff rates via the following regression specification:

$$\Delta \log(v_{ijt}) = \alpha_j + \tau_{it} + \beta \Delta \log(1 + \text{Tariff}_{ijt}) + \omega_{ijt}, \tag{D.1}$$

where i indicates foreign countries, j denotes products, and t corresponds to time; α_j is a product fixed effect; τ_{it} is a country-time fixed effect; and ω_{ijt} is a stochastic error. We denote import values by v_{ijt}. We estimate equation (D.1) separately for intermediate and final goods using both log differences and the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$.[9] The results are presented in Table D.1.

Table D.1: Impact of US Tariffs on Import Values

	Intermediate Goods		Final Goods	
	(1) Log Change Import Value $\Delta\log(v_{ijt})$	(2) Inv. Hyperb. Import Value $\Delta\log(v_{ijt})$	(3) Log Change Import Value $\Delta\log(v_{ijt})$	(4) Inv. Hyperb. Import Value $\Delta\log(v_{ijt})$
log change tariff $\Delta\log(1 + \text{Tariff}_{ijt})$	-1.20***	-2.45***	-1.65***	-3.59***
	(0.07)	(0.41)	(0.18)	(0.79)
N	1538067	2619150	753535	1395205
R2	0.025	0.043	0.026	0.045

Notes: Observations are at the country-HTS10-month level for the period January 2018 to December 2019. Since the specification is in 12-month changes, the data includes observations from January 2017 onwards. Robust standard errors in parentheses. Variables are in twelve-month log change. All columns include product-level and country-time fixed effects. The dependent variables are the log change and the change in the inverse hyperbolic sine of US import values of intermediate and final goods, respectively. We use the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

Column 1 (3) suggests that a one percent increase in tariffs on intermediate (final) goods is associated with a 1.20 (1.65) percent decrease in import value. However, since tariffs can lead to zero imports, which will be dropped from the regression, columns 2 and 4 perform the same regression this time using the inverse hyperbolic sine instead of the log change. This adjustments leads to greater trade elasticities for both types of goods. A one percent increase in tariffs on intermediate (final) goods is associated with a 2.45 (3.59) percent decrease in import value. Note that the estimates from this specification correspond to an elasticity of substitution between intermediate (final) goods of 2.45 (3.59).

Sectoral Markup Approach Information on firm-level markups allows us to derive elasticities in a straightforward manner since markup $= \frac{\text{elasticity}}{\text{elasticity}-1}$. We thus compile data for this exercise as follows. We obtain upstream/downstream sector classifications using WIOD. We use 2014 sales of the US to the US and RoW to calculate the share of total sales per sector that goes to final consumers. We then classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median. This yields a dataset which shows upstream and downstream classifications for 87 sectors at the 2-digit NACE level (European industry classification). We combine this 2-digit NACE with a NACE-NAICS concordance file that maps 4-digit NACE (we only use the first 2 digits) to 6-digit NAICS. If there are multiple NACE 2-digit codes for a NAICS 6-digit code, we choose the NACE 2-digit code that has larger total US sales. This yields a final dataset that shows upstream and

[9]Note that regression coefficients based on the hyperbolic sine transformation are sensitive to the scale of the import values. This is, results vary depending on whether import values are measured in thousands, millions, etc. Following Amiti et al. (2019), we measure import values in single US dollars.

downstream classifications for 1,175 different NAICS 6-digit codes. We combine these data with data kindly provided by Baqaee and Farhi (2020) (BF) based on 6-digit NAICS codes. The BF data list markups and sales for 31,683 different firms from 1978 – 2018. They provide three different types of markups calculated based on a user cost, a production function, or an accounting profits method. We select their data between 2012 and 2017 and focus on the markups calculated using the production function estimation approach. We further exclude firms that have markups smaller than 1 (14% of all firm-year observations).

Table D.2: Elasticities

	mean	sd	min	p5	p25	p50	p75	p95	max	count
Upstream	4.43	4.26	1.10	1.15	1.60	2.75	5.04	16.50	16.50	11045
Downstream	6.44	6.05	1.29	1.46	2.44	4.03	7.49	22.24	22.24	14773

Notes: The table shows weighted mean elasticities for upstream and downstream sectors between 2012 and 2017 across all firms in the WIOD that have markups greater than 1. Elasticities stem from the production function estimation approach. Weights represent the share of firm sales in total sales. We winsorize elasticities and sales at the 5-95th percentile by sector.

We then calculate firm-level elasticities as elasticity $= \frac{\text{markup}}{\text{markup}-1}$ and winsorize elasticities and sales at the 5-95th percentile by sector. Finally, we calculate weighted mean elasticities for upstream and downstream sectors across all firms where weights represent the share of firm sales in total sales. Table D.2 presents elasticities for upstream and downstream sectors pooling all years from 2012 to 2017.

D.3 Scale Elasticity

Data on scale elasticities comes from Bartelme et al. (2021). The authors provide 2SLS estimates on scale elasticities for 15 manufacturing industries presented in Table D.3. We classify these industries into upstream and downstream industries following the same procedure as in the *Sectoral Markup Approach* and then calculate the average scale elasticity in those sectors. For the upstream sector we obtain an average scale elasticity of 0.166 and for the downstream sector an average scale elasticity of 0.170.

D.4 Share of Inputs in the Downstream Sector

As in the *Sectoral Markup Approach*, we classify sectors into upstream and downstream depending on whether the share of total sales to final consumers is below or above the median across all sectors. From the WIOD in 2014 we calculate the share of inputs in the downstream sector as the ratio of intermediate inputs to sales in the downstream sectors leading to an estimate of $1 - \alpha = 0.45$.

Table D.3: Scale Elasticity Estimates from Bartelme et al. (2021)

Industry	NACE Rev. 2	WIOD class.	Scale elast.
Food products, beverages and tobacco	10, 11, 12	downstream	0.22
Textiles	13, 14, 15	downstream	0.12
Wood and products of wood and cork	16	upstream	0.13
Paper products and printing	17, 18	upstream	0.15
Coke and refined petroleum products	19	upstream	0.09
Chemicals and pharmaceutical products	20, 21	upstream	0.24
Rubber and plastic products	22	upstream	0.42
Other non-metallic mineral products	23	upstream	0.17
Basic metals	24	upstream	0.09
Fabricated metal products	25	upstream	0.12
Computer, electronic and optical products	26	downstream	0.08
Electrical equipment	27	upstream	0.08
Machinery and equipment, nec	28	downstream	0.24
Motor vehicles, trailers and semi-trailers	29	downstream	0.18
Other transport equipment	30	downstream	0.18

Notes: Industries and 2SLS scale elasticities stem from Bartelme et al. (2021). Upstream and downstream classifications stem from WIOD where we classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median.