Trade Policy and Global Sourcing: An Efficiency Rationale for Tariff Escalation*

Pol Antràs
Harvard University, CEPR, and NBER

Teresa C. Fort
Tuck School at Dartmouth, CEPR, and NBER

Agustín Gutiérrez
University of Chicago

Felix Tintelnot
University of Chicago, CEPR, and NBER

October 2022

Abstract

Import tariffs tend to be higher for final goods than for inputs, a phenomenon commonly referred to as tariff escalation. We show that tariff escalation can be rationalized on efficiency grounds in the presence of scale economies. When both downstream and upstream sectors produce under increasing returns to scale, a unilateral tariff in either sector boosts the size and productivity of that sector, raising welfare. While these forces are reinforced up the chain for final-good tariffs, input tariffs may drive final-good producers to relocate abroad, mitigating their potential productivity benefits. The welfare benefits of final-good tariffs thus tend to be larger, with the optimal degree of tariff escalation increasing in the extent of downstream returns to scale. A quantitative evaluation of the US-China trade war demonstrates that any welfare gains from the increase in US tariffs are overwhelmingly driven by final-good tariffs.

*We are grateful to Lorenzo Caliendo, Arnaud Costinot, Dave Donaldson, Ahmad Lashkaripour, Steve Redding, Andrés Rodríguez-Clare, Iván Werning, and various seminar audiences for insightful comments, and to Joe Shapiro for sharing data with us. Ignacia Cuevas, Evgenii Fadeev, Jack Liang, Nicolas Wesseler, and Shuhan Zou provided excellent research assistance. A previous version of this paper was circulated under the title "Import Tariffs and Global Sourcing."

1 Introduction

Import tariffs tend to be lower on intermediate inputs than on final goods. This pattern has been documented in multiple studies spanning numerous countries across five decades (Travis, 1964; Balassa, 1965; Bown and Crowley, 2016; Shapiro, 2020), and is commonly referred to as 'tariff escalation,' a term that captures the fact that tariffs 'escalate' down the production chain. Figure 1 illustrates the prevalence of tariff escalation across trading partners in 2007: for almost every country-pair, the simple average of final-good tariffs is higher than average input tariffs.

Empirical research suggests that these relatively low input tariffs improve downstream firm and worker outcomes. Early papers in this area document significant productivity gains from lower input tariffs (Amiti and Konings, 2007; Goldberg et al., 2010; Topalova and Khandelwal, 2011), while new evidence shows that recent US input tariff hikes harmed US manufacturing employment (Flaaen and Pierce, 2019) and exports (Handley et al., 2020).[1]

Despite the ubiquity of tariff escalation and mounting evidence on the benefits of relatively low input tariffs, existing trade theory provides little guidance for why tariff escalation might be welfare enhancing. Early neoclassical models with homogeneous goods analyze input versus final-good tariffs explicitly, but do not show that optimal tariffs should be lower for inputs. Modern Ricardian trade models stress that optimal tariffs should be uniform across sectors, and that tariff *de*-escalation may maximize welfare in second-best settings without export taxes.

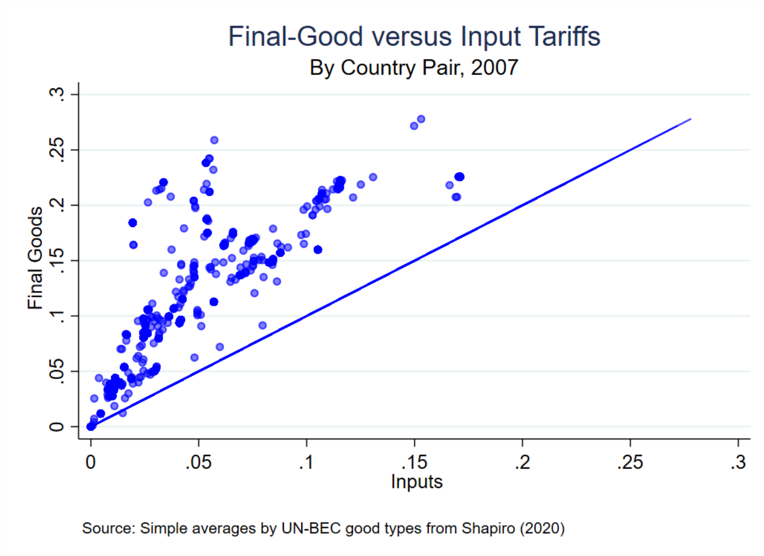
The leading explanation for tariff escalation relies on political counter-lobbying (Cadot et al., 2004; Gawande et al., 2012), in which all firms lobby for protection of their output, but final-good producers counter-lobby against tariffs on their imported inputs. Although consumers would also prefer low final-good tariffs, this explanation assumes that collective action disincentives preclude them from lobbying. The assumption that final-good importers face a lobbying disadvantage is often cited, however it is increasingly at odds with the well-documented and rising concentration of consumer-good imports by a few large wholesale and retail firms (Basker and Van, 2010; Ganapati, 2018; Smith and Díaz, 2020).[2] Moreover, the welfare implications are similar to those from modern-Ricardian trade models: uniform tariffs across sectors raise welfare.

In this paper, we analyze the market structures under which tariff escalation emerges as a social-welfare maximizing policy. Since neoclassical models with constant returns to scale do not predict that lower input tariffs are optimal, we use a general-equilibrium framework with an upstream and downstream sector that produce differentiated intermediate and final goods, respectively, both under increasing returns to scale. These scale economies provide an efficiency motive for shifting expenditure towards domestic varieties in each sector, but final-good versus input tariffs differ in their ability to do so and raise welfare. While a final-good tariff is unique in that it can be used to achieve the first-best allocation without any other downstream instruments, an input tariff is never

[1] Evidence on the costs of raising prices on imported inputs is also documented for anti-dumping duties (Bown et al., 2020; Barattieri and Cacciatore, 2020) and the Bush steel tariffs (Cox, 2021). Relatedly, reductions in US trade policy uncertainty with China on firms' inputs led to relatively higher US export growth (Breinlich et al., 2021).

[2] For example, in September 2018 Walmart responded to the Trump tariffs on consumer goods with a direct letter to US Trade Representative Robert Lighthizer warning of price hikes and consumer harm.

Figure 1: Average Final-Good versus Input Tariffs



sufficient. Although input tariffs shift expenditure towards domestic varieties, they also tend to drive final-good producers to relocate abroad. This relocation reduces the size of the downstream sector, which is detrimental to welfare when downstream production features increasing returns to scale. As a result, we find that optimal trade policy generally involves higher tariffs on final goods than inputs, with the degree of tariff escalation closely related to the downstream scale elasticity (i.e., the elasticity of industry-level productivity to scale).

We model both sectors as monopolistically competitive with scale economies that are internal to firms, but show that there is an isomorphic model with perfect competition and external economies of scale that generates identical tariff motives. In both cases, the presence of increasing returns in the *upstream* sector results in a domestic inefficiency whenever final goods are produced using labor and inputs. The upstream sector is too small, and a production subsidy to inputs shifts labor upstream, thereby expanding that sector and raising its efficiency. The optimal size of this subsidy depends *only* on the extent of increasing returns upstream, and despite pulling labor from the downstream sector, also increases final-good production. This downstream growth arises because the expanded availability of inputs raises final-good producers' labor efficiency. Intuitively, the market fails to deliver efficiency because upstream firms do not internalize their impact on downstream production, and this has a bigger impact when the increasing returns to scale upstream are larger.[3] In the closed-economy, this is the only inefficiency and it is only present when both sectors use labor. There is no motive for a downstream subsidy because there is no misallocation in consumers' expenditure allocation.

To analyze optimal tariffs, we first consider a small, open economy with a Home and Foreign

[3] In the monopolistically competitive model, the optimal subsidy is increasing in the elasticity of substitution across inputs. The relationship between this elasticity and the degree of returns to scale is pinned down in our isomorphic model such that the optimal subsidy is identical in both settings.

country, and trade in both inputs and final goods. Home has market power over its exported varieties, but takes import prices (net of tariffs) as given. By holding Foreign price indices fixed, we can derive analytic solutions for optimal trade policy following the primal approach (Lucas and Stokey, 1983; Costinot et al., 2015).

In the open-economy, the social planner has the well-known motives to exploit her country's market power abroad (Gros, 1987), and to shift expenditure towards Home varieties to benefit from increasing returns to scale in production (Venables, 1987; Ossa, 2011). The social planner can always achieve the first-best allocation with a combination of production subsidies and export taxes in both sectors (e.g., as in Lashkaripour and Lugovskyy, 2021). This implementation is useful for understanding policy motives. The subsidies increase the sizes of each sector, which raises their efficiency (due to increasing returns to scale), and thus also welfare. The sizes of the optimal subsidies depend only on the degree of increasing returns (i.e., the scale elasticity) in each sector. Export taxes allow Home to exercise its market power abroad, and thus optimal taxes depend only on Foreign's elasticity of demand for each sector. There is no welfare-motive for a tariff in either sector under this implementation, though Lerner Symmetry (see Costinot and Werning, 2019) implies that they could be set to any (arbitrary) uniform level with an offsetting adjustment to the subsidies and taxes.

A key distinction between final-good versus input tariffs is that a final-good tariff is a perfect substitute for the combined downstream production subsidy and export tax, whereas an input tariff is not. A final-good tariff that depends only on the extent of scale economies downstream shifts domestic expenditures towards Home varieties, thereby increasing the size of the sector and constraining exports such that Home exploits it market power abroad optimally. By contrast, an input tariff is *never* sufficient to achieve the first-best allocation. Even when the input tariff raises upstream productivity and thus lowers Home input prices, it is less efficient than the combined subsidy and export tax, since it necessarily pulls labor from downstream firms to do so through higher wages. The first-best can thus also be achieved using a tariff as the sole downstream instrument, but always requires an upstream export tax. When the downstream sector uses labor, the first-best also requires an upstream production subsidy that depends only on the degree of increasing returns to scale in that sector.

A main goal of this paper is to assess whether real-world tariffs reflect social welfare-maximizing policies. Since export taxes are often illegal and production subsidies face strict World Trade Organization (WTO) limitations, analyzing optimal policies that rely solely on import tariffs is crucial.[4] When the downstream sector uses only inputs to produce, all labor is already employed upstream, so there is no scope to generate efficiency gains in upstream production. The Home government now exploits its market power abroad using an input tariff, but we show that tariff escalation persists, and is in fact larger, than it would have been if the upstream export tax were available. This increased escalation arises because the input tariff raises downstream firms' costs, which would lead them to relocate to Foreign without an offsetting higher downstream tariff.

[4]For example, Article I, Section 9, Clause 5 of the US Constitution explicitly bans export taxes.

When the downstream sector uses both labor and inputs to produce, the social planner now uses an input tariff to alleviate (imperfectly) the misallocation of labor identified in the closed-economy. As a result, tariff escalation is no longer necessarily optimal. Although we cannot derive a simple characterization of tariff escalation in this setting, we discuss how it is shaped by the relative sizes of increasing returns in the two sectors, and the share of labor used in downstream production. First, the optimal input tariff tends to be larger when the downstream labor share is high, because the difference between the market allocation of labor upstream versus the planner's allocation is increasing in the labor share. Second the relative size of upstream versus downstream returns to scale is now also relevant, since a tariff serves to increase a sector's size and thus its efficiency. To demonstrate the importance of these distinct channels, we solve for optimal, second-best tariffs as a function of a wide range of values for these three key parameters. In our grid search, tariff escalation is optimal in more than 90 percent of cases, with de-escalation being optimal only when the downstream labor share is high, and the input sector's scale elasticity is equal to or greater than the final-good sector's scale elasticity. For all empirically plausible parameter combinations of the downstream labor share and the scale elasticities, we always obtain tariff escalation as a solution to the planner's second-best problem in which tariffs are the only policy instruments.

We further highlight the key role of increasing returns to scale in downstream production for explaining tariff escalation by characterizing optimal policy when production features constant returns to scale. When inputs are produced under constant returns to scale but final-good production features increasing returns, optimal tariffs feature escalation across all the cases described above, including in second-best settings without subsidies or export taxes. By contrast, when downstream goods are produced under constant returns to scale, tariff escalation is only optimal when demand for downstream goods is more elastic than upstream demand.

The calibrated model is particularly helpful for analyzing second-best policies that rely solely on tariffs. In this setting, the optimal final-good tariff is 30.6 percent, versus an optimal input tariff of only 17.0 percent.[5] Tariff escalation is decreasing in the extent to which the downstream sector uses labor in production, consistent with it mimicking the upstream subsidy that is not available under a second-best implementation. As before, the solution to the social planner's second-best problem features tariff escalation for all empirically plausible parameter combinations for the downstream labor share and the scale elasticities.

Finally, we use the model to study the welfare effects of the US trade war in 2018 to 2019. Approximately 60 percent of the 2018 tariffs were on inputs, affecting nearly 20 percent of all US imports of intermediate inputs (Bown and Zhang, 2019). Our quantitative results indicate that, absent any foreign retaliation, these tariff increases would have raised US welfare by 0.12 percent, with the positive effect overwhelmingly driven by higher final-good tariffs. Once we include foreign retaliatory tariffs, the increase in US welfare shrinks to 0.02 percent. By comparison, the welfare effects net of retaliation would have been negative if input tariffs alone had been used.

[5]The finding that optimal tariffs are much larger than tariff rates in the data is a well-known feature of the quantitative trade policy literature (see Costinot and Rodríguez-Clare 2014; Ossa 2014).

Our paper contributes to the literature in several ways. First, we provide an efficiency rationale for tariff escalation in an environment with a benevolent social planner.[6] Early neoclassical models with homogeneous final goods and inputs explicitly modeled both types of tariffs, but did not show that relatively lower input tariffs would raise welfare, despite their effects on downstream costs (Ruffin, 1969; Casas, 1973; Das, 1983). Recent work on optimal trade policy in multi-sector competitive Ricardian models predicts that optimal tariffs should be uniform across sectors (Costinot et al., 2015; Beshkar and Lashkaripour, 2020).[7] Blanchard et al. (2021) demonstrate that the terms-of-trade motive for final-good tariffs persists, but is dampened, in a competitive model when a country's final-good imports contain its domestic value added. We contribute to this work by analyzing input and output tariffs separately under different market structures. While uniform tariffs are optimal when both sectors produce under constant returns to scale, increasing returns to scale in downstream production provide an efficiency rationale tariff escalation, with the extent of optimal escalation increasing in the degree of these returns.[8]

Our departure from perfect competition and constant returns to scale highlights the potential for final-good and input tariffs to affect welfare by changing the mass of firms in both sectors. These production relocation effects are studied in Venables (1987) and Ossa (2011), who show that with imperfect competition and scale economies, an increase in a final-good import tariff attracts firms to a country, which in turn lowers prices and thus raises welfare. We extend the analysis by adding input trade and moving from a partial- to a general-equilibrium framework. Prior work finds that in general equilibrium with roundabout production, agglomeration forces may lead countries to specialize in manufacturing production (Krugman and Venables, 1995; Puga and Venables, 1999). Amiti (2004) introduces a two-sector model with agglomeration forces and uses numerical simulations to argue that tariff de-escalation is optimal. Our contribution is to model input versus final-good sectors separately, to provide closed-form solutions for the first-best welfare allocations that demonstrate why final-good versus input tariff motives differ, and to demonstrate that tariff escalation is present for all empirically plausible combinations of the model's parameter space.

Finally, we add to a growing body of work that studies optimal trade policy in the presence of market power and domestic distortions. Early work shows that market power provides an incentive for final-good tariffs, even when countries are too small to affect world prices (Gros, 1987; Demidova and Rodiguez-Clare, 2009).[9] Caliendo et al. (2021) demonstrate that when domestic subsidies

[6]Our result on tariff escalation differs from the celebrated production efficiency result in Diamond and Mirrlees (1971), since they study a closed-economy setting in which a planner seeks to raise government revenue at the minimum efficiency cost. Tariff escalation is also distinct from the 'cascading trade protection' in Erbahar and Zi (2017), who study the effects of upstream tariffs on the *demand* for downstream tariffs, regardless of efficiency considerations.

[7]Since the terms-of-trade motive is decreasing in a sector's export supply elasticity, the tariff escalation present in real-world trade policy might appear to be consistent with existing neoclassical theory if export supply elasticities were to be higher for intermediate inputs than for final goods. However, the data used in Shapiro (2020) indicate a weak positive correlation of 0.049 between the measure of upstreamness in Antràs et al. (2012) and the inverse export supply elasticities in Soderbery (2015).

[8]Recent papers also study the effects of input tariffs in frameworks with *relational* GVCs and incomplete contracts (Ornelas and Turner, 2008; Antràs and Staiger, 2012; Ornelas and Turner, 2012; Grossman and Helpman, 2020).

[9]See Campolmi et al. (2014, 2018) for other recent work on optimal trade policy in the presence of domestic

are unavailable, the optimal tariff tends to be lower when imports are used in production in a roundabout setting because the lower tariff mitigates a double marginalization inefficiency that arises from markups in the tradable sector. Our results differ because we model trade in both inputs and final-goods and show that tariff escalation *decreases* when an upstream subsidy is not available. We also exploit an isomorphism between the model with market power and one with external economies of scale, similar to Kucheryavyy et al. (2017), to show analytically that the first-best allocation using the minimal number of instruments features tariff escalation. This may seem at odds with Lashkaripour and Lugovskyy (2021), who characterize optimal tariffs as independent of the economy's input-output structure when optimal production subsidies are available. While we also find that a downstream production subsidy can be used instead of a downstream tariff, its use then requires an additional downstream export tax. Crucially, a final-good tariff is a perfect substitute for those instruments, while an input tariff is not. In sum, we show that optimal tariffs on inputs are lower than those on final-goods when implementing the first-best with the minimum number of domestic instruments. Extensive quantitative exploration also indicates that second-best import tariffs (which rule out all subsidies and export taxes) feature tariff escalation for all empirically plausible parameter combinations.[10]

The rest of the paper is structured as follows. In Section 2, we use a closed-economy version of our Krugman-style model expanded to include an intermediate-input sector to analyze domestic distortions and optimal policy. In Section 3, we develop the open-economy version of the model, as well as the isomorphism to a competitive economy with external economies of scale. Optimal policy for a small open economy is developed in sections 4 and 5, and we provide quantitative results for a large open economy in Section 6. In Section 7, we study the impact of small input and final-good tariffs for a large open economy, we perform a counterfactual analysis of the welfare impacts of the Trump tariffs in Section 8, and we conclude in Section 9.

2 A Krugman Economy with an Input Sector

In this section, we introduce a closed-economy model with an upstream (input) sector and a downstream (final-good) sector, both featuring increasing returns to scale, product differentiation, and monopolistic competition. Our framework is a simple extension of the closed-economy version of the classical model in Krugman (1980), expanded to include an intermediate-good sector. We begin with this relatively simple framework because we can derive analytical solutions to both the market equilibrium and the social planner's problem, which provide intuition for how imperfect competition and scale economies in vertical sectors lead to welfare-reducing distortions.

distortions. Other papers study the desirability of tariff escalation under various market structures (e.g., Spencer and Jones, 1991, 1992; McCorriston and Sheldon, 2011; Hwang et al., 2017).

[10]In this sense, our paper contributes to an active literature studying the quantitative implications of trade policy (Eaton and Kortum, 2002; Alvarez and Lucas Jr, 2007; Costinot and Rodríguez-Clare, 2014; Ossa, 2014).

2.1 Environment

Consider an economy in which the representative consumer values the consumption of differentiated varieties of manufacturing goods according to the utility function

$$U = \left(\int_0^{M^d} q^d\left(\omega\right)^{\frac{\sigma-1}{\sigma}} d\omega \right)^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \tag{1}$$

where M^d is the endogenous measure of final-good varieties produced in the economy, $q^d\left(\omega\right)$ is the quantity consumed of variety ω, and σ is the elasticity of substitution across varieties. Individuals supply one unit of labor inelastically, with L denoting the total labor force. There are no other factors of production, so labor should be interpreted as representing "equipped" labor.

Labor is used for the production of intermediate inputs (the upstream sector) and (possibly) in producing final goods (the downstream sector). More specifically, we represent technologies in the upstream and downstream sectors with

$$f^u + x^u(\varpi) = A^u \ell^u(\varpi), \qquad \varpi \in [0, M^u], \tag{2}$$

and

$$f^d + x^d(\omega) = A^d \ell^d(\omega)^{\alpha} Q^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M^d], \quad \alpha \in [0,1], \tag{3}$$

respectively. In these expressions, f^s denotes the fixed output requirements for entry in sector $s \in \{D, U\}$, $x^s(\omega)$ is the output produced for sale by variety ω in sector $s \in \{D, U\}$, A^s is a sector-specific technology parameter, and $Q^u(\omega)$ is a composite of all intermediate goods, which is in turn given by

$$Q^u(\omega) = \left(\int_0^{M^u} q^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right)^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \tag{4}$$

where $q^u(\varpi)$ is the quantity consumed of input variety ϖ. In words, the upstream sector uses only labor, and technology features increasing returns to scale due to the presence of a fixed overhead cost. The downstream sector combines labor with a continuum of intermediate inputs of measure M^u, where M^u is endogenous, and technology again exhibits increasing returns stemming from a fixed overhead cost. Notice that $\theta > 1$ governs the degree of substitutability across inputs, while $\alpha \in [0, 1]$ corresponds to the downstream labor (or value-added) intensity in production.[11]

There is an endogenous measure, M^d, of manufacturing firms in the downstream sector, each producing a single final-good variety. Analogously, there is an endogenous measure, M^u, of manufacturing firms in the upstream sector, each producing a single intermediate-input variety. All entrants have access to the same technologies in (2), (3) and (4). Market structure in both sectors is characterized by monopolistic competition and free entry.

[11] Note that we specify the fixed costs of production in terms of output rather than labor. This assumption is immaterial for our main results, and it avoids introducing additional sources of inefficiency into our framework (see also Costinot and Rodríguez-Clare, 2014, footnote 20).

2.2 Equilibrium and Efficiency

Given the CES assumptions built into our framework and the lack of strategic interactions, firms in both sectors charge a constant markup over their marginal cost, which combined with free entry, pins down firm size according to (see Appendix A for details):

$$x^u = (\theta - 1) f^u, \qquad x^d = (\sigma - 1) f^d. \tag{5}$$

Naturally, in equilibrium we must have $x^d = q^d$ and $x^u = M^d q^u$. Invoking households' demand for downstream goods and labor-market clearing (see Appendix A), we can determine the measure of upstream and downstream firms in the economy:

$$M^u = \frac{(1-\alpha)A^u L}{f^u \theta}; \tag{6}$$

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((\theta - 1) f^u\right)^{1-\alpha} \left(\frac{(1-\alpha)A^u}{f^u \theta}\right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}}. \tag{7}$$

Finally, welfare of the representative consumer is simply given by $U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} q^d$, where M^d is given in (7) and $q^d = x^d$ in (5). When $\alpha \to 1$, we obtain

$$U = \left(\frac{A^d}{f^d \sigma} L\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f^d,$$

which is the standard formula in Krugman (1980).[12] Welfare is increasing in market size with an elasticity equal to $\sigma/(\sigma-1) > 1$, reflecting the variety gains associated with living in an economy that provides a larger number of final-good varieties.

As in the "Krugman" benchmark, our model features scale effects, with welfare increasing in the size of the labor force L. In addition, these scale effects are even larger when the upstream sector is active (i.e., $\alpha < 1$). To see this, we can write welfare as

$$U = \left(\frac{(\sigma-1)A^d/\sigma}{\left((\sigma-1)f^d\right)^{\frac{1}{\sigma}}} \left(\frac{(\theta-1)A^u/\theta}{\left((\sigma-1)f^u\right)^{1/\theta}}\right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}}\right)^{\frac{\sigma}{\sigma-1}} \xi_\alpha, \tag{8}$$

where ξ_α is a function of only α and θ. Note that $\frac{\theta-\alpha}{\theta-1} \geq 1$, and thus the elasticity of welfare with respect to L is larger when $\alpha < 1$.

To gain a better understanding of the role of imperfect competition and increasing returns to scale in shaping welfare in our closed economy, in Appendix A.2 we characterize the social optimum in our model, and explore conditions under which the above market equilibrium is efficient. There, we prove the following result:

[12] A small and largely immaterial point of departure from Krugman (1980) is the fact that we have modeled the productivity terms A^d and A^u as shaping both the marginal and fixed costs of production. As a result, firm size is independent of these productivity parameters, but these parameters affect welfare directly.

Proposition 1. In the decentralized equilibrium, firm-level output is at its socially optimal level in both sectors, but the market equilibrium features too little entry into both the downstream and upstream sectors unless $\alpha = 1$ (so the upstream sector is shut down) or $\alpha = 0$ (so the downstream sector does not use labor directly in production).

Why is the market equilibrium typically inefficient? It might seem intuitive that this inefficiency is associated with upstream markups leading to a double-marginalization inefficiency. However, combining equations (5), (6) and (7), one can show that the aggregate market allocation of labor to the upstream sector is given by:

$$M^u \ell^u = (1 - \alpha)L; \tag{9}$$

and is in fact independent of the degree of input substitutability (θ) and thus of the level of upstream markups. In other words, lower input substitutability – and thus higher markups – do *not* depress the market allocation of labor to the upstream sector. Intuitively, although markups reduce the demand for intermediate inputs, they also induce entry of new firms into the upstream sector, and these two effects cancel each other, as often occurs with CES preferences.

Instead, lower input substitutability and higher markups matter for welfare because they *increase* the social-welfare maximizing allocation of labor to that sector. More specifically, in Appendix A.2 we show that the social planner would allocate a share of labor to that sector equal to:

$$M^u \ell^u = \frac{\theta}{\theta - \alpha}(1 - \alpha)L > (1 - \alpha)L, \tag{10}$$

which is decreasing in θ. The intuition for this result is as follows: in the market equilibrium, upstream firms do not internalize the fact that their entry generates positive spillovers for firms in the downstream sector, with the size of this vertical spillover decreasing in the degree of input substitutability θ.

To reinforce this interpretation, in Appendix A.4, we show that the equilibrium of our vertical Krugman economy is isomorphic to that of a competitive vertical economy with external economies of scale under specific relationships between the elasticities of substitution and the external economies of scale parameters (cf. Kucheryavyy et al., 2017). In this variant of our model, there are no markups and it is clear that the market inefficiency is due only to upstream suppliers failing to internalize the positive productivity effects of their entry on downstream firms. This isomorphism will also prove useful in characterizing optimal trade policy in the open economy (see, in particular, Section 3.2).

Although this vertical closed economy is generically inefficient, Proposition 1 highlights the fact that efficiency is achieved when $\alpha = 1$ or $\alpha = 0$. The intuition for this result is straightforward: in those cases, all labor is allocated to either the downstream sector (when $\alpha = 1$) or to the upstream sector (when $\alpha = 0$), and because firm-level output is always efficient (see Proposition 1), there is no scope for a market inefficiency.

2.3 Optimal Policy

To analyze how a government can restore efficiency, suppose that it has the ability to provide production subsidies (or charge production taxes). We denote these taxes by s^d and s^u in the downstream and upstream sectors, respectively, and assume that subsidy proceeds are extracted from households (or tax revenue is rebated to households) in a lump-sum manner.

In Appendix A.3 we show that downstream subsidies s^d have no impact on the market allocation, while the following implementation result applies:

Proposition 2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $(s^u)^* = 1/\theta$.

This upstream subsidy serves the role of increasing the size of the upstream sector. To do so, it must reallocate labor upstream, and in our two-sector model, this necessarily draws labor from downstream firms. While increasing labor upstream clearly raises upstream output, notice that the optimal upstream subsidy also increases downstream output, despite the fact that the sector now employs less labor. This is due to the increasing returns to scale upstream. By increasing the size of the upstream sector, the optimal subsidy also raises its efficiency, which provides the downstream sector with more inputs such that it also grows.

Crucially, the optimal amount of labor reallocation from the downstream to the upstream sector depends only on the upstream elasticity of substitution. Intuitively, the only 'net' source of inefficiency in our framework is the vertical spillover from entry upstream to productivity downstream, and this effect is mediated by upstream substitutability (see also Appendix A.4). Because our downstream sector captures all consumer spending regardless of downstream substitutability, downstream subsidies are redundant instruments in our framework, and the upstream subsidy is independent of the downstream elasticity of substitution, σ.

In Appendix A.5, we briefly develop two extensions of the simple model in this section. First, we allow the upstream sector to use the same bundle of inputs Q^u used in the final-good sector, while letting labor intensity upstream (denoted by β) differ from that downstream. Second, we outline a multi-stage extension of the model, in which the input bundle used in upstream production aggregates varieties from a yet more upstream sector, which in turns uses inputs from an even more upstream sector, and so on. In both extensions, we show that efficiency again calls for the use of subsidies in all input sectors, but not in the most downstream sector.[13]

Having explored the equilibrium and efficiency properties of a "Krugman" closed economy with an input sector, we next turn to exploring the open-economy implications of this framework. We are particularly interested in shedding light on the consequences of trade protection in both upstream and downstream sectors, and also on the optimal design of these trade policies. Beyond the double-marginalization and vertical-spillovers mechanism discussed above, the determination of

[13] Unlike in the work of Liu (2019), we do not find that optimal subsidies should necessarily be monotonically increasing in the upstreamness of a sector. The reason for this is that, unlike in Liu's work, we solve for the first-best *vector* of subsidies: when the government can set subsidies in only one sector, the size of the subsidy is indeed higher, the more upstream the sector.

trade policies in our framework is shaped by both terms-of-trade considerations (as in Neoclassical Trade Theory), as well as relocation effects (as in New Trade Theory).

3 Open Economy Equilibrium: A Useful Isomorphism

We now consider a two-country extension of our two-sector model, which allows for (costly) international trade in both final goods and intermediate inputs. Throughout the rest of the paper, whenever the downstream sector uses labor and inputs (i.e., $\alpha \in (0,1)$), we focus on equilibria with incomplete specialization.

3.1 Environment with Internal Economies of Scale

There are now two countries (Home and Foreign), indexed by i or j (and sometimes by H and F), each populated by L_i consumers/workers. Trade is costly due to the presence of both iceberg trade costs and import tariffs. We denote the symmetric iceberg trade costs that apply to final goods and inputs by $\tau^d > 1$ and $\tau^u > 1$, respectively, and we denote the tariffs set by country i on imports of final goods and intermediate inputs by t_i^d and t_i^u, respectively.

We also consider additional instruments, namely domestic production subsidies (s_i^d and s_i^u) and export taxes (v_i^d and v_i^u) in both sectors. Prior work has shown how these combined instruments can be used to maximize social welfare, and we show that the same is true in our setting. Since our goal is to analyze if and when tariff escalation may arise as a real-world, welfare-maximizing policy, however, we are particularly interested in implementations that use instruments that are actually available to governments. As such, considering cases without export taxes is important, since they are rarely used in practice, and in fact are expressly prohibited in some countries, such as the United States (see U.S. Constitution, Article 1, Section 9, Clause 5). Similarly, studying situations without production subsidies is important for understanding observed policies since the WTO Agreement on Subsidies and Countervailing Measures ('SCM Agreement') significantly limits governments use of such instruments without punishment. We also note that, perhaps due to these institutional constraints, ruling out production subsidies is a widespread practice in theoretical analyses of trade policy determination in which production subsidies could well improve welfare, as exemplified in the work of Grossman and Helpman (1994) or Ossa (2011), among many others.

Denoting country $i \in \{H, F\}$ variables with i subindices, technologies upstream and downstream are now characterized by equations

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H, F\},$$

and

$$f_i^d + x_i^d(\omega) = A_i^d (\ell_i^d(\omega))^{\alpha} Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H, F\}.$$

Because intermediate inputs are tradable, the bundle of inputs now includes both domestic and foreign input varieties:

$$Q_i^u(\omega) = \left[\sum_{j\in\{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right) \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H, F\}.$$

The representative consumer in country i derives utility according to:

$$U_i = \left[\sum_{j\in\{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}}, \qquad \sigma > 1, \quad i \in \{H, F\}, \tag{11}$$

where M_j^d is the endogenous measure of firms in country j. Implicit in these equations is the fact that because trade costs are all ad-valorem and preferences are CES, all firms in all sectors find it profitable to sell in both markets. As in our closed-economy model, market structure in both sectors and both countries is characterized by monopolistic competition and free entry.

As mentioned above, the government imposes tariffs on imports of both final goods and inputs. Given symmetry across firms, the tariff revenue collected by the government is rebated to households via lump-sum transfers in an amount

$$R_i = \frac{t_i^d}{1+t_i^d} M_j^d p_{ji}^d q_{ji}^d + \frac{t_i^u}{1+t_i^u} M_i^d M_j^u p_{ji}^u q_{ji}^u + \frac{v_i^d}{1-v_i^d} M_i^d \tilde{p}_{ij}^d q_{ij}^d + \frac{v_i^u}{1-v_i^u} M_j^d M_i^u \tilde{p}_{ij}^u q_{ij}^u, \tag{12}$$

where p_{ji}^d and p_{ji}^u are the prices paid by consumers in i for final goods and by firms in i for inputs, and where $\tilde{p}_{ij}^d$ and $\tilde{p}_{ij}^u$ are the prices collected by producers in i when selling final goods and inputs in country j. When the government also levies production subsidies, this government balance condition needs to be modified in a straightforward manner.

3.2 An Isomorphic Competitive Economy with External Economies of Scale

It is not complicated to derive the equations characterizing the equilibrium of the above two-country economy as a function of the parameters of the model and the policy choices of a given country i, which we associate with Home. In our analysis of optimal policy, however, it is much more convenient and tractable to work with the equilibrium conditions of an isomorphic competitive economy with external rather than internal economies of scale.[14] In the remainder of this section, we develop such an isomorphic characterization. We provide a derivation of the equilibrium conditions of the above 'Krugman' economy with internal economies of scale in Appendix B.1.

Consider a simpler economy in which there are only four goods: a Home final good, a Foreign final good, a Home intermediate input, and a Foreign intermediate input. Preferences in country

[14]This isomorphism is inspired by the work of Kucheryavyy et al. (2017). We thank Steve Redding and Iván Werning for the helpful suggestion to pursue this direction.

$i = \{H, F\}$ are given by

$$U\left(Q_{ii}^d, Q_{ji}^d\right) = \left(\left(Q_{ii}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{ji}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}, \tag{13}$$

where Q_{ii}^d is i's consumption of its local good, and Q_{ji}^d are imports by i of country j's good. As in our baseline model, the parameter σ governs the substitutability between the Home and Foreign goods.

The final good in each country is produced combining local labor (ℓ_i^d), the Home intermediate input (q_{ii}^u), and the Foreign intermediate input (q_{ji}^u). Technology is given by

$$x_i^d = \hat{A}_i^d F^d\left(\ell_i^d, q_{ii}^u, q_{ji}^u\right) = \hat{A}_i^d\left(\ell_i^d\right)^{\alpha}\left(\left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}(1-\alpha)},$$

where $\hat{A}_i^d$ is downstream productivity, α determines the labor intensity of final-good production, and θ governs the substitutability between the Home and the Foreign inputs. Downstream productivity $\hat{A}_i^d$ is in turn given by

$$\hat{A}_i^d = \bar{A}_i^d\left(F^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right)\right)^{\gamma^d} = \bar{A}_i^d\left(\left(L_i^d\right)^{\alpha}\left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d}, \tag{14}$$

and is thus an endogenous function of (i) country i's aggregate allocation of labor L_i^d to the downstream sector, (ii) its aggregate use of country i's intermediate input, and (iii) its aggregate use of country j's intermediate input. The parameter γ^d governs the degree of external economies of scale in the downstream sector, and is often referred to as the scale elasticity of this sector.

The intermediate input in each country is produced using local labor ℓ_i^u according to

$$x_i^u = \hat{A}_i^u F_i^u\left(\ell_i^u\right) = \hat{A}_i^u \ell_i^u,$$

where upstream productivity is also endogenous and given by

$$\hat{A}_i^u = \bar{A}_i^u\left(L_i^u\right)^{\gamma^u}, \tag{15}$$

where L_i^u is country i's aggregate allocation of labor to the upstream sector, and where the parameter γ^u governs the scale elasticity of the upstream sector.

We assume that the above technologies are available to a competitive fringe of producers in each country and sector. These producers take prices of all goods as given, and do not internalize the effects of their choices on the productivity terms $\hat{A}_i^d$ and $\hat{A}_i^u$. Given symmetry, it should be clear that in equilibrium, $\ell_i^d = L_i^d$, $\ell_i^u = L_i^u$, $q_{ii}^u = Q_{ii}^u$, and $q_{ji}^u = Q_{ji}^u$ for all firms. We can thus ignore lower-case variables hereafter.

The key conditions characterizing the decentralized market equilibrium in a given country i

consist of four resource constraints and four optimality conditions. The four resource constraints are (i) an aggregate labor market constraint

$$L_i = L_i^u + L_i^d, \tag{16}$$

(ii)-(iii) two equations equating output produced in each sector to its use (for domestic consumption or for export)

$$\begin{aligned} \hat{A}_i^u L_i^u &= Q_{ii}^u + Q_{ij}^u; & (17) \\ \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left((Q_{ii}^u)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}} \right)^{\frac{\theta(1-\alpha)}{\theta-1}} &= Q_{ii}^d + Q_{ij}^d, & (18) \end{aligned}$$

and (iv) a trade-balance condition

$$P_{ji}^d Q_{ji}^d + P_{ji}^u Q_{ji}^u = P_{ij}^d Q_{ij}^d + P_{ij}^u Q_{ij}^u. \tag{19}$$

In this last condition, the prices P_{ji}^d and P_{ji}^u reflect import prices collected by foreign exporters, so the domestic (country i) prices paid by the buyers of those goods are $(1+t_i^d)P_{ji}^d$ and $(1+t_i^u)P_{ji}^u$, respectively, where remember that t_i^d and t_i^u are the import tariffs set by country i. Similarly, the export prices P_{ji}^d and P_{ji}^u in the trade balance condition (19) correspond to the prices paid by foreign buyers, so the domestic (country i) price collected by the sellers of those goods are $(1-v_i^d)P_{ij}^d$ and $(1-v_i^u)P_{ij}^u$, respectively, where remember that v_i^d and v_i^u denote country i's downstream and upstream export taxes.

Turning to the four optimality conditions characterizing the decentralized equilibrium, the first two equations simply equate the marginal rate of substitution in final-good and intermediate-input consumption to the domestic (country i) relative price faced by the buyers of these goods, or

$$\begin{aligned} \frac{U_{Q_{ii}^d}\left(Q_{ii}^d, Q_{ji}^d\right)}{U_{Q_{ji}^d}\left(Q_{ii}^d, Q_{ji}^d\right)} &= \frac{\left(1-v_i^d\right)}{\left(1+t_i^d\right)} \frac{P_{ij}^d}{P_{ji}^d}; & (20) \\ \frac{F_{Q_{ii}^u}^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right)}{F_{Q_{ji}^u}^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right)} &= \frac{(1-v_i^u)}{(1+t_i^u)} \frac{P_{ij}^u}{P_{ji}^u}. & (21) \end{aligned}$$

In these equations, subindices on the functions U and F^d denote partial derivatives of these functions with respect to the argument in the denominator. The next optimality condition ensures the equality between the benefits of exporting the domestic intermediate input to the benefits of using that amount of domestic inputs to produce an additional amount of the final good that is in turn exported:

$$\hat{A}_i^d F_{Q_{ii}^u}^d\left(L_i^d, Q_{ii}^u, Q_{ji}^u\right) = \frac{(1-v_i^u)\, P_{ij}^u}{(1-v_i^d)\, P_{ij}^d}. \tag{22}$$

The final efficiency condition equates the marginal product of labor in both sectors in terms of a common good (i.e., the final good)

$$F^d_{L^d_i}\left(L^d_i, Q^u_{ii}, Q^u_{ji}\right) = \hat{A}^u_i F^u_{L^u_i}\left(L^u_i\right) F^d_{Q^u_{ii}}\left(L^d_i, Q^u_{ii}, Q^u_{ji}\right). \tag{23}$$

We have developed equilibrium conditions (16) through (23) in a competitive model without meaningful firm-level decisions on entry, exporting, importing and pricing. Nevertheless, as anticipated above, in Appendix B.2 we prove that our baseline Krugman-style model with internal economies of scale and imperfect competition delivers the exact same set of equilibrium conditions for an appropriate choice of the primitive productivity terms $\bar{A}^d_i$ and $\bar{A}^u_i$ in equations (14) and (15), and as long as the scale elasticities γ^d and γ^u are set to $\gamma^d = 1/(\sigma - 1)$ and $\gamma^u = 1/(\theta - 1)$, respectively.

We summarize this discussion as follows (the proof is in Appendix B.2):

Proposition 3. The decentralized equilibrium of the two-country model in Section 3.1 featuring internal scale economies, product differentiation, and monopolistic competition can be reduced to a set of equations identical to equations (16) through (23) applying to the competitive model with external economies of scale developed in this section.

4 Optimal Trade Policy for a Small Open Economy with No Domestic Distortions

In this section, we consider the impact and optimal design of trade policies upstream and downstream for the special case in which the Home country is a small open economy, and in which the downstream sector does not employ labor (i.e., $\alpha = 0$). The first assumption allows us to ignore any impact of Home policies on aggregate, world price indices (though note that Home still faces a downward sloping demand curve for its differentiated products). The second assumption implies that the allocation of labor across sectors is necessarily efficient, since it is all employed upstream (see Proposition 1) and independent of trade-policy choices. Furthermore, because firm-level output levels are also socially efficient (see Appendix B.1), trade policies can only affect the measure of final-good producers that enter in each country, and countries' relative wages. This allows us to compare our results more cleanly to those in the important contributions of Gros (1987), Venables (1987), and Ossa (2011), which feature "horizontal" models without an input sector. As in those frameworks, a combination of trade taxes applied to imports or exports is sufficient to implement the first-best allocation, so domestic subsidies are redundant, as we show later in this section.

We focus on a small, open economy because, despite the simplification on the labor side, characterizing optimal trade policy is still involved. This relatively simple example allows us to derive analytic solutions for the first- and second-best optimal policies, and to highlight the key role of the degree of scale economies in the downstream sector for our results. In addition, the solutions we derive here are an extremely good approximation for our quantitative findings in Section 6.

We proceed in two steps. First, we consider the (unrestricted) set of trade policies that implement the first-best in our small open economy. Because such an optimal mix necessarily involves export taxes, and often these are not available to governments (e.g., they are forbidden in the US Constitution), we also analyze the choice of second-best policies when only import tariffs are allowed. As explained in Section 3.2, we conduct our analysis based on our isomorphic competitive economy featuring external economies of scale because this greatly simplifies the derivations.

4.1 First-Best Policies

To solve for the first-best policies, we closely follow the primal approach in Costinot et al. (2015).[15] More specifically, we first consider an environment in which a (fictitious) social planner directly controls consumption and output decisions, and we derive three key conditions characterizing the structure of the optimal allocation. We then compare these conditions to those derived from a decentralized market equilibrium in which the government imposes trade taxes (see Section 3.2), and we finally show how the optimal allocation can be implemented through a simple combination of these taxes. We later consider the case in which the government also has access to domestic policy instruments, such as production or consumption subsidies.

A. Optimal Allocation

Consider the problem of a Home social planner who seeks to maximize welfare in equation (13), subject to the labor-market constraint (16), the output-market constraints (17) and (18), and the trade balance condition (19). The planner is assumed to control domestic consumption (Q^d_{HH}, Q^u_{HH}), imports (Q^d_{FH}, Q^u_{FH}) and exports (Q^d_{HF}, Q^u_{HF}) of both final goods and intermediate inputs. Based on the recasting of our model in Section 3.2, for the case $\alpha = 0$, this problem reduces to choosing $\left\{Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}\right\}$ to

$$\begin{aligned} \max \quad & U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \\ s.t. \quad & \hat{A}^u_H\left(L_H\right)L_H = Q^u_{HH} + Q^u_{HF} \\ & \hat{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)F^d\left(Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\ & P^d_{FH}Q^d_{FH} + P^u_{FH}Q^u_{FH} = Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}}P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)$ and $\hat{A}^u_H\left(L_H\right)$ are given in equations (14) and (15) for $\alpha = 0$, respectively, and where

$$F^d\left(Q^u_{HH}, Q^u_{FH}\right) = \left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}. \tag{24}$$

The first two constraints in the program above simply equate the output of each sector to its uses (domestic consumption or exports). The third constraint, a trade balance constraint, requires a bit more explanation. First, notice that this equation is derived from (19), after substituting

[15] See Costinot et al. (2020) and Kortum and Weisbach (2021) for other recent applications of this approach.

$P_{HF}^{d} = P_{FF}^{d}\left(Q_{HF}^{d}/Q_{FF}^{d}\right)^{-1/\sigma}$ and $P_{HF}^{u} = P_{FF}^{u}\left(Q_{HF}^{u}/Q_{FF}^{u}\right)^{-1/\theta}$, which correspond to Foreign's inverse demand for the Home final and intermediate-input goods, respectively.[16] Although we have assumed that Home is a small open economy, the fact that it produces *differentiated* final goods and *differentiated* intermediate inputs still confers some market power to the Home government, since it perceives a downward sloping demand for its goods. Second, it may seem non-standard to introduce prices in the constraint of a planner problem, but this is precisely where our assumption of Home being a small open economy is useful. More specifically, we assume that Home is small in the sense that its policy choices have *no impact* on Foreign's domestic prices P_{FF}^{d} and P_{FF}^{u}, or on the prices P_{FH}^{d} and P_{FH}^{u} (before import tariffs) collected by Foreign exporters. As a result, the Home government treats these prices as parameters in the planner problem above.

Working with the first-order conditions of this problem (see Appendix C.1), we characterize the first-best allocations via the three following conditions. First, on the consumption side, the Home social planner seeks to equate the representative consumers' marginal rate of substitution with the *social* relative cost of domestic versus foreign goods

$$\frac{U_{Q_{HH}^{d}}\left(Q_{HH}^{d}, Q_{FH}^{d}\right)}{U_{Q_{FH}^{d}}\left(Q_{HH}^{d}, Q_{FH}^{d}\right)} = \frac{\frac{\sigma-1}{\sigma} P_{HF}^{d}}{P_{FH}^{d}}. \tag{25}$$

Note that the private cost of consuming domestic goods (which is equal to the opportunity cost P_{HF}^{d} of exporting these goods) exceeds its social cost. This wedge reflects a fairly standard rationale for terms-of-trade manipulation. In particular, for a given level of final-good production, an increase in domestic consumption Q_{HH}^{d} necessarily reduces exports, and this in turn raises Home's export prices and thus improves its terms of trade, even when Home is a small open economy (see Gros, 1987). Raising the private cost of imported goods or decreasing the private benefit of exporting goods by a factor $\sigma/(\sigma-1)$ restores the equality of the *relative* private and social costs of domestic and foreign goods.[17]

The second key efficiency condition is analogous to equation (25) and equates the marginal rate of substitution between domestic and foreign inputs in the production of final goods to the ratio of social costs of these inputs, or

$$\frac{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u}, Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u}, Q_{FH}^{u}\right)} = \frac{\frac{\theta-1}{\theta} P_{HF}^{u}}{P_{FH}^{u}}. \tag{26}$$

The ratio of social costs of these inputs is again distinct from the ratio of their private costs. The

[16] These equations can in turn be derived based on the optimality conditions (20) and (21) applying when $i = F$ and $j = H$.

[17] One may wonder whether an alternative interpretation of this result relates it to the fact that the markups charged by domestic firms on domestic consumers generate profits that remain in the Home country (or, more precisely, they lead to increased labor demand via firm entry seeking to dissipate those profits), while for imported goods, markups are collected by foreign firms, and thus the private and social marginal cost of those foreign goods coincide. Since we derive our results in a competitive economy with no markups, our preferred intuition is that this condition reflects market power in export markets driven by product differentiation, and has little to do with domestic market structure or scale economies. Appendix Section C.2 provides a formal illustration of this intuition.

reason for this is analogous to the one in equation (25): the social cost of domestic inputs is lower than the private cost because the Home government perceives a downward sloping demand for its goods, and thus raising the private cost of imported inputs or decreasing the private benefit of exporting them is socially desirable because it allows Home to exploit its market power abroad.

The final efficiency condition is given by

$$\left(1+\gamma^{d}\right)\hat{A}_{H}^{d}F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)=\frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}}\frac{P_{HF}^{u}}{P_{HF}^{d}}, \tag{27}$$

and equates the benefits of exporting domestic inputs to the benefits of using them to produce final goods that are in turn exported. This third equation takes into account both the productivity enhancing effects of boosting domestic production of final goods (the first term $1+\gamma^{d}$ on the left-hand side), as well as the relative potential for Home to exploit market power in its inputs versus final goods, which is mediated by the ratio $\left(\sigma/\left(\sigma-1\right)\right)/\left(\theta/\left(\theta-1\right)\right)$ on the right-hand-side of equation (27). Note that when σ and θ go to infinity, Home's market power disappears, and Home becomes a small open economy in the traditional sense, i.e, in the sense of being unable to affect its terms of trade through relative price effects.

B. First-Best Trade Policies

We now compare these optimal allocations to those from the decentralized equilibrium in which the government can set import tariffs or export taxes, as derived in Section 3.2. It should be clear that there is a close connection between equations (20)–(22) for the decentralized equilibrium, and equations (25)–(27) characterizing the socially optimal allocations.[18]

There are two key differences between these two sets of equations. First, the market equilibrium conditions naturally incorporate the effect of taxes in shaping individual consumers' and firms' private choices. Second, these decentralized-market equations do *not* incorporate the positive impact of downstream output expansion on productivity, or the positive effect of curtailing exports of final goods or inputs on Home's terms of trade.

A simple comparison of these sets of equations indicates that a combination of import tariffs and export taxes can achieve the first-best allocation as long as it satisfies:

$$\begin{aligned}
1+t_{H}^{d} &= \left(1+\gamma^{d}\right)\left(1+\bar{T}\right); && (28)\\
1+t_{H}^{u} &= 1+\bar{T}; && (29)\\
1-v_{H}^{d} &= \frac{\sigma-1}{\sigma}\left(1+\gamma^{d}\right)\left(1+\bar{T}\right); && (30)\\
1-v_{H}^{u} &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right), && (31)
\end{aligned}$$

for any arbitrary constant such that $1+\bar{T}\geq 0$.

[18] In Section 3.2, we identified a fourth optimality condition – equation (23) – associated with the allocation of labor across sectors, but this condition is irrelevant when the downstream sector does not use labor (i.e. $\alpha=0$).

A few comments are in order. First, note that the level of optimal import tariffs and export taxes is indeterminate in our setting. This is a manifestation of Lerner's symmetry: optimal policies featuring a common ratio of gross import tariffs and export taxes (i.e., $(1+t_H^s)/(1-v_H^s)$ for $s=\{d,u\}$) deliver the exact same market allocations. Second, notice that the ratio of optimal gross import tariffs on final goods and on inputs, or the "tariff escalation wedge" *is* pinned down in our model and given by

$$\frac{1+t_H^d}{1+t_H^u}=1+\gamma^d>1.$$

Third, the optimal allocation *cannot* be achieved with only import tariffs, since implementing the first-best requires distinct export taxes downstream and upstream.[19] Fourth, noting that our isomorphism applies only when $1+\gamma^d=\sigma/(\sigma-1)$, setting $\bar{T}=0$ minimizes the set of instruments necessary to achieve the first-best. In such a case, the first-best policies involve only two instruments: a downstream import tariff at a level $t_H^d=\gamma^d=1/(\sigma-1)$ and an upstream export tax v_H^u equal to $1/\theta$. In sum, we have derived the following result:

Proposition 4. When $\alpha=0$, the first-best allocation can be achieved with a combination of import and export trade taxes. Although, the levels of trade taxes are not uniquely pinned down, the tariff escalation wedge is necessarily given by $\left(1+t_H^d\right)/\left(1+t_H^u\right)=1+\gamma^d>1$. Furthermore, under the isomorphism condition $\gamma^d=1/(\sigma-1)$ the first-best can be achieved with just a downstream import tariff t_H^d equal to $1/(\sigma-1)$ and an upstream export tax v_H^u equal to $1/\theta$.

Why do optimal policies involve higher import tariffs on final goods than on inputs? And why does the government choose to tax imports of final goods while taxing exports of inputs when using the minimum set of instruments? The key distinction between trade taxes on final goods and on inputs is as follows. A downstream import tariff or export tax shifts consumers' expenditures towards Home final-good varieties, thereby improving Home's terms of trade (Gros, 1987). The increased demand for Home's final goods in turn raises downstream productivity by expanding the size of the sector. While this mechanism is similar to prior work on relocation effects (Venables, 1987; Ossa, 2011), when $\alpha=0$ the expansion of the downstream sector is due to increased input expenditures rather than a reallocation of labor to that sector.

Although an upstream tariff similarly redirects expenditure towards Home inputs and improves its terms of trade, it also raises Home's downstream producers' costs, which reduces their output and thus efficiency. This asymmetry between final-good and input tariffs arises because inputs are sold to firms that produce under increasing returns to scale and can relocate to Foreign, whereas final-goods are sold to consumers who do not produce under increasing returns and cannot move. As a result, the Home government has a disproportionate incentive to manipulate its terms of trade in the input sector via an export tax because it shifts expenditure on inputs towards Home firms without raising downstream firms' input costs. This is clear from equations (30) and (31), which show that the incentive to use upstream export taxes is magnified by a factor $1+\gamma^d$ relative to the incentive to use downstream export taxes. In other words, the returns to scale in downstream

[19] By Lerner symmetry, export taxes are redundant only if they can be set at the same level in all sectors.

production govern the benefits from increasing the size of that sector. Because the relative size of sectoral import tariffs and export taxes is constrained by (20) and (21), this in turn manifests itself in the form of a lower import tariff upstream than downstream.

This special case of our model highlights the combined importance of three key factors in the optimality of tariff escalation: (i) downstream firms' costs depend on input costs, and thus on input tariffs, (ii) downstream production features increasing returns to scale, and (iii) downstream firms can relocate to Foreign while consumers cannot. As a result, input tariffs raise downstream firms' costs, which decreases the size of the downstream sector (as firms relocate), and thus lowers its efficiency. By contrast, final-good tariffs raise consumer costs, but do not decrease the number of consumers or affect their production efficiency (in fact, downstream tariffs raise downstream efficiency by expanding the size of the sector).[20] In the next subsection, we further demonstrate why increasing returns to scale downstream are essential for optimal policy to feature escalation by analyzing a comparable model with constant returns to scale.

C. The Case of No Scale Economies Downstream

A key advantage of our isomorphic competitive economy with external economies of scale is that when $\gamma^d \to 0$, this economy converges to a competitive economy with no scale economies.[21] Using the expressions above, it is straightforward to derive first-best trade policies in this case. Specifically, equations (28)–(31) now reduce to

$$\begin{aligned}
1+t_H^d &= 1+\bar{T}; \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right),
\end{aligned}$$

for any arbitrary constant such that $1+\bar{T} \geq 0$. It is then immediate that:

Proposition 5. When $\alpha = 0$, in the absence of scale economies downstream, the first-best can be attained with a combination of import and export taxes. Although, the levels of trade taxes are not uniquely pinned down, the tariff escalation wedge $\left(1+t_H^d\right)/\left(1+t_H^u\right)$ necessarily equals 1. Furthermore, the first-best can be achieved with just a downstream export tax at a level v_H^d equal to $1/\sigma$ and an upstream export tax v_H^u equal to $1/\theta$.

This result shows that the emergence of tariff escalation is directly tied to the existence of scale economies in the downstream sector. In their absence, we obtain a result analogous to that derived by Costinot et al. (2015) and by Beshkar and Lashkaripour (2020), namely that optimal trade policy

[20] In this special case of the model, input tariffs cannot increase upstream efficiency because the sector produces using only labor and is already employing all of it.

[21] In this setting with all labor employed upstream by assumption, there is no scope for efficiency gains in the upstream sector, and thus increasing returns upstream are irrelevant.

involves uniform import tariffs across sectors (regardless of their differentiation or whether they are inputs or final goods) and differential export taxes that optimally exploit Home's market power. Conversely, the presence or absence of scale economies in the upstream sector is irrelevant for the desirability of trade policies featuring tariff escalation in this setting.

We should briefly mention a caveat with the above argument. In particular, the isomorphism between our economies with internal economies of scale and with external economies imposes $\gamma^d \to 0 = 1/(\sigma - 1)$. So it would appear that as $\gamma^d \to 0$, we must necessarily have $\sigma \to \infty$, which would imply that the Home economy has no market power in downstream markets. Note, however, that even in such a case, the model without scale economies would still not generate tariff escalation.

D. General Functional Forms

It is interesting to note that even when $\sigma \to \infty$ and $\theta \to \infty$, so that Home ceases to have any market power in exports, our model continues to rationalize tariff escalation as long as $\gamma^d > 0$. The reason for this is that in deriving the first-best policies in equations (28)-(31) , we have not invoked the fact that Home preferences $U_H\left(Q^d_{HH}, Q^d_{FH}\right)$ in (13) or that the aggregator $F^d\left(Q^u_{HH}, Q^u_{FH}\right)$ of inputs in (24) are CES aggregators governed by σ and θ, respectively. As a result, the parameters σ and θ in the first-best policies are solely related to parameters governing preferences and technology *in Foreign*, not at Home; only the scale elasticity parameter γ^d in these formulas is associated with features of the Home economy. This reinforces our interpretation that the appearance of σ and θ in the formulas above is associated with standard terms-of-trade-manipulation incentives, rather than with the markups faced by domestic buyers.[22]

E. Alternative First-Best Implementations

A natural question about our results is whether and how domestic tax instruments, such as consumption or production subsidies, affect optimal tariff escalation. It is well understood that, in some settings, it is straightforward to replicate the effects of an import tariff using a combination of consumption taxes and production subsidies. In Appendix C.3, we analyze first-best policies when the set of instruments is expanded to include these instruments. We summarize the results here.

First, the only way to implement the first-best using *only* two domestic instruments (analogously to the two trade policy instruments in Proposition 4) is via the use of discriminatory consumption subsidies on domestic purchases of final goods and inputs. More specifically, the first-best can be achieved via a subsidy to the consumption of domestic intermediate inputs at a rate equal to $s^u_{HH} = 1/\theta$ (which coincides with the level that implements the first-best in the closed economy), and a subsidy to the consumption of domestic final goods equal to $s^d_{HH} = 1/\sigma$. These subsidies shift domestic consumption towards Home varieties, which boosts the sectors' sizes and thus their

[22] Although optimal trade policies seem to depend only on foreign demand elasticities and the extent of increasing returns in Home's downstream production, we acknowledge that the CES functional forms for $U_H\left(Q^d_{HH}, Q^d_{FH}\right)$ and $F^d\left(Q^u_{HH}, Q^u_{FH}\right)$ are crucial for the isomorphism in Section 3.2. As a result, the role of γ^d implies a role for the Home elasticity σ_H, since $\gamma^d = 1/\left(\sigma_H - 1\right)$ in that model (see Appendix C.2 for more details).

efficiencies. At the same time, the discriminatory nature of the subsidies effectively constrains Home sales in Foreign, such that firms in both sectors optimally exploit their market power abroad.

Second, an import tariff downstream is a perfect substitute for a discriminatory consumption subsidy downstream, while an export tax upstream is a perfect substitute for a discriminatory consumption subsidy upstream. It then follows that the first-best can also be achieved with a combination of a subsidy in one sector and a trade instrument in the other sector. Whether tariff escalation remains a feature of the first-best policies is sensitive to the instruments used in the implementation.

Third, when discriminatory consumption subsidies are not available – as they typically are not – the first-best *cannot* be achieved with just two instruments, unless these two instruments are trade policy instruments, as in our implementation in Proposition 4. If the government chooses to rely on non-discriminatory production or consumption subsidies, it can achieve the first-best combining an appropriate level of these non-discriminatory subsidies with either export taxes or import tariffs in both sectors to offset the portion of the subsidy that accrues to Foreign firms or consumers. In particular, production subsidies require export taxes to offset the portion of the subsidy that accrues to Foreign, while consumption subsidies require import tariffs to offset the subsidy to Home consumers' and firms' purchases of Foreign goods. The implied tariff escalation level is naturally sensitive to which precise instruments are used in the implementation of the first-best. To reiterate, however, any implementation of the first-best involving a domestic subsidy requires at least three tax instruments, rather than just two, as in Proposition 4. In addition, it is the use of those redundant subsidies themselves that motivates the use of tariffs that do not feature escalation, rather than the underlying structure of the economy.

4.2 Second-Best Import Tariffs

We now consider an environment in which the only policies available to the Home government are import tariffs on final goods and on inputs. As shown in the previous sections, these tariffs are not sufficient to achieve the first-best allocation, so a natural question is whether second-best import tariffs continue to feature tariff escalation.

A. Second-Best Import Tariffs with Scale Economies

For the case of a small open economy, the primal approach developed in Costinot et al. (2015) is again very useful to characterize the second-best allocation and how they can be implemented with only import tariffs. The second-best optimal allocation seeks to solve the same problem laid out in Section 4.1, expanded to include an additional constraint. Specifically, while the Home planner can always ensure that the optimality conditions (25) and (26) are satisfied via an appropriate choice of import tariffs, the same is not true about the optimality condition (27). To see this, note that absent export taxes ($v_H^d = v_H^d = 0$), equation (22) reduces to

$$\hat{A}_H^d F_{Q_{HH}^u}^d \left(Q_{HH}^u, Q_{FH}^u\right) = \frac{P_{HF}^d}{P_{HF}^u} = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}, \tag{32}$$

which cannot be affected directly via import tariffs. As in the first-best case, the Home government will internalize (but private agents will not) the fact that the ratio of export prices is shaped by Home's relative export supply of inputs and final goods.

In Appendix C.4, we work with the first-order conditions of the planner problem, compare them those applying to a decentralized market equilibrium with only import tariffs – which corresponds to equations (20)–(22) setting $v_H^d = v_H^u = 0$ – and establish that:

Proposition 6. When $\alpha = 0$, the second-best optimal combination of import tariffs involves an import tariff on final goods t_H^d higher than $1/(\sigma - 1)$ and a tariff escalation wedge larger than the first-best one, so $(1 + t_H^d)/(1 + t_H^u) > 1 + \gamma^d = \sigma/(\sigma - 1) > 1$.

To understand the intuition behind this result, it is useful to remember the role that export taxes serve in the set of first-best trade policies. First, export taxes are essential for manipulating differential terms-of-trade effects in final-good versus input markets. Second, upstream export taxes also provide a tool to manipulate the terms of trade for inputs in a less distortionary way (with respect to the size and productivity of the downstream sector) than upstream import tariffs. In the absence of export taxes, the Home government finds it optimal to use import tariffs on inputs to manipulate its terms of trade upstream (since it cannot rely on export taxes to do so), but the motive to do so is attenuated for inputs relative to final goods. This is for precisely the same reason that tariff escalation is present in optimal first-best trade policy. Input tariffs raise final-good producers' costs, which shrinks the size of the sector, and thus lowers its efficiency. As a result, even in the second-best without access to export taxes or subsidies, the tariff escalation wedge $(1 + t_H^d)/(1 + t_H^u)$ remains above one, and is in fact higher than in the first-best, as a higher downstream tariff is now necessary to compensate for the negative impact of upstream import tariffs on entry in the Home downstream sector.

B. Second-Best Import Tariffs with No Scale Economies

It is also instructive to characterize second-best import tariffs in the absence of scale effects. Characterizing these policies is again quite straightforward, since we need only consider the case when $\gamma^d \to 0$ in our competitive economy with external scale economies. In Appendix C.4, we prove the following result:

Proposition 7. In the absence of scale economies, the second-best optimal combination of import tariffs involves tariff escalation (i.e., $\left(1 + t_H^d\right)/(1 + t_H^u) > 1$) if and only if $\sigma > \theta$.

To understand this result, it is useful to focus on the case $\sigma = \theta$. In a competitive Ricardian model, as long as optimal export taxes are common across sectors (i.e., $\sigma = \theta$ in our setting), the first-best can be implemented via either export taxes or import tariffs (Costinot et al., 2015; Beshkar and Lashkaripour, 2020). As a result, second-best import tariffs are sufficient to achieve the first-best allocations, and will necessarily be equal across sectors (regardless of their upstreamness). In such a case, this competitive model with no scale effects deliver *no* tariff escalation.

Starting from this benchmark, when $\sigma \neq \theta$, the first-best can no longer be implemented with only import tariffs. Beshkar and Lashkaripour (2017) show that in a horizontal economy without vertical links across sectors, second-best import tariffs continue to be common across sectors. In the presence of vertical links, Beshkar and Lashkaripour (2020) instead point out that import tariffs can partly mimic the effects export taxes by raising the relative price of downstream sectors. If the planner would like to set a higher export tax in one sector relative to another sector, it can adjust the relative size of the second-best import tariff on inputs to achieve the desired differential terms-of-trade manipulation. When $\sigma < \theta$, the desired export tax is higher downstream, so the government will actually implement tariffs featuring higher tariffs upstream (i.e., tariff de-escalation). Conversely, when $\sigma > \theta$, the planner would prefer a lower export tax downstream, which can be partly achieved by setting a lower tariff on intermediate inputs (i.e., tariff escalation), since this reduces the relative price of exports of the downstream good.[23]

5 Optimal Trade Policy for a Small Open Economy with Domestic Distortions

In this section, we consider environments in which final-good production uses both inputs and labor. As a result, the two sectors compete for workers, and as we show in Section 2, the intersectoral allocation of labor in the decentralized equilibrium is inefficient because too little labor is allocated to the upstream sector. This labor misallocation naturally has ramifications for the set of first-best policies – as trade taxes are no longer sufficient to achieve the optimal allocation – and also for the second-best import tariffs, since tariffs will now be used to alleviate this inefficiency.

5.1 First-Best Policies

We begin by studying the optimal structure of first-best policies. As in Section 4, we follow the primal approach in Costinot et al. (2015) and first characterize the optimal allocation, which we then show how to implement via trade taxes and domestic instruments. Because many of the derivations are analogous to those in Section 4, we relegate details to Appendix D.1.

Determining the optimal allocation in this setting is analogous to the problem in Section 4.1, except that (i) the planner also controls the allocation of labor across sectors subject to a labor-market constraint, and (ii) productivity in *both* sectors is endogenous and shaped by the allocation of labor to each sector. More precisely, the planner chooses $\left\{L_H^u, L_H^d, Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u\right\}$

[23] Interestingly, Proposition 7 continues to hold unaltered when $\alpha > 0$, which is the reason why its statement does not impose the proviso $\alpha = 0$ (see Section 5).

to

$$\max \quad U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned} s.t. \quad & L^u_H + L^d_H = L_H \\ & \hat{A}^u_H\left(L^u_H\right)L^u_H = Q^u_{HH} + Q^u_{HF} \\ & \hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\ & P^d_{FH}Q^d_{FH} + P^u_{FH}Q^u_{FH} = Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}}P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{HF}\right)\right)$ and $\hat{A}^u_H\left(L^u_H\right)$ are given in (14) and (15), respectively, and where

$$F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(L^d_H\right)^{\alpha}\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}.$$

As we show in Appendix D.1, manipulating the first-order conditions of this problem produce three optimality conditions identical to those in equations (25)-(27), except that L^d_H now appears as an argument of the partial derivative terms associated with the function $F^d(\cdot)$ in equations (26) and (27). More substantively, the optimal allocation now also includes the following fourth optimality condition,

$$F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(1+\gamma^u\right)\hat{A}^u\left(L^u_H\right)F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right), \tag{33}$$

which equates the social value of the marginal product of labor in both sectors in terms of a common good (i.e., the final good). More specifically, the left-hand-side includes terms associated with the social marginal product of directly allocating labor to the production of final goods, while the right-hand-side contains terms related to the social marginal product of allocating labor to the upstream sector, and then using the resulting intermediate inputs to increase the production of final goods.[24]

We next compare these optimal allocations to those from a decentralized equilibrium in which the government can set taxes or subsidies on all transactions. In Section 4.1, and in particular, in equations (20)–(22), we show how trade taxes affect the market-equilibrium analogues of conditions (25)–(27). Because condition (33) is an internal optimality condition involving only domestic transactions, trade taxes cannot possibly affect it. In fact, the *only* type of policy instruments that can affect it are taxes or subsidies affecting the production or consumption of domestic inputs. In particular, denoting the subsidy for intermediate inputs by s^u_H, the market-equilibrium analogue of equation (33) is

$$F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \frac{1}{1-s^u_H}\hat{A}^u\left(L^u_H\right)F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right). \tag{34}$$

[24]The left-hand-side and right-hand-side of (33) do not *exactly* capture the social marginal return to labor because we have cancelled terms capturing the endogenous increase in productivity associated with the expansion of the final-good sector (see Appendix D.1).

Comparing equations (33) and (34), it is clear that the implementation of the optimal allocation necessarily requires an upstream subsidy equal to

$$s_H^u = \frac{\gamma^u}{1+\gamma^u} = \frac{1}{\theta},$$

which is identical to the optimal subsidy in the closed-economy version of our model (see Proposition 2).

How does the use of this upstream domestic subsidy affect the nature of the other first-best policies? As in our analysis in Section 4, we first focus on the case in which the government minimizes the use of non-trade taxes. Because downstream domestic subsidies are redundant instruments in our model, we initially rule them out (though we will consider them below). If the government has access to an upstream production subsidy s_H^u, we show in Appendix D.2 that the first-best instruments implementing the social optimality conditions (25)–(27) must satisfy the exact same conditions (28)–(31) as in the case when $\alpha = 0$, namely

$$\begin{aligned} 1+t_H^d &= \left(1+\gamma^d\right)\left(1+\bar{T}\right); \\ 1+t_H^u &= 1+\bar{T}; \\ 1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\ 1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right), \end{aligned}$$

for any arbitrary constant such that $1+\bar{T} \geq 0$. As a result, we can conclude that:

Proposition 8. When $\alpha > 0$, the first-best allocation can be achieved with a production subsidy for inputs, and (at least two) trade taxes associated with a tariff escalation wedge $(1+t_H^d)/(1+t_H^u) = 1+\gamma^d = \sigma/(\sigma-1) > 1$. Furthermore, the first-best can be achieved with just an upstream production subsidy s_H^u equal to $1/\theta$, a downstream import tariff at a level t_H^d equal to $1/(\sigma-1)$, and an upstream export tax v_H^u equal to $1/\theta$.

In words, once the domestic distortion identified in the closed-economy version of our model is corrected using an upstream production subsidy, the first-best can be attained with the same trade instruments used in the simpler case in which all labor is employed upstream and there are no domestic distortions ($\alpha = 0$). As a result, first-best policies continue to feature tariff escalation for the same reasons explained in Section 4.

Alternative Implementations As described in the introduction, the first-best can also be achieved using domestic subsidies. To provide further intuition on the differing forces behind final-good versus input tariffs, we analyze the range of domestic policies that can achieve the first-best in Appendix D.2 and summarize the results here.

First, the simplest way to achieve the first-best allocation is by using a *discriminatory* upstream consumption subsidy equal to $1/\theta$, and a comparable discriminatory downstream subsidy equal to

$1/\sigma$. Although discriminatory subsidies are generally illegal under WTO rules precisely because they act as trade barriers, this implementation highlights the government's objective to shift Home consumption towards its own varieties, which boosts each sector's size and thus its productivity. In this implementation, there is no need for tariffs and thus no measure of escalation.

Second, and crucially for understanding the distinct motives for upstream versus downstream tariffs, a downstream tariff is a perfect substitute for this discriminatory subsidy, whereas an upstream tariff is *not*. In other words, Home can maximize social welfare using a final-good tariff equal to $1/(\sigma-1)$, and a discriminatory consumption subsidy on inputs equal to $1/\theta$. A comparable input tariff will not achieve the first-best. Although it also shifts expenditure towards Home inputs, which increases the sector's size and efficiency, in our GE framework it does so via increased labor demand upstream and thus higher wages. These higher wages raise downstream firms' costs, which leads them to relocate to Foreign thereby reducing the size and efficiency of the downstream sector, and thus welfare.[25] In this implementation, tariff escalation is again $1+\gamma^d = \sigma/\left(\sigma-1\right) > 1$, as in Proposition 8 above.[26]

Third, if the government cannot use discriminatory subsidies, the first-best can also be achieved using a combination of production subsidies *and* export taxes. This implementation is studied in Lashkaripour and Lugovskyy (2021), who argue that optimal tariffs are uniform, regardless of scale economies and input-output relationships. Their argument also applies in our setting. A downstream production subsidy equal to $1/\sigma$ and export tax equal to $1/\sigma$, along with an upstream production subsidy equal to $1/\theta$ and an export tax equal to $1/\theta$ are sufficient to achieve the first-best, and there is in fact no welfare motive for tariffs of any size, and thus no measure of escalation.[27] Crucially, and in-line with the intuition above, a final-good tariff is a perfect substitute for the combined final-good subsidy and export tax. By contrast, an input tariff is never sufficient to satisfy the constraints in equation (34). In other words, and as captured in Proposition 8, the first-best allocation requires a combined production subsidy and export tax upstream, while a downstream tariff is sufficient. In this implementation, tariff escalation is again as defined in Proposition 8. Even in cases in which the government has access to a full range of instruments, the set of required instruments is always minimized when downstream domestic subsidies are *not* used, in which case achieving the first-best entails optimal tariffs that feature tariff escalation.

The Case of No Scale Economies As in Section 4, we analyze optimal policy in the absence of scale economies. This simply amounts to setting $\gamma^d = \gamma^u = 0$. In such a case, it is straightforward to verify that Proposition 5 continues to apply: although, the levels of first-best trade taxes are not uniquely pinned down, the tariff escalation wedge $\left(1+t_H^d\right)/\left(1+t_H^u\right)$ necessarily equals one, and

[25] Note that the final-good tariff not only increases labor demand, but also increases input demand, which counterbalances its higher wage effects.

[26] Note also that an upstream export tax can no longer replicate the effects of an upstream domestic discriminatory subsidy when $\alpha > 0$. Only the latter instrument can ensure that condition (33) is satisfied in the decentralized equilibrium.

[27] By Lerner Symmetry, they can be set to any uniform level as long as the production subsidies and export taxes are adjusted to cancel out the tariffs' impact.

the first-best can be achieved with just two export taxes. In fact, for optimal tariffs to be uniform, it is sufficient to set $\gamma^d = 0$, again highlighting the crucial role of downstream scale economies for generating tariff escalation.

5.2 Second-Best Trade Policies

We now analyze the more realistic case in which the Home government only has access to import tariffs. All implementations of the first-best in the previous section involved at least an upstream subsidy and an export tax, so it follows that the first-best *cannot* be achieved using tariffs alone. Furthermore, in the absence of subsidies and export taxes, import tariffs will seek to mimic the role they played in the first-best implementation.

As in Section 4, it is straightforward to see that when upstream export taxes are ruled out, the Home government seeks to manipulate its terms of trade via upstream import tariffs. Second-best policies thus involve positive upstream import tariffs. Nevertheless, and as formalized in Proposition 6, this force is not sufficient on its own to undo the desirability of tariff escalation.

When both sectors use labor ($\alpha > 0$), however, the upstream sector's size and thus productivity are directly affected by the amount of labor it employs. As a result, the Home government can increase upstream efficiency, and potentially welfare, by shifting domestic expenditure towards Home inputs using an input tariff. The size of the downstream labor share generates two opposing forces on the welfare effects of this efficiency gain. On the one hand, the difference between the amount of labor the social planner would like to allocate upstream versus the amount used upstream in the competitive market is increasing in the final-good sector's labor share, as reflected by the difference between equations (9) and (10). A high labor share thus generates a stronger subsidy (and hence input tariff in the second-best) motive. On the other hand, as the downstream labor share rises, inputs are relatively less important for final-good output, such that the gains from a larger and thus more efficient input sector are smaller.

These countervailing forces have precluded us from obtaining an analytical solution to the second-best policies. We therefore assess the prevalence of tariff escalation as a welfare-maximizing policy in this more realistic, second-best setting by computing numerical solutions to the planner problem for a wide range of parameter values. For the key elasticities of substitution across final goods (σ) and inputs (θ), we consider discrete values from 2 to 8. Similarly, we solve the model using a labor share for downstream production (α) ranging from 0 to 0.9. We obtain solutions for 92 percent of these 490 cases.[28] The cases for which we cannot obtain a solution tend to have a high labor share ($\alpha \approx 0.90$) or low values of the downstream elasticity ($\sigma \approx 2$). Technical details and a full discussion of this analysis are presented in Appendix E.1.

The model delivers tariff escalation in 91 percent of the solved cases. The average value of tariff escalation is 1.25 and the median is 1.16. To understand why tariff escalation tends to maximize

[28] Solving for optimal tariffs in this second-best setting is quite involved since we need to provide values for import prices, export demand shifters, and productivities in both sectors. We construct these using guidance from our quantitative analysis in Section 6 (see Appendix E.1 for details).

welfare in this second-best setting, we analyze how it varies across different values of the downstream labor share and the upstream and downstream elasticities.

Figure 2 plots the resulting tariff escalation in this second-best setting as a function of the labor share for the range of upstream elasticities, and delivers two key messages. First, there is a strong, negative relationship between tariff escalation and the downstream labor share. This is due to the fact that the difference between the social planner's first-best labor allocation upstream versus the competitive market's allocation is increasing in α (see equations (9) and (10) in Section 2). When the planner cannot address this inefficiency directly with an upstream subsidy, she instead relies on an input tariff. The tariff shifts Home expenditure towards domestic inputs, which increases the upstream sector's size and thus its efficiency. Notice that unlike for the downstream sector, which can grow by accessing more/cheaper inputs, the upstream tariff can only expand via increased use of labor.[29] Second, and directly related to the efficiency gains from reallocating labor upstream, tariff escalation is less likely when the upstream elasticity is lower, which in our isomorphic model implies more returns to scale upstream. This result is precisely in-line with the fact that in the second-best, an input tariff now substitutes for an input subsidy by increasing the size of the upstream sector. When the returns to scale upstream are higher, this motive for an input tariff increases. When $\sigma = 5$, $\theta =$, 2, 3, or 4 and $\alpha = 0.9$, the model indicates that tariff de-escalation is optimal. Though we hold the downstream elasticity in this figure constant at 5, the same patterns are evident using the full range of values from 2 to 8 (see Appendix Figure E.1).

Figure 3 depicts the importance of the relative sizes of the downstream versus upstream elasticities, again for different values of the downstream labor share. Consistent with the role of downstream returns as a motive for tariff escalation, the extent of optimal escalation is increasing in the relative size of downstream versus upstream returns to scale. In fact, tariff escalation is always optimal when the downstream returns are higher than the upstream returns. By contrast, tariff de-escalation may be optimal when the upstream returns are larger. As explained above, this is because an input tariff shifts Home expenditure towards domestic inputs, which raises the size of the sector and thus its efficiency. As evident in Figure 3, this efficiency motive is strongest for higher values of α. In this case, the social planner has a larger motive to reallocate labor from downstream to upstream production, and a higher input tariff can help with this objective in the absence of subsidies.[30]

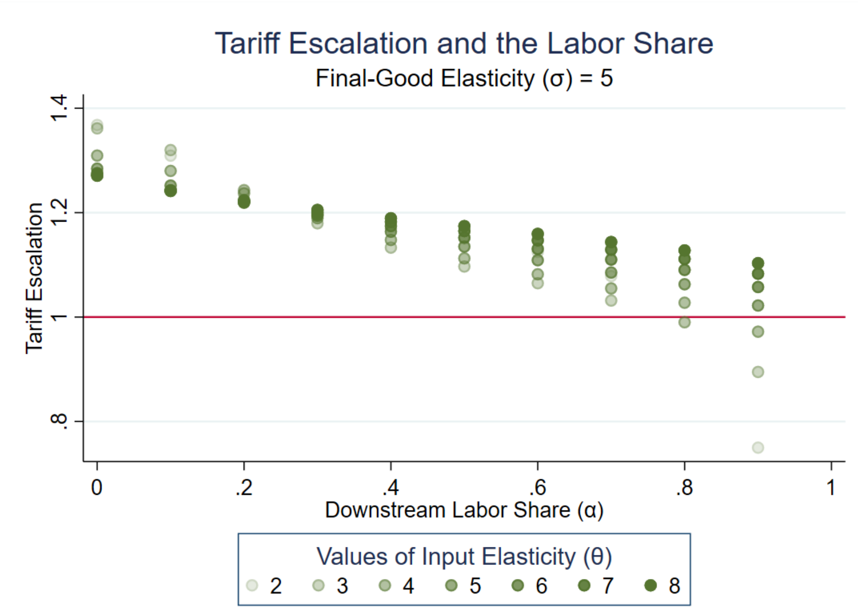
We also evaluate tariff escalation for different values of A_d, A_u, τ_d, and τ_u, while holding the downstream and upstream elasticities fixed at 5 and the downstream labor share at 0.55.[31] We obtain solutions in 92 percent of the 625 cases we consider. The average value of the ratio of optimal downstream to upstream tariffs is 1.12 and the median is also 1.12. These optimal tariffs feature

[29] Although we do not model roundabout production in inputs for simplicity, the results from Caliendo et al. (2021) suggest that tariff escalation would be even more prevalent in that case. Those authors analyze optimal second-best tariffs when only inputs are traded, and find that they tend to be lower when input production is roundabout.

[30] When the downstream sector does not employ much labor, the potential to increase upstream efficiency by reallocating labor is small, so there is less motive for a larger input tariff on efficiency grounds, and tariff escalation remains more likely. By contrast, when the downstream sector employs a high share of labor, there is more potential for the input sector to grow from reallocating this labor and thus become more efficient.

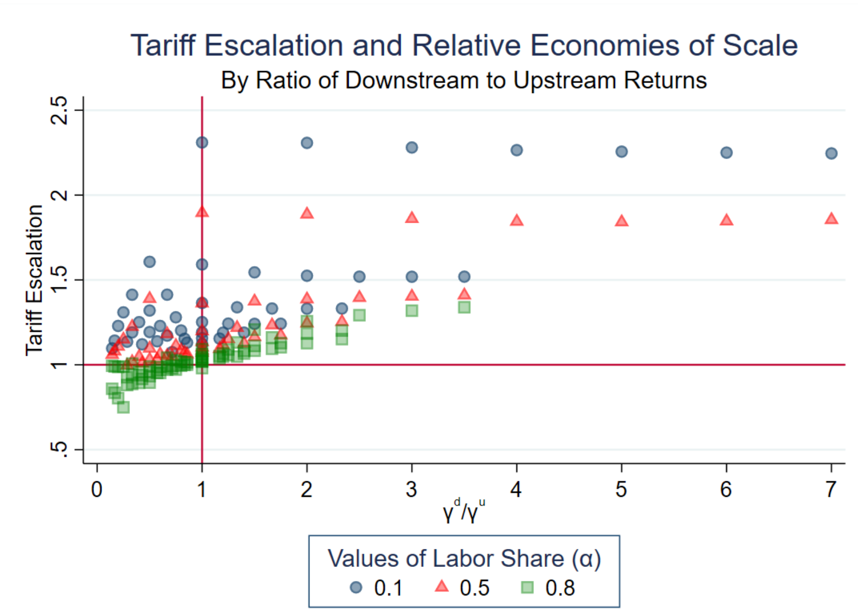
[31] The median values of these parameters are chosen based on the results of the structural estimation of our model in Section 6.

Figure 2: Second-Best Tariff Escalation and α



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the downstream labor share (α) and upstream elasticity of substitution (θ). The downstream elasticity of substitution (σ) is fixed at 5. Appendix E.1 shows similar patterns for additional values of σ.

Figure 3: Second-Best Tariff Escalation and Relative Scale Economies



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the relative returns to scale in downstream versus upstream production (γ^d/γ^u) and the downstream labor share (α).

escalation in 96 percent of the solved cases (see Appendix E.1).

The Case of No Scale Economies Finally, we provide additional intuition for the key role played by increasing returns to scale in determining optimal tariffs by analyzing special cases of our model with no scale economies. When the upstream sector does not feature increasing returns to scale (i.e., $\gamma^u = 0$), the model is much more tractable and we provide analytic results that show tariff escalation is always optimal in the second-best, even when $\alpha > 0$. In this case, there is no efficiency rationale for taxing inputs, whereas increasing returns to scale downstream continue to motivate a downstream import tariff. The result in Proposition 6 in fact continues to hold even when $\alpha > 0$, and thus import tariffs result in a tariff escalation wedge larger than the first-best one, or $(1+t_H^d)/(1+t_H^u) > 1+\gamma^d = \sigma/(\sigma-1) > 1$. This highlights again the role of downstream scale economies in generating tariff escalation. Indeed, when we further set $\gamma^d = 0$, so the model features constant returns to scale in both sectors, optimal tariffs only feature tariff escalation when $\sigma > \theta$, just as in our previous Proposition 7 for the case $\alpha = 0$. Details and derivations are in Appendix D.3.

6 Quantitative Results for a Large Open Economy

In this section, we relax the 'small open economy' assumption by allowing for Home and Foreign prices to change, and quantitatively solve for the social-welfare maximizing levels of input and final-good tariffs. We perform the analysis by mapping world data to our two-country model, interpreting the Home country as the United States, and the Foreign country as the Rest of the World (RoW, hereafter). We first analyze the implementation of the first-best allocation, which as in our theoretical analysis of the small-open economy case requires an upstream subsidy and at least one export tax. We then analyze optimal tariffs when only import tariffs are feasible. In this real-world setting, tariff escalation appears to be a robust feature of the structure of optimal import tariffs. In the first-best allocation, optimal tariffs continue to feature escalation as long as downstream subsidies are not used.

Although we provide quantitative results for a set of parameters anchored on US data, the qualitative nature of our results – most notably, the fact that second-best optimal tariffs on final goods are larger than on inputs – remains unaffected when exploring a wider range of parameter values as demonstrated in Section 6.3. These results line up well with our theoretical results in Sections 4 and 5 obtained for a 'small open economy.' Since the results in this section are quantitative, we revert to the 'Krugman' version of our model with internal economies of scale, monopolistic competition, and firm entry. This allows us to provide additional intuition on the role of firm reallocation in explaining tariff escalation, which we do in the next section using the estimated results from this section.

6.1 Data and Parameters

In order to discipline our model quantitatively, we need to take a stance on a number of parameters and ensure that they provide values for key equilibrium variables consistent with those observed in the data. The main parameters of the model are the elasticities of substitution upstream and downstream (θ and σ), the downstream labor share (α), iceberg trade costs upstream and downstream (τ^u and τ^d), productivity upstream and downstream in each country (A^u_{US}, A^u_{RoW}, A^d_{US} and A^d_{RoW}), fixed costs upstream and downstream (f^u and f^d), and the Home and Foreign labor endowments (L_{US} and L_{RoW}). Perhaps not too surprisingly given the isomorphism we develop in previous sections, the fixed cost parameters turn out to be irrelevant for our quantitative conclusions, so we do not discuss them below.

Our quantitative approach constitutes a blend of calibration and estimation. We first discuss various approaches to estimating the key elasticities of substitution θ and σ, we then back out the downstream labor share α and the labor forces L_{US} and L_{RoW} from readily available public data, and we finally estimate trade costs (τ^u and τ^d) and the productivity parameters (A^u_{US}, A^u_{RoW}, A^d_{US} and A^d_{RoW}) by minimizing the distance between our model and a series of moments obtained from standard sources.

Elasticities of Substitution (θ and σ) We consider four alternative approaches to quantifying the elasticities of substitution across varieties in the upstream and downstream sectors (θ and σ, respectively). We summarize these approaches here and provide additional details in Appendix F.2. The first approach is to treat these elasticities as symmetric across sectors. In this first approach, we fix the values of the elasticities of substitution across varieties in each sector to 5 ($\sigma = \theta = 5$), as in Costinot and Rodríguez-Clare (2014). We first consider this symmetric case to rule out the possibility that differences in demand elasticities across good types are the only source of variation in the response of welfare to changes in input versus final-good tariffs.

The second approach is to calibrate these parameters from data on mark-ups. Recall that under monopolistic competition and CES preferences, the optimal firm-level mark-up is equal to $\theta/(\theta-1)$ upstream and $\sigma/(\sigma-1)$ downstream. Using sales and mark-up data from Baqaee and Farhi (2020) based on publicly listed firms in Compustat, we compute the sales-weighted average mark-ups of firms which we assigned to either upstream or downstream based on their primary sector. This approach leads to estimates of $\theta = 4.43$ for the elasticity of substitution upstream and of $\sigma = 6.44$ for the elasticity of substitution downstream.

The third approach is to estimate these parameters based on the response of trade flows to the US-China trade war in 2018 to 2019. Specifically, we follow Amiti et al. (2019) and calculate 12-month changes in US imports and US import tariffs at the product-country level. Under the CES demand structure, regressing the changes in trade flows on the changes in tariffs provides estimates of the trade elasticity. Our preferred specification from this approach leads to estimates of $\theta = 2.35$ for the elasticity of substitution upstream, and $\sigma = 3.08$ for the elasticity of substitution downstream. The small magnitude of the trade elasticities is consistent with the findings in Amiti et al. (2020) and could reflect that the response in trade flows was diminished by uncertainty about the persistence of these tariff changes.

The fourth (and final) approach is to exploit the isomorphism of our model to a competitive model with external economies of scale. As discussed in Section 2, the isomorphism places the following restrictions between the external economies of scale parameters and the elasticities of substitution across varieties hold: $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$. We use estimates of scale elasticities from Bartelme et al. (2019). We note two important caveats. First, they estimate these parameters only for 15 manufacturing sectors (we classify nine of these as upstream and six as downstream). Second, their framework abstracts from intermediate inputs and therefore their estimates may not be perfectly compatible with our setup. With these caveats in mind, the average (unweighted) scale elasticities are 0.133 upstream and 0.135 downstream. Exploiting the isomorphism between this setup and our framework with monopolistic competition and free entry, we convert these to $\theta = 8.52$ and $\sigma = 8.41$ for this fourth approach.

Downstream Labor Intensity, Trade and Expenditure Shares and Labor Endowments We measure the share of inputs in production, $1 - \alpha = 0.45$, from usage of intermediate inputs by downstream sectors based on the WIOD database (see Appendix F.3 for details). Similarly, we calculate trade and expenditure shares for the upstream (intermediate-input) and downstream (final consumption) sectors based on trade flow data provided in the WIOD, taking into account whether a trade flow is used for final consumption or as an intermediate input).[32] We infer the labor endowment of each country from population data published by CEPII.[33]

Estimation of Productivity Parameters and of Trade Costs Finally, we normalize US productivity in both sectors to one, $A^d_{US} = A^u_{US} = 1$. This leaves us with four parameters to estimate: trade costs in each sector $\{\tau^d, \tau^u\}$, and sectoral productivity in the rest of the world $\{A^d_{RoW}, A^u_{RoW}\}$.[34] To estimate the model, we search for the vector of parameters $\{\tau^d, \tau^u, A^d_{RoW}, A^u_{RoW}\}$ that minimizes the sum of squares of the differences between model-generated and empirical moments, subject to our equilibrium constraints. Panel B of Table 1 lists the set of moments we target in the estimation. The moments correspond to those that are necessary to solve for the changes in equilibrium outcomes in response to a counterfactual change in tariffs (i.e., the hat algebra approach) and are all retrieved from the World Input-Output Database (WIOD).

Panel A in Table 1 presents the estimated values of the RoW's productivities and iceberg trade costs in each sector obtained under symmetric elasticities upstream and downstream, $\theta = \sigma = 5$. Trade costs appear slightly higher in the downstream sector, but within the range of standard estimates of trade barriers. The estimates indicate that the United States is about three times more

[32] We use data for 2014 which is the latest available year in the WIOD.

[33] Specifically, we set $L^{us} = 10 \times \frac{Pop^{us}}{Pop^{us}+Pop^{row}} = 0.45$ and $L^{row} = 10 \times \frac{Pop^{row}}{Pop^{us}+Pop^{row}} = 9.55$.

[34] We restrict entry costs f^d and f^u to be symmetric across sectors and countries and fix those values to 1. As anticipated, this restriction is without loss of generality, as we find that both the model fit and counterfactuals are invariant to changing the entry costs to arbitrary (and possibly asymmetric) values.

Table 1: Calibrated Parameters and Moments

A. Calibrated Parameters	
Productivity in final-good sector, RoW relative to US, A^d_{row}	0.325
Productivity in input sector, RoW relative to US, A^u_{row}	0.142
Iceberg cost for final goods from US to RoW, τ^d	2.375
Iceberg cost for inputs from US to RoW, τ^u	2.032

B. Moments	Data	Model
Sales share to US from US in final goods	0.943	0.964
Sales share to RoW from RoW in final goods	0.988	0.985
Sales share to US from US in intermediate good	0.897	0.889
Sales share to RoW from Row in intermediate good	0.982	0.978
Expenditure share in US final goods for the US	0.960	0.946
Expenditure share in RoW final good for the RoW	0.981	0.989
Expenditure share in US int. good for the US	0.906	0.921
Expenditure share in RoW int. good for the RoW	0.980	0.967
Total US sales (int. goods) to total US expenditure (final goods)	0.771	0.466
Total RoW sales (int. goods) to total RoW expenditure (final goods)	1.242	0.446
Total US sales (final goods) to total US expenditure (final goods)	1.018	0.997
Total RoW sales (final goods) to total RoW expenditure (final goods)	0.993	0.999
Total expenditure in final goods by the US relative to RoW	0.303	0.285

Sources: World Input Output Database and authors' calculations. Notes: Panel A presents the estimated values of the RoW's productivities and iceberg trade costs in each sector obtained under symmetric elasticities upstream and downstream, $\theta = \sigma = 5$. Panel B presents the targeted moments in the estimation. Column 1 presents moments from the data and column 2 presents their estimated counterparts. Note that in the model, total sales upstream to total expenditure downstream cannot be larger than 1 since the upstream sector is pure value added.

efficient in final-good production than the rest of the world, and seven times more efficient in terms of input production. Despite only estimating four parameters, the fit of the model is quite good for most moments, except for the ratio of total sales in the upstream sector to total expenditure in the downstream sector. Note that in the data, the upstream sector uses intermediate inputs in production as well – which for simplicity we abstract from in our framework.

6.2 Optimal Tariffs

In this section, we use the estimated parameters to compute the optimal tariff levels on final goods and intermediate inputs for the United States when the rest of the world sets a zero tariff on US goods.

Optimal Import Tariffs under First-Best Policies We begin by considering optimal policy in an environment in which the Home government has the necessary instruments to achieve the first-best allocation. We focus on the set of instruments discussed in Proposition 8, namely import tariffs and export taxes, as well as an upstream production subsidy. After normalizing the downstream export tax to zero (by Lerner's symmetry), we find that the optimal vector of policies is given by

$$\left(t_H^d, t_H^u, s_H^u, v_H^u\right) = (0.253, 0.003, 0.200, 0.200).$$

In words, even when accounting for general equilibrium effects due to the United States not being a small open economy, we find that the first-best policies are remarkably consistent with the results from Proposition 8. Tariff escalation is close to $\sigma/(\sigma-1) = 5/4$, and the optimal domestic production subsidy upstream and the optimal upstream export tax are essentially indistinguishable from $1/\theta = 0.2$. Note also that the upstream import tariff is virtually zero.

Optimal Import Tariffs under Second-Best Policies Real world trade policies rarely feature export taxes—they are in fact outlawed by the US Constitution—and production subsidies are rarely used systematically. For explaining the observed tariff escalation, second-best policies that only feature import tariffs are therefore of particular interest. We again maximize US welfare, taking as given that the rest of the world places no tariffs on the United States. In this case, the vector of optimal import tariffs is given by

$$\left(t_H^d, t_H^u\right) = (0.306, 0.170).$$

Tariff escalation thus prevails under second-best policies. Here, under $\alpha = 0.55$, tariff escalation is smaller in magnitude under second-best import tariffs compared to the first-best policy results above. Recall that in the $\alpha = 0$ case tariff escalation is larger under second-best policies as shown in Proposition 4, but this need not be the case for $\alpha > 0$. Interestingly, however, optimal tariff escalation under second-best policies (around 1.11) is a bit larger in magnitude than the observed tariff escalation of around 1.04 observed in the US in 2018 (see Figure 5 in Section 8).

6.3 Optimal Tariffs: Robustness

We next explore the robustness of our findings to alternative parameter values. We analyze optimal import tariffs under the three alternative procedures of estimating θ and σ as well as for different values for α (i.e., downstream value-added intensity). When changing these parameters, we recalibrate the trade and productivity parameters to provide the best fit of the moments from Panel B of Table 1.

Table 2 presents the results of first-best policies that include an upstream production subsidy and export tariffs (panel A), and the second-best results using only import tariffs (panel B). As is clear, the level of the tax instruments is quite sensitive to changes in the parameters. However, for all parameter values, we have that $\frac{1+t^d}{1+t^u} > 1$, and therefore optimal final-good tariffs are higher than input tariffs. Under the elasticity parameters shown in column 4, the optimal second-best tariff escalation is close in magnitude to the observed tariff escalation from Figure 5 (though the level of

Table 2: Optimal Tax Policy - Robustness for Various Parameter Values

Parameter Values	$\alpha = 0.55$					$\sigma = \theta = 5$		
	$\theta = 4.43$ $\sigma = 6.44$	$\theta = 2.35$ $\sigma = 3.08$	$\theta = 8.52$ $\sigma = 8.41$	$\theta = 2.5$ $\sigma = 4$	$\theta = 5.5$ $\sigma = 4$	$\alpha = 0.75$	$\alpha = 0.25$	$\alpha = 0$
A. Optimal Trade & Tax Policies								
t^d	0.187	0.488	0.137	0.338	0.339	0.254	0.258	0.265
t^u	0.003	0.002	0.002	0.003	0.003	0.003	0.003	0.002
v^u	0.227	0.428	0.118	0.426	0.182	0.200	0.205	0.211
s^u	0.226	0.425	0.118	0.400	0.182	0.200	0.200	0
$\frac{1+t^d}{1+t^u}$	1.184	1.484	1.136	1.334	1.335	1.249	1.255	1.262
B. Optimal Import Tariffs								
t^d	0.224	0.505	0.162	0.365	0.388	0.260	0.334	0.349
t^u	0.176	0.314	0.091	0.301	0.151	0.182	0.111	0.056
$\frac{1+t^d}{1+t^u}$	1.042	1.145	1.065	1.049	1.205	1.065	1.201	1.278

Notes: Each column presents optimal tariffs and taxes for alternative values of the parameters described in Section 6.1 and their corresponding, re-estimated values of τ^d, τ^u, A^d_{row} and A^u_{row}. The set of calibrated parameters that corresponds to each column is displayed in Table E.8 in Appendix E.3. Panels A and B present optimal tariffs and taxes for the cases of policy instruments in Section 6.2. Tariff escalation ($\frac{1+t^d}{1+t^u} > 1$) is a robust feature across all specifications.

the import tariffs is much larger). We thus conclude that for all empirically plausible parameter combinations for the downstream labor share and the scale elasticities, the solution to the social planner's second-best problem always features tariff escalation.

Panel A of Table 2 also reveal a systematic property of the tariff escalation consistent with the results from Proposition 8 derived for a small open economy. Comparing across columns, $\frac{1+t^d}{1+t^u} \approx \frac{\sigma}{\sigma-1}$. The pattern is striking, though note that this relationship is not exact, and can vary for a given level of σ as other parameters (e.g., α) are changed.

7 Decomposing the Welfare Effects from Tariffs

In this section, we analyze the distinct welfare effects of input versus final-good tariffs both analytically and numerically. To do so, we decompose the first-order welfare effects of *small tariffs* levied by the 'Home' government on imported final goods or imported inputs into six, distinct channels. As in Section 6, we allow for endogenous prices in Home and Foreign, and use the 'Krugman' firm-level model so that we can analyze firm-level relocation effects. We first derive a theoretical decomposition of the welfare effects of small tariffs, and then evaluate this decomposition quantitatively.

7.1 First-Order Welfare Effects of Small Import Tariffs

The key innovation in this subsection relative to our theoretical results in Sections 4 and 5 is that we no longer restrict Home to be a small, open economy.

Because welfare at Home corresponds to the representative household's real income, we have

$$U_H = \frac{w_H L_H + R_H}{P_H^d},$$

where R_H is tariff revenue in equation (12), and where P_H^d is the ideal price index at Home.

We are interested in the change in Home's welfare associated with a change in the tariff schedule $\{t_H^d, t_H^u\}$ starting from an equilibrium with zero tariffs. For simplicity, and without loss of generality, we set the Home wage to be the numéraire, so we can focus on the effect of tariffs on tariff revenue and the price index. The change in Home's welfare, dU_H, around $t_H^d = 0$ and $t_H^u = 0$ (and thus $R_H = 0$), can then be written as:

$$\frac{dU_H}{U_H} = \left[-\frac{dP_H^d}{P_H^d} + \frac{dR_H}{w_H L_H} \right], \tag{35}$$

with

$$\frac{dR_H}{w_H L_H} = b_F^H \times dt_H^d + \lambda_H^d \times \Omega_{FH} \times dt_H^u, \tag{36}$$

where $b_F^H \equiv \frac{M_F^d p_{FH}^d q_{FH}^d}{w_H L_H}$ is the share of Home income spent on foreign varieties, $\lambda_H^d \equiv \frac{M_H^d p_H^d x_H^d}{w_H L_H}$ is the ratio of domestic final-good revenue to national income (with $R_H = 0$) in country H, and $\Omega_{FH} \equiv \frac{M_F^u M_H^d p_{FH}^u q_{FH}^u}{M_H^d p_H^d x_H^d}$ is the share of Home final-good revenue spent on intermediate input varieties from F.

Consider next the change in Home's ideal price index. Given the formula for this price index – see equations (B.6) and (B.8) in Appendix B.1 – and given firm symmetry, we have:

$$\frac{dP_H^d}{P_H^d} = b_H^H \times \left(\frac{1}{1-\sigma} \frac{dM_H^d}{M_H^d} + \frac{dp_{HH}^d}{p_{HH}^d} \right) + b_F^H \times \left(\frac{dM_F^d}{M_F^d} \frac{1}{1-\sigma} + \frac{dp_{FH}^d}{p_{FH}^d} + dt_H^d \right). \tag{37}$$

The ideal (downstream) price index changes because in equilibrium the total measure of firms, in both Home and Foreign, responds to the change in tariff. At the same time, the change in relative prices also affects the price charged by downstream producers. Each factor's contribution to the change in the price index depends on the importance of foreign and domestic goods in the consumption basket, b_j^H. The change in the unit price of downstream goods is given by:

$$\frac{dp_{ii}^d}{p_{ii}^d} = \alpha \frac{dw_i}{w_i} + (1-\alpha) \frac{dP_i^u}{P_i^u}, \tag{38}$$

with

$$(1-\alpha) \frac{dP_i^u}{P_i^u} = \left(\frac{dM_i^u}{M_i^u} \frac{1}{1-\theta} + \frac{dp_{ii}^u}{p_{ii}^u} \right) \Omega_{i,i} + \left(\frac{dM_j^u}{M_j^u} \frac{1}{1-\theta} + \frac{dp_{ji}^u}{p_{ji}^u} + dt_i^u \right) \Omega_{ji}. \tag{39}$$

This latter equation captures the change in the upstream price index in each country, which is in turn shaped by the change in the measure of upstream firms in each country, the change in the price of individual input varieties, and the relative importance of domestic and foreign inputs in production, as captured by the terms Ω_{ii} and Ω_{ji}. Since we have set Home wages as the numéraire, we have $\frac{dw_H}{w_H} = 0$. Also, since we hold iceberg trade costs fixed in this exercise, we have $\frac{dp^d_{ji}}{p^d_{ji}} = \frac{dp^d_{ii}}{p^d_{ii}}$. Finally, since upstream goods only use labor in production, we have $\frac{dp^u_{FF}}{p^u_{FF}} = \frac{dp^u_{FH}}{p^u_{FH}} = \frac{dw_F}{w_F}$.

Putting all the pieces together – that is, combining equations (35)-(39) – we finally obtain the following expression for the first-order effect of tariffs on Home welfare:

$$\begin{aligned}\frac{dU_H}{U_H} &= -\left(b^H_H \Omega_{FH} + b^H_F \left(\Omega_{FF} + \alpha\right)\right) \frac{dw_F}{w_F} \\ &+ \left(\frac{b^H_H \Omega_{HH} + b^H_F \Omega_{HF}}{\theta - 1}\right) \frac{dM^u_H}{M^u_H} + \left(\frac{b^H_H \Omega_{FH} + b^H_F \Omega_{FF}}{\theta - 1}\right) \frac{dM^u_F}{M^u_F} \\ &+ \left(\frac{b^H_H}{\sigma - 1}\right) \frac{dM^d_H}{M^d_H} + \left(\frac{b^H_F}{\sigma - 1}\right) \frac{dM^d_F}{M^d_F} \\ &+ \left(\lambda^d_H - b^H_H\right) \Omega_{FH} (dt^u_H) \mathbb{I}_{\{t = t^u\}}.\end{aligned} \tag{40}$$

This expression contains six terms.[35] The first one captures 'factorial' *terms-of-trade* benefits from raising tariffs, which in our Ricardian model operate via changes in relative wages (or foreign wages, given our choice of numéraire). The next four terms capture *relocation effects* due to changes in the masses of domestic and foreign firms in both the upstream and downstream sectors. All terms enter positively, reflecting the positive effect of increased varieties upstream and downstream on welfare, but it should be clear that general-equilibrium constraints will preclude all these measures of firms from increasing in reaction to Home import tariffs. How each of these relocation effects influences welfare is in turn given by the relative importance of these four types of firms in the purchases of Home consumers and Home firms.

To provide intuition for the quantitative results to come, notice that due to home bias, we typically have $b^H_H > b^F_H$ and also $b^H_H > b^H_F$. Furthermore, $\Omega_{HH} \leq 1 - \alpha < 1$, and b^F_H will be small unless Home is a large economy. As a result, it will typically be the case that $b^H_H > b^H_H \Omega_{HH} + b^H_F \Omega_{HF}$. This carries two significant implications. First, a given percentage increase in the measure of domestic downstream firms (dM^d_H / M^d_H) has a larger impact on Home welfare than the same percentage increase in the measure of foreign downstream firms (dM^d_F / M^d_F). Second, a given percentage increase in the measure of domestic downstream firms (dM^d_H / M^d_H) has a larger impact on Home welfare than the same percentage increase in the measure of domestic upstream firms (dM^u_H / M^u_H). This suggests that, on account of relocation effects, (i) the Home government will have an incentive to levy import tariffs downstream to attract the entry of final-good producers into its economy – as highlighted in the work of Venables (1987) and Ossa (2011) –, and (ii) although such an incentive also exists with regard to entry of upstream firms, the net welfare effects of input producers' entry are smaller.

[35] Note that while the downstream tariff has direct effects on the price index and on tariff revenues, these two effects are exactly offsetting, so that the only net effects on welfare operate indirectly through equilibrium variables.

The sixth and final term in equation (40) is more subtle and relates to a key term identified in the work of Beshkar and Lashkaripour (2020). More specifically, notice that $\lambda_H^d - b_H^H$ represents the value of exported downstream goods as a share of Home's GDP. This last term then captures the extent to which an input tariff is passed on to foreign consumers, thereby mimicking an export tax, which also improves Home's terms of trade (Costinot et al., 2015; Beshkar and Lashkaripour, 2020). For this same reason, this last term *only* applies to changes in input tariffs.

Because changes in tariffs do not enter the other terms in equation (40) explicitly, it would appear that (small) intermediate-input import tariffs increase welfare by more than (small) final-good tariffs on account of this last extra term. Nevertheless, we have already indicated above that final-good and input tariffs generate differential effects on relative wages (dw_F/w_F) and on relocation effects (dM_i^s/M_i^s for $s = d, u$), and we will show in the next subsection that these channels are quantitatively dominant. More precisely, and anticipating the quantitative results to come, relocation effects seem to be the quantitatively dominant force in leading small final-good tariffs to generate larger welfare gains than small input tariffs.

7.2 Quantitatively Decomposing the Welfare Effects of Tariffs

To assess the relative magnitudes of the channels above, we use the calibrated model to quantify each of them. To do so, we first solve for the zero-tariff equilibrium, so that we can compute the statistics Ω_{ij}, b_j^i and λ_i^d in this environment.[36]

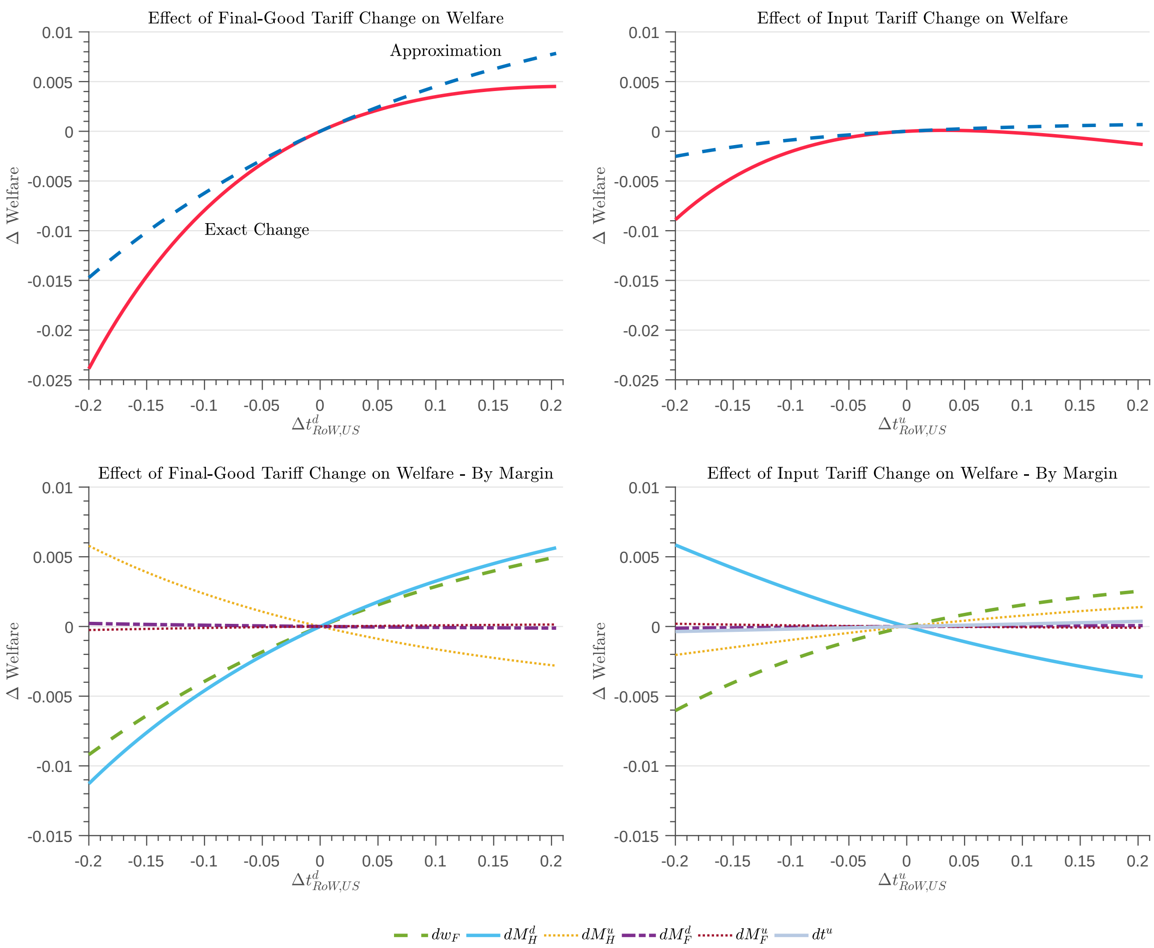
Figure 4 depicts the welfare effects of changes in a final-good tariff (left panels) versus changes in an input tariff (right panels). The top two panels compare the percentage changes in welfare starting from the zero-tariff equilibrium (solid red line) to the percentage changes predicted by our first-order approximations around zero (dashed-blue line). The first-order approximation works well for small changes in both final-good and intermediate-input tariffs. Starting from zero tariffs, the welfare effects of small import tariffs are positive for both types of goods, but turn negative for input tariffs at much lower rates than for final-good tariffs.

The bottom panels of Figure 4 decompose the approximation of the aggregate effects into their component parts, as shown in equation (40). Specifically, we decompose changes in welfare into changes due to: (i) changes in relative wages (dashed green); (ii) the relocation of final good producers to the United States (solid cyan); (iii) the relocation of input producers to the United States (dotted yellow); (iv) changes in the mass of final-good producers in the RoW (short-dash purple); (v) changes in the mass of input producers in the RoW (dash-dot magenta); and (vi) the gain from passing part of the input tariff onto final consumers in the RoW (solid gray). Although in the figure we label these by dw_F, dM_H^u, and so on, it should be understood that we are plotting the full value of each of the six terms in equation (40), with the labels identifying only one element of each term.

Several observations are in order. First, notice that by raising tariffs on final goods, US welfare

[36] Under zero tariffs, these statistics take the values $\Omega_{H,H} = 0.41$, $\Omega_{F,H} = 0.04$, $\Omega_{F,F} = 0.44$, $\Omega_{H,F} = 0.02$, $b_H^H = 0.93$, $b_F^H = 0.07$, $\lambda_H^d = 0.98$.

Figure 4: First Order Decomposition of Welfare Changes



Notes: Figure depicts the welfare effects of changes in a final-good tariff (left panel) versus changes in an input tariff (right panel). The top panels compare the percentage changes in utility starting from the zero-tariff equilibrium (solid red line) to the percentage changes predicted by our first-order approximations around zero (dashed blue line). The bottom two panels decompose the approximation of the aggregate effects into the component parts in equation (40). These are changes in welfare due to: (i) changes in relative wages (dashed green); (ii) the relocation of final good producers to the United States (solid cyan); (iii) the relocation of intermediate producers to the United States (dotted yellow); (iv) changes in the mass of final-good producers (short-dash purple); (v) changes in the mass of intermediate good producers (short-dash magenta); and (vi) the gain from passing part of the tariff onto foreign consumers (solid gray).

increases not only because it tilts the factorial terms of trade in its favor – i.e., a reduction in w_F – but also because it induces a relocation of final-good producers into its own country. In addition, notice that the magnitude of the term associated with dM^d_H is on average as large as the term dw_F. Hence, this relocation effect is as important as the factorial terms-of-trade channel usually emphasized in the literature. Nevertheless, the net entry of final-good producers is accompanied by a net exit of input producers (yellow-dotted curve), with welfare effects that are about half as large as those associated with the relocation of final-good producers. The other three effects are largely negligible (the last one is exactly zero for the case of final-good tariffs). These results suggest that the United States is similar to a small, open economy relative to the RoW, and provides quantitative support for the similarity between our analytical solutions for a small, open economy and the quantitative solutions for a LOE.

Turning to the results for input tariffs, the relocation effects are again similar in magnitude to the factorial terms-of-trade effects, though these relocation effects now entail a net *negative* effect on welfare. Higher import tariffs on intermediate inputs increase entry upstream, but reduce entry of final-good producers. The negative welfare effect of the latter dominates quantitatively, which is intuitive based on a comparison of the terms multiplying dM_H^d and dM_H^u in equation (40), as explained in Section 7. These terms contain the exposure of the consumer price index to changes in the measure of downstream and upstream producers, respectively.[37] The effect on welfare from changes in the mass of firms in the rest of the world is largely negligible, while the last term – the gain from passing intermediate-input tariffs on to foreign consumers – is small in magnitude.

8 Counterfactuals: Evaluation of the 2018-2019 Tariff Increases

We close the paper by evaluating how the tariff increases during the 2018 to 2019 trade war affected US welfare. Figure 5 illustrates the general pattern of tariff escalation for both US tariffs on other countries and for foreign tariffs applied on the United States, both before and after the recent trade war. Due to their low initial levels, average input tariffs increased the most on a percent basis, though tariff escalation remains a distinct feature of the data.[38]

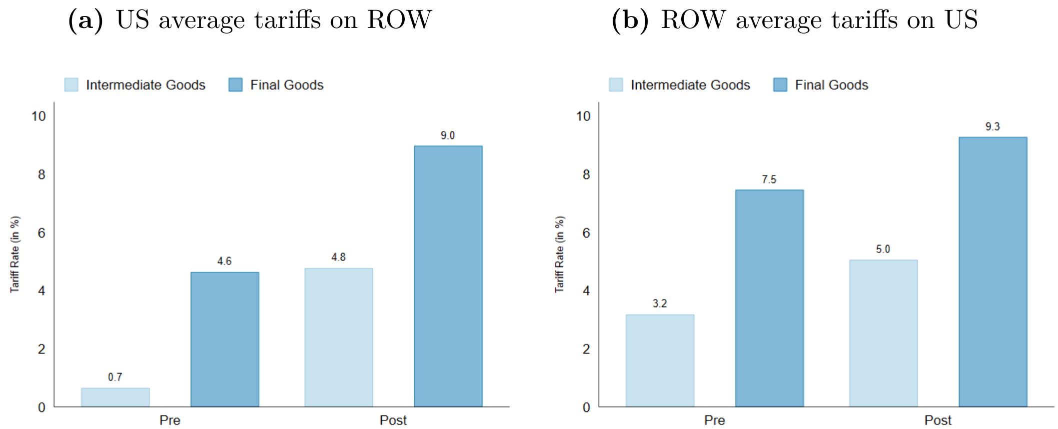
These tariff increases largely arose from the US-China trade war, but also include the US tariffs on washing machines, solar panels, aluminum, and steel, as well as the retaliatory tariffs from the rest of the world (RoW).

To evaluate the welfare effects of these tariff increases, we calculate US and RoW welfare under five scenarios. First, we use the observed 2017 tariffs and no other domestic instruments. Second, we apply the observed 2019 US tariffs. Third, we evaluate the distinct welfare effects of downstream versus upstream tariffs by changing tariffs in each sector individually. Fourth, we construct a counterfactual tariff that generates the same revenue as the 2019 US tariffs, but only applies to either the downstream or the upstream sector. Finally, we evaluate the welfare implications of optimal second-best tariffs and optimal first-best policies. For each of these scenarios, we first hold RoW tariffs constant at their 2017 values, and then re-calculate welfare using the observed 2019

[37] Note that $\Omega_{H,H}$ and $\Omega_{H,F}$ are bounded above by $1-\alpha$. The welfare effect associated with a percentage increase in downstream firms is 0.31, whereas the term multiplying the percentage change in upstream firms is only 0.13 in equation (40).

[38] Figure 5 displays the trade-weighted average tariffs on intermediate and final goods imposed by the United States on the Rest of the World (RoW) and by the RoW on US imports. Tariff data at the HTS 6-digit level for ROW are from the WTO Tariff Download Facility. Tariff data at the HTS 8-digit level for the United States are from the USITC. Import and export data are from the US Census Bureau. We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC). For details on data construction and for a version of Figure 5 without trade weights, see Appendix F.1.

Figure 5: Tariff Escalation Before and After the US-China Trade War



Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC, 2015 annual import and export data for weighted averages from US Census Bureau. Goods are classified as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods start with BEC code 41, 521, 112, 321, 522, 61, 62, 63 (including capital goods). All other goods have no classification.

retaliatory tariff implemented by the RoW.

To approximate the US economy as closely as possible, we use elasticity estimates from US data on mark-ups. These estimates correspond to an upstream elasticity of $\theta = 4.43$ and a downstream elasticity of $\sigma = 6.44$ (see Section 6.1). Under the isomorphic model, these imply values of $\gamma^u = 0.29$ and $\gamma^d = 0.18$, such that the upstream sector has greater increasing returns to scale.

Table 3 displays our results, with panels A and B showing the cases with and without retaliation, respectively. Average US import tariffs increased from 0.7 to 4.8 percentage points for intermediate goods, and from 4.6 to 9 percentage points for final goods. We find that without tariff retaliation by the RoW, US welfare would have increased by 0.12% from these tariff changes. This gain is consistent with our estimates of the sizable unilateral optimal tariff rates for the US in Section 6 (in the absence of any export taxes or domestic subsidies).

We next evaluate the extent to which this gain was due to larger final-good versus input tariffs. In the third row of Table 3, we consider only the increase in the final good tariff, whereas row four considers the case if only the tariffs on intermediate goods had increased. The comparison reveals that the US welfare gains are driven overwhelmingly by higher final-good tariffs.

The dominant role of final-good tariffs on welfare increases is even starker when considering a counterfactual increase in US final-good tariffs (row 5 of Table 3) that would have (naively) raised the same revenue as the observed tariff increases based on the average tariff rate change and the 2017 trade flows. In this case, US welfare – absent any foreign retaliation – would have risen by 0.11%. If instead the tariff increases had been adjusted so to apply only to intermediate inputs, US welfare would have increased by only 0.02%. foreign retaliation.

In Panel B of Table 3, we repeat these exercises but now take into consideration the observed retaliation by the RoW, which increased its import tariffs on US intermediate inputs from 3.2 to 5.0 percentage points and on US final goods from 7.5 to 9.3 percentage points. In this case, the

Table 3: Counterfactual Welfare Effects of US-China Trade War

	A. RoW tariff at 2017 level			**B.** RoW tariff at 2019 level		
	U_{US}	U_{RoW}	$\frac{U_{US}}{U_{US,2017}}$	U_{US}	U_{RoW}	$\frac{U_{US}}{U_{US,2017}}$
1. US tariff - 2017 level	0.057586	0.172282				
2. US tariff - 2019 level	0.057657	0.172125	1.001228	0.057598	0.172152	1.000217
3. 2019 US tariff only Downstream	0.057632	0.172177	1.000791	0.057573	0.172206	0.999791
4. 2019 US tariff only Upstream	0.057604	0.172235	1.000314	0.057548	0.172263	0.999343
5. Counterfactual Tariff only Downstream	0.057650	0.172097	1.001122	0.057592	0.172125	1.000106
6. Counterfactual Tariff only Upstream	0.057597	0.172145	1.000194	0.057541	0.172171	0.999233
7. Optimal US Import Tariffs	0.057733	0.171737	1.002544	0.057671	0.171757	1.001482
8. Optimal US Trade & Tax Policies	0.058374	0.171715	1.013684	0.058312	0.171736	1.012608

Notes: Table presents US welfare (U_{US}), RoW welfare (U_{RoW}), and US welfare relative to its initial 2017 level ($\frac{U_{US}}{U_{US,2017}}$). Panel A computes welfare holding the RoW tariffs at their 2017 levels, while Panel B uses the observed 2019 RoW retaliatory tariffs. *1. US tariff - 2017 level* provides baseline welfare values using actual 2017 tariff values; *2. US tariff - 2019 level* uses actual 2019 US tariffs; *3. 2019 US tariff only Downstream* uses 2017 upstream but 2019 downstream tariffs; *4. 2019 US tariff only Upstream* uses 2017 downstream but 2019 upstream tariffs; *5. Counterfactual Tariff only Downstream (6. Upstream)* uses a counterfactual US downsteam (upstream) tariff that generates the same revenue as the actual 2019 US tariffs, based on the observed trade flows in 2017. Counterfactual tariffs are $\tilde{t}^d = 0.131$ or $\tilde{t}^u = 0.156$; *7. Optimal US Import Tariffs* uses the second-best optimal import tariffs in response to RoW's trade policy in 2017 (Panel A) or 2019 (Panel B). The optimal policy vector for panel A is $\left(t_H^d, t_H^u\right) = (0.231, 0.183)$ and $\left(t_H^d, t_H^u\right) = (0.232, 0.184)$ for panel B; *8. Optimal US Trade & Tax Policy*: US chooses optimal import tariffs, export tax, and production subsidy, as described in Section 4, in response to RoW's trade policy from 2017 (Panel A) or 2019 (Panel B). The optimal policy vector is $\left(t_H^d, t_H^u, v_H^u, s_H^u\right) = (0.186, 0.003, 0.227, 0.226)$ for panel A and $\left(t_H^d, t_H^u, v_H^u, s_H^u\right) = (0.186, 0.003, 0.227, 0.226)$ for panel B.

US welfare gain from the tariff increases shrinks to only 0.02%. Therefore, tariff retaliation by the RoW largely undermines the US welfare gains from higher tariffs, which in turn are overwhelmingly driven by higher final-good (and not input) tariffs. If the US had only placed the tariffs on final goods, while (naively) raising the same revenue (row 5), US welfare would have increased by 0.01%, even accounting for retaliation. If instead those tariffs had only been placed on intermediate inputs (row 6), US welfare would have declined by 0.06%.

Rows seven and eight present the potential welfare gains from implementing optimal import tariffs with and without other US policy instruments. The gains from second-best, optimal import tariffs (without production subsidies or export taxes) are displayed in row seven of Table 3. Optimal US tariffs absent any foreign retaliation achieve a welfare gain of 0.25 percent, with a tariff escalation wedge of 1.04.[39] Row eight allows for a full set of instruments that includes both import and export taxes, as well as production subsidies.[40] The optimal trade policy with domestic subsidies and export taxes (and assuming no foreign retaliation) leads to a 1.4 percent increase in welfare, and features close to zero tariffs on inputs and an escalation wedge of 1.18. As in our analysis in the

[39] The levels of these tariffs are much higher than the ones observed in the data $(t_d^*, t_u^*) = (0.2313, 0.1831)$.

[40] Optimal policy has import taxes $(t_d^*, t_u^*) = (0.1860, 0.0027)$ and export taxes upstream, $v_u^* = 0.2269$, as well as an input production subsidy $s_u = 0.2261$.

SOE, the tariff escalation wedge is now notably higher since the government uses the input subsidy rather than the tariff to address the domestic labor misallocation. We note that these calculations assume no foreign retaliation (or, in panel B, no retaliation above the observed changes from RoW tariffs from 2017 to 2019).

9 Conclusion

In this paper, we provide an efficiency rationale for the fact that import tariffs on final good are systematically higher than those on intermediate inputs. This so-called tariff escalation has been widely documented across time and space, but there is little support in the literature for the notion that the pattern constitutes a social welfare-maximizing policy.

We develop a two-sector model with a final-good sector and an intermediate input sector, both featuring increasing returns to scale, and show that (i) first-best trade policies are consistent with tariff escalation, and that (ii) second-best import tariffs generally feature tariff escalation. A key result is that tariff escalation is driven by the presence and extent of increasing returns to scale in downstream production. Access to cheaper inputs expands the downstream sector's size and thus raises its efficiency. Relatively higher input tariffs are only optimal in a small set of second-best settings in which an upstream production subsidy cannot be used, and upstream efficiency and thus input prices are sufficiently sensitive to the upstream sector's size.

Although our model generically features domestic distortions related to the existence of scale economies upstream, the optimality of relatively lower input tariffs is *not* explained by a (second-best) correction of these domestic distortions. If anything, domestic distortions reduce the desirability of tariff escalation. Instead, input tariffs are less beneficial because they impact the size of the final-good sector *and* because the size of the final-good sector shapes its productivity under increasing returns to scale. It is thus scale economies downstream, rather than upstream, that shape the optimality of tariff escalation.

Our results are based on a parsimonious model featuring a single factor of production, only two sectors of production, and homogeneous firms. Future research should elucidate the robustness of our results to more realistic settings, which should also provide a helpful lens through which to interpret the drivers of tariff escalation in the data.

References

Alvarez, Fernando and Robert E Lucas Jr, "General equilibrium analysis of the Eaton–Kortum model of international trade," *Journal of monetary Economics*, 2007, *54* (6), 1726–1768.

Amiti, Mary, "Are Uniform Tariffs Optimal?," in "Trade Theory, Analytical Models and Development" 2004, p. 47.

_ **and Jozef Konings**, "Trade liberalization, intermediate inputs, and productivity: Evidence from Indonesia," *American Economic Review*, 2007, *97* (5), 1611–1638.

_ **, Stephen J. Redding, and David E. Weinstein**, "The Impact of the 2018 Tariffs on Prices and Welfare," *Journal of Economic Perspectives*, 2019, *33* (4), 187–210.

_ **, Stephen J Redding, and David E Weinstein**, "Who's paying for the US tariffs? A longer-term perspective," in "AEA Papers and Proceedings," Vol. 110 2020, pp. 541–46.

Antràs, Pol and Robert W Staiger, "Offshoring and the role of trade agreements," *American Economic Review*, 2012, *102* (7), 3140–83.

_ **, Davin Chor, Thibault Fally, and Russell Hillberry**, "Measuring the upstreamness of production and trade flows," *American Economic Review*, 2012, *102* (3), 412–16.

Balassa, Bela, "Tariff protection in industrial countries: an evaluation," *Journal of Political Economy*, 1965, *73* (6), 573–594.

Baqaee, David and Emmanuel Farhi, "Productivity and Misallocation in General Equilibrium," *Quarterly Journal of Economics*, February 2020, *135* (1), 105–163.

Barattieri, Alessandro and Matteo Cacciatore, "Self-Harming Trade Policy? Protectionism and Production Networks," 2020. NBER Working Paper No. 27630.

Bartelme, Dominick G, Arnaud Costinot, Dave Donaldson, and Andres Rodriguez-Clare, "The textbook case for industrial policy: Theory meets data," Technical Report, National Bureau of Economic Research 2019.

Basker, Emek and Pham Hoang Van, "Imports 'R' Us: Retail Chains as Platforms for Developing Country Imports," *American Economic Review*, May 2010, *100* (2), 414–18.

Beshkar, Mostafa and Ahmad Lashkaripour, "Interdependence of Trade Policies in General Equilibrium," *Center for Applied Economics and Policy Research (CAEPR) Working Paper*, 2017, *3*.

_ **and** _ , "The Cost of Dissolving the WTO: The Role of Global Value Chains," 2020.

Blanchard, Emily J, Chad P Bown, and Robert C Johnson, *Global supply chains and trade policy* 2021.

Bown, Chad and Eva Zhang, "Measuring Trump's 2018 Trade Protection: Five Takeaways," *Peterson Institute Trade and Investment Policy Watch*, February 2019.

_ , Paola Conconi, Aksel Erbahar, and Lorenzo Trimarchi, "Trade Protection Along Supply Chains," 2020. mimeo.

Bown, C.P. and M.A. Crowley, "Chapter 1 - The Empirical Landscape of Trade Policy," 2016, *1*, 3 – 108.

Breinlich, Holger, Elsa Leromain, Dennis Novy, and Thomas Sampson, "Import Liberalization as Export Destruction? Evidence from the United States," 2021.

Cadot, Olivier, Jaime De Melo, and Marcelo Olarreaga, "Lobbying, counterlobbying, and the structure of tariff protection in poor and rich countries," *The World Bank Economic Review*, 2004, *18* (3), 345–366.

Caliendo, Lorenzo, Robert C Feenstra, John Romalis, and Alan M Taylor, "A Second-best Argument for Low Optimal Tariffs," 2021. NBER Working Paper 28380.

Campolmi, Alessia, Harald Fadinger, and Chiara Forlati, "Trade policy: Home market effect versus terms-of-trade externality," *Journal of International Economics*, 2014, *93* (1), 92–107.

_ , _ , _ et al., "Trade and domestic policies in models with monopolistic competition," 2018.

Casas, Francisco R, "Optimal effective protection in general equilibrium," *The American Economic Review*, 1973, *63* (4), 714–716.

Costinot, Arnaud and Andrés Rodríguez-Clare, "Trade Theory with Numbers: Quantifying the Consequences of Globalization," in "Handbook of international economics," Vol. 4, Elsevier, 2014, pp. 197–261.

_ and Iván Werning, "Lerner symmetry: A modern treatment," *American Economic Review: Insights*, 2019, *1* (1), 13–26.

_ , Andrés Rodríguez-Clare, and Iván Werning, "Micro to macro: Optimal trade policy with firm heterogeneity," *Econometrica*, 2020, *88* (6), 2739–2776.

_ , Dave Donaldson, Jonathan Vogel, and Iván Werning, "Comparative advantage and optimal trade policy," *The Quarterly Journal of Economics*, 2015, *130* (2), 659–702.

Cox, Lydia, "The Long-Term Impact of Steel Tariffs on U.S. Manufacturing," Technical Report, Mimeo, Harvard University 2021.

Das, Satya P, "Optimum tariffs on final and intermediate goods," *International Economic Review*, 1983, pp. 493–508.

Demidova, Svetlana and Andres Rodiguez-Clare, "Trade Policy Under Firm-level Heterogeneity in a Small Economy," *Journal of International Economics*, 2009, *78*, 100–12.

Diamond, Peter A and James A Mirrlees, "Optimal taxation and public production I: Production efficiency," *The American economic review*, 1971, *61* (1), 8–27.

Eaton, Jonathan and Samuel Kortum, "Technology, geography, and trade," *Econometrica*, 2002, *70* (5), 1741–1779.

Erbahar, Aksel and Yuan Zi, "Cascading trade protection: Evidence from the US," *Journal of International Economics*, 2017, *108*, 274–299.

Fajgelbaum, Pablo D., Pinelopi K. Goldberg, Patrick J. Kennedy, and Amit K. Khandelwal, "The Return to Protectionism," *Quarterly Journal of Economics*, 2020, *135* (1), 1–55.

Flaaen, Aaron and Justin Pierce, "Disengtangling the Effects of the 2018-2019 Tariffs on a Globally Connected US Manufacturing Sector," Finance and Economics Discussion Series Working Paper 086, Board of Governors of the Federal Reserve System 2019.

Ganapati, Sharat, "The Modern Wholesaler: Global Sourcing, Domestic Distribution, and Scale Economies," Working Papers 18-49, Center for Economic Studies, U.S. Census Bureau December 2018.

Gawande, By Kishore, Pravin Krishna, and Marcelo Olarreaga, "Lobbying competition over trade policy," *International Economic Review*, 2012, *53* (1), 115–132.

Goldberg, Pinelopi, Amit Khandelwal, Nina Pavcnik, and Petia Topalova, "Imported Intermediate Inputs and Domestic Product Growth: Evidence from India," *Quarterly Journal of Economics*, 2010, *125*, 1727–67.

Gros, Daniel, "A Note on the Optimal Tariff, Retaliation, and the Welfare Loss from Tariff Wars in a Framework with Intra-Industry Trade," *Journal of International Economics*, 1987, *23*, 357–67.

Grossman, Gene and Elhanan Helpman, "Protection for Sale," *American Economic Review*, 1994, *84* (4), 833–50.

Grossman, Gene M and Elhanan Helpman, "When Tariffs Disturb Global Supply Chains," Technical Report, National Bureau of Economic Research 2020.

Handley, Kyle, Fariha Kamal, and Ryan Monarch, "Rising Import Tariffs, Falling Export Growth: When Modern Supply Chains Meet Old-Style Protection," Working Paper 26611, NBER 2020.

Hwang, Hong, Chao-Cheng Mai, and Shih-Jye Wu, "Tariff escalation and vertical market structure," *The World Economy*, 2017, *40* (8), 1597–1613.

Kortum, Samuel S and David A Weisbach, "Optimal unilateral carbon policy," 2021.

Krugman, Paul, "Scale Economies, Product Differentiation, and the Pattern of Trade," *American Economic Review*, 1980, *70* (5), 950–959.

_ **and Anthony J Venables**, "Globalization and the Inequality of Nations," *The quarterly journal of economics*, 1995, *110* (4), 857–880.

Kucheryavyy, Konstantin, Gary Lyn, and Andrés Rodríguez-Clare, "Grounded by gravity: A well-behaved trade model with external economies," Technical Report, Mimeo, University of California, Berkeley 2017.

Lashkaripour, Ahmad and Volodymyr Lugovskyy, "Profits, Scale Economies, and the Gains from Trade and Industrial Policy," Technical Report, Indiana University 2021.

Li, Minghao, "CARD trade war tariffs database," *Center for Agriculture and Rural Development, Report. https://www. card. iastate. edu/china/trade-war-data*, 2018.

Liu, Ernest, "Industrial policies in production networks," *The Quarterly Journal of Economics*, 2019, *134* (4), 1883–1948.

Lucas, Robert E and Nancy L Stokey, "Optimal fiscal and monetary policy in an economy without capital," *Journal of monetary Economics*, 1983, *12* (1), 55–93.

McCorriston, Steve and Ian Sheldon, "Tariff (De-) Escalation with Successive Oligopoly," *Review of Development Economics*, 2011, *15* (4), 587–600.

Ornelas, Emmanuel and John L. Turner, "Trade Liberalization, Outsourcing, and the Hold-up Problem," *Journal of International Economics*, 2008, *74* (1), 225–241.

_ **and** _ , "Protection and International Sourcing," *The Economic Journal*, 2012, *122* (559), 26–63.

Ossa, Ralph, "A New Trade Theory of GATT/WTO Negotiations," *Journal of Political Economy*, 2011, *119* (1), 122–152.

_ , "Trade wars and trade talks with data," *American Economic Review*, 2014, *104* (12), 4104–46.

Puga, Diego and Anthony J Venables, "Agglomeration and economic development: Import substitution vs. trade liberalisation," *The Economic Journal*, 1999, *109* (455), 292–311.

Ruffin, Roy J, "Tariffs, intermediate goods, and domestic protection," *The American Economic Review*, 1969, *59* (3), 261–269.

Shapiro, Joseph S, "The Environmental Bias of Trade Policy*," *The Quarterly Journal of Economics*, 12 2020, *136* (2), 831–886.

Smith, Dominic and Sergio Ocampo Díaz, "The Evolution of US Retail Concentration," Technical Report 2020.

Soderbery, Anson, "Estimating import supply and demand elasticities: Analysis and implications," *Journal of International Economics*, 2015, *96* (1), 1–17.

Spencer, Barbara J and Ronald W Jones, "Vertical foreclosure and international trade policy," *The Review of Economic Studies*, 1991, *58* (1), 153–170.

_ **and** _ , "Trade and protection in vertically related markets," *Journal of international Economics*, 1992, *32* (1-2), 31–55.

Topalova, Petia and Amit Khandelwal, "Trade Liberalization and Firm Productivity: The Case of India," *The Review of Economics and Statistics*, 2011, *93* (3), 995–1009.

Travis, William Penfield, *The Theory of Trade and Protection* number 121, Harvard University Press, 1964.

Venables, Anthony J., "Trade and Trade Policy with Differentiated Products: A Chamberlinian-Ricardian Model," *The Economic Journal*, 1987, *97* (387), 700–717.

Trade Policy and Global Sourcing:
A Rationale for Tariff Escalation

Pol Antràs, Teresa C. Fort, Agustín Gutiérrez, and Felix Tintelnot

Online Appendix (Not for Publication)

A Closed-Economy Model: Details on Derivations

A.1 Equilibrium

Given the CES assumptions built into our framework and the lack of strategic interactions, firms in both sectors charge a constant markup over their marginal cost, which delivers

$$p^u = \frac{\theta}{\theta - 1} \frac{w}{A^u} \tag{A.1}$$

and

$$p^d = \frac{\sigma}{\sigma - 1} \frac{1}{A^d} \frac{w^\alpha \left(P^u\right)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}, \tag{A.2}$$

where P^u is the price index of intermediate inputs associated with Q^u, or

$$P^u = \left(\int_0^{M^u} p^u(\varpi)^{1-\theta} d\varpi \right)^{\frac{1}{1-\theta}}.$$

Firms make zero profits due to free entry, which pins down firm size according to:

$$x^u = (\theta - 1) f^u, \qquad x^d = (\sigma - 1) f^d. \tag{A.3}$$

Naturally, in equilibrium we must have $x^d = q^d$ and $x^u = M^d q^u$. The total measure of firms in the economy can be determined as follows. First, note that from the household's demand for downstream goods we have

$$M^d p^d q^d = wL, \tag{A.4}$$

which plugging in (A.2) and (A.3) delivers

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((1-\alpha) \frac{\theta - 1}{\theta} A^u \right)^{1-\alpha} \left(M^u\right)^{\frac{1-\alpha}{\theta-1}} L. \tag{A.5}$$

Next, note that labor-market clearing imposes

$$L = M^u \frac{(f^u + x^u)}{A^u} + M^d \frac{\alpha p^d x^d}{w}. \tag{A.6}$$

Plugging in equations (A.3) and (A.4), we can solve for the total measure of firms in the upstream sector:

$$M^u = \frac{(1-\alpha) A^u L}{f^u \theta}. \tag{A.7}$$

Then, equations (A.5) and (6) determine the measure of firms in the downstream sector:

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((\theta - 1) f^u \right)^{1-\alpha} \left(\frac{(1-\alpha) A^u}{f^u \theta} \right)^{\frac{(1-\alpha)\theta}{\theta - 1}} (L)^{\frac{\theta - \alpha}{\theta - 1}}. \tag{A.8}$$

Finally, aggregate welfare is simply given by $U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} q^d$, where M^d is given in (7) and $q^d = x^d$ in (A.3).

When $\alpha \to 1$, we obtain

$$U = \left(\frac{A^d}{f^d \sigma} L\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f^d,$$

which is the standard formula in Krugman (1980).[1] Welfare is increasing in market size with an elasticity equal to $\frac{\sigma}{\sigma-1} > 1$, reflecting the variety gains associated with living in an economy that provides a larger number of final-good varieties.

Relative to this "Krugman" benchmark, in the presence of an active upstream sector (i.e., $\alpha < 1$), our model continues to feature scale effects, and in fact the elasticity of welfare to market size is larger than in the model with only a final-good sector. To see this, we can write welfare as

$$U = \left(\frac{(\sigma-1) A^d/\sigma}{((\sigma-1) f^d)^{\frac{1}{\sigma}}} \left(\frac{(\theta-1)\, A^u/\theta}{((\sigma-1) f^u)^{1/\theta}} \right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}} \right)^{\frac{\sigma}{\sigma-1}} \xi_\alpha, \tag{A.9}$$

where ξ_α is a function of only α and θ. Note that $\frac{\theta-\alpha}{\theta-1} \geq 1$, and thus this framework features larger scale effects than our model without an input sector.

To gain a better understanding of the role of imperfect competition and increasing returns to scale on welfare in our closed economy, we next turn to characterizing the social optimum in our model, and explore conditions under which the above market equilibrium is efficient.

A.2 Social Planner Problem

The social planner maximizes the objective in (1), choosing M^d, M^u, ℓ^d, ℓ^u, x^d and x^u subject to feasibility, or:

$$\begin{aligned}
\max_{M^d, M^u, \ell^d, \ell^u, x^d, x^u} \quad & U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} x^d \\
s.t. \quad & L = \ell^u M^u + \ell^d M^d \\
& f^u + x^u = A^u \ell^u \\
& f^d + x^d = A^d \left(\ell^d\right)^\alpha \left((M^u)^{\frac{\theta}{\theta-1}} \frac{x^u}{M^d} \right)^{1-\alpha}.
\end{aligned}$$

Working with the first-order conditions of this problem, we find that

$$(x^u)^* = (\theta - 1) f^u \tag{A.10}$$

and

$$(M^u)^* = \frac{\theta}{\theta - \alpha} \frac{(1-\alpha) A^u L}{\theta f^u} \tag{A.11}$$

in the upstream sector, and

$$\left(x^d\right)^* = (\sigma - 1) f^d \tag{A.12}$$

[1] A small and immaterial point of departure from Krugman (1980) is the fact that we have modeled the productivity terms A^d and A^u as shaping both the marginal and fixed costs of production. As a result, firm size is independent of these productivity parameters, but these parameters affect welfare directly.

and

$$\left(M^{d}\right)^{*}=\left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha}\left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{\alpha^{\alpha}A^{d}}{\sigma f^{d}}\left(\left(\theta-1\right)f^{u}\right)^{1-\alpha}\left(\frac{(1-\alpha)A^{u}}{\theta f^{u}}\right)^{\frac{(1-\alpha)\theta}{\theta-1}}(L)^{\frac{\theta-\alpha}{\theta-1}} \tag{A.13}$$

in the downstream sector. Comparing equations (A.10)-(A.13) to the corresponding ones in the market equilibrium, we conclude that:[2]

Proposition 1. In the decentralized equilibrium, firm-level output is at its socially optimal level in both sectors, but the market equilibrium features too little entry into both the downstream and upstream sectors unless $\alpha = 1$ (so the upstream sector is shut down) or $\alpha = 0$ (so the downstream sector does not use labor directly in production).

Why is the market equilibrium typically inefficient? At first glance, it may appear that the only source of inefficiency is the markup charged by upstream producers, which distorts the choice between labor and the bundle of input varieties for downstream firms. More specifically, this upstream markup makes inputs relatively more expensive and, in response, downstream firms substitute towards labor. At the same time, that markup also incentivizes entry upstream, which generates a variety-productivity effect downstream. To disentangle these two opposing forces, it is useful to compare the market allocation of labor to the social planner's optimal allocation.

Combining equations (2), (A.3), and (A.7), the aggregate decentralized market allocation of labor to the upstream sector is given by

$$M^{u}\ell^{u}=(1-\alpha)L,$$

while from equations (A.10) and (A.11), the social planner would allocate a share of labor to that sector equal to

$$M^{u}\ell^{u}=\frac{\theta}{\theta-\alpha}(1-\alpha)L>(1-\alpha)L.$$

Thus, the market equilibrium is inefficient, in the sense that it allocates too little labor to the upstream sector. It might seem intuitive that this inefficiency is associated with upstream markups leading to a double-marginalization inefficiency. However, note that the market allocation of labor to the upstream sector is actually *independent* of the degree of input substitutability (θ), and thus, of the level of upstream markups. In other words, lower input substitutability – and thus higher markups – do *not* depress the market allocation of labor to the upstream sector; instead, they increase the social-welfare maximizing allocation of labor to that sector. This fact does not necessarily rule out the relevance of a double marginalization inefficiency, but it does suggest that the market inefficiency may alternatively be interpreted as reflecting that, in the market equilibrium, upstream firms do not internalize the fact that their entry generates positive spillovers for firms in the downstream sector, with the size of this spillover decreasing in the degree of input substitutability θ.[3]

When $\alpha = 1$ or $\alpha = 0$, all labor is allocated to either the downstream sector (when $\alpha = 1$) or to the upstream sector (when $\alpha = 0$), and because firm-level output is always efficient, there is no scope for a market inefficiency in those two cases.

[2] Notice that for $\theta > 1$, $\left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha}\left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \geq 1$, with equality when α is either 0 or 1.

[3] This can be verified from the fourth constraint of the social planner problem above, which indicates that downstream productivity is proportional to $(M^{u})^{\frac{\theta(1-\alpha)}{\theta-1}}$.

A.3 Optimal Policy

Suppose we endow a government with the ability to provide subsidies (or charge taxes) on the purchases of final goods or intermediate inputs. Denote these taxes by s^d and s^u in the downstream and upstream sectors, respectively. We assume that subsidy proceeds are extracted from households (or tax revenue is rebated to households) in a lump-sum manner.

Once we introduce taxes, price indexes become:

$$P^u = (M^u)^{\frac{1}{1-\theta}} (1 - s^u)p^u, \qquad P^d = \left(M^d\right)^{\frac{1}{1-\sigma}} (1 - s^d)p^d$$

and household disposable income becomes,

$$I = wL - M^d s^d p^d x^d - M^u s^u p^u x^u.$$

It is straightforward to show that taxes and subsidies do not alter the equilibrium firm size, which is still pinned down by free entry at the (efficient) levels given in (A.3). Turning to the determination of the measure of firms in each sector, we first invoke households' demand for downstream goods combined with goods-market clearing and household total income to obtain

$$M^d = \frac{wL - s^u M^u p^u x^u}{p^d x^d}.$$

Next, labor market clearing ensures that equation (A.6) still holds. The equilibrium measure of firms, given subsidies s^d and s^u, is then:

$$\begin{aligned} M^u &= \frac{1}{1 - \alpha s^u} \frac{(1-\alpha)A^u L}{\theta f^u} \\ M^d &= (1 - s^u)^\alpha \left(\frac{1}{1 - \alpha s^u}\right)^{\frac{\theta-\alpha}{\theta-1}} \frac{\alpha^\alpha A^d}{\sigma f^d} \left[\frac{(1-\alpha)A^u}{\theta f^u}\right]^{(1-\alpha)\frac{\theta}{\theta-1}} ((\theta - 1) f^u)^{1-\alpha} L^{\frac{\theta-\alpha}{\theta-1}}. \end{aligned}$$

Notice that downstream subsidies s^d have no impact on the market allocation. Because they are a *redundant* instrument, we can safely set them to 0. From the above expressions, it is then clear that:

Proposition 2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $(s^u)^* = 1/\theta$.

Notice that the subsidy corresponds to the reciprocal of the elasticity of substitution across inputs. As a result, this subsidy encourages the entry of upstream suppliers especially when the inputs they produce are relatively less substitutable. There are two potential (and non-exclusive) explanations for this result. First, the lower is θ, the larger is the market power of and thus the markup charged by input suppliers, and thus the larger the subsidy required to undo this double marginalization inefficiency. Second, the lower is θ, the larger are the variety gains associated with upstream entry on the productivity of downstream firms, so to the extent that those gains are not internalized by input suppliers, again the larger is the required subsidy upstream.

A.4 Double Marginalization versus External Effects

We next dig a little bit deeper into the source of the market inefficiency. More specifically, we show that our vertical Krugman economy is isomorphic to a competitive vertical economy with external economies of scale. In this variant of our model, it is clear that the market inefficiency is due only to upstream suppliers failing to internalize the positive productivity effects of their entry on downstream firms (since there are no markups), and an upstream subsidy is again sufficient to restore efficiency.

The vertical economy with external economies of scale features consumers that spend their income on a single homogeneous final good. On the production side, this final good is produced combining labor and a homogeneous intermediate input, which is in turn produced with labor. The homogeneous intermediate input and final good are produced according to the technologies

$$\begin{aligned} x^u &= A^u \ell^u \left(L^u\right)^{\gamma^u} \\ x^d &= A^d \left(\ell^d\right)^{\alpha} \left(q^u\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{\gamma^d}, \end{aligned}$$

where L^u and L^d are the aggregate allocations of labor to the upstream and downstream sector, Q^u is total production upstream, and γ^u and γ^d measure the magnitude of external economies of scale.

Individual firms are symmetric, competitive, and infinitesimal, so they take the aggregates as given and price at marginal cost. The resulting prices for the upstream and downstream sector are given by

$$P^u = \frac{w}{A^u} \left(L^u\right)^{-\gamma^u}$$

and

$$P^d = \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$, $Q^u = A^u \left(L^u\right)^{1+\gamma^u}$ and $Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d}$, it is straightforward to infer that the equilibrium allocation of labor across sectors is given by

$$L^u = (1-\alpha) L, \quad \text{and} \quad L^d = \alpha L,$$

just as in our "Krugman" vertical economy with internal economies of scale. In addition, one can also show that whenever $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$, with an appropriate choice of the technological parameters A^d and A^u, the equilibria of the two models are fully isomorphic, not just in terms of the allocation of labor across sectors, but also in terms of prices and welfare.

As in the case of internal economies of scale, this market equilibrium can easily be shown to be inefficient. In particular, setting up the planner problem,

$$\begin{aligned} \max_{L^u, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d} \\ s.t. \quad & Q^u = A^u \left(L^u\right)^{1+\gamma^u} \\ & L^u + L^d = L, \end{aligned} \tag{A.14}$$

it is straightforward to see that this delivers

$$\left(L^u\right)^* = \frac{1+\gamma^u}{\gamma^u (1-\alpha) + 1} (1-\alpha) L, \quad \text{and} \quad \left(L^d\right)^* = \frac{1}{\gamma^u (1-\alpha) + 1} \alpha L. \tag{A.15}$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$. Finally, one can also verify that an upstream subsidy equal to $\left(s^u\right)^* = \gamma^u / (1 + \gamma^u)$ is sufficient to restore efficiency. While it is perhaps surprising that the planner need not make any correction for the external economies in the downstream sector, this result is due to the fact that all of that sector's output is sold to consumers. By contrast, the fact that inputs are all sold to firms means that their under provision requires a subsidy so that it does not distort final-good producers' purchases of inputs versus labor.

In sum, we have shown that a model with external economies of scales is isomorphic to our model with internal economies of scale as long as $\gamma^u = 1/(\theta - 1)$, and that the rationale for the use of upstream subsidies to restore efficiency can be tied to a love-for-variety productivity effect, rather than it being necessarily driven by a double marginalization inefficiency.

A.5 Extensions

In this Appendix, we briefly develop two extensions of our closed-economy model, both featuring a more complex input sector.

I. Roundabout Production Upstream

We first allow the upstream sector to use not only labor in production, but also the same bundle of inputs Q^u used in the final-good sector. More specifically, and focusing on the isomorphic economy with perfect competition, homogeneous goods, and external economies of scale developed in Section A.4, we now assume

$$\begin{aligned} x^u &= A^u (\ell^u)^\beta (q^u)^{1-\beta} \left((L^u)^\beta (Q^u)^{1-\beta} \right)^{\gamma^u} \\ x^d &= A^d \left(\ell^d\right)^\alpha (q^u)^{1-\alpha} \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{\gamma^d}, \end{aligned}$$

where β governs the labor intensity of production upstream. It is clear from the second of these expressions that firms in the downstream sector will spend a fraction of its costs on the upstream sector, or

$$P^u q^u = (1-\alpha) P^d x^d.$$

Noting that, due to symmetry, $x^u = Q^u$ and $x^d = Q^d$, and that the decentralized market prices for the downstream sector is given by

$$P^d = \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$ and $Q^d = A^d \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{1+\gamma^d}$ we thus obtain

$$\frac{1}{A^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u Q^u}{1-\alpha}\right)^{1-\alpha} A^d \left(L^d\right)^\alpha = wL$$

or

$$\left(\frac{w}{\alpha}\right)^\alpha (wL)^{1-\alpha} \left(L^d\right)^\alpha = wL,$$

from which it is immediate that

$$L^u = (1-\alpha) L, \quad \text{and} \quad L^d = \alpha L,$$

just as in our baseline model

We next consider the planner problem,

$$\begin{aligned}
\max_{L^u,L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} (Q^u)^{1-\alpha} \right)^{1+\gamma^d} \\
s.t. \quad & Q^u = A^u \left((L^u)^{\beta} (Q^u)^{1-\beta} \right)^{1+\gamma^u} \\
& L^u + L^d = L.
\end{aligned}$$

Noting that the second constraint can be written as

$$Q^u = \tilde{A}^u (L^u)^{1+\tilde{\gamma}^u},$$

where

$$\begin{aligned}
\tilde{A}^u &= (A^u)^{\frac{1}{1-(1-\beta)(1+\gamma^u)}} \\
\tilde{\gamma}^u &= \frac{\gamma^u}{1-(1-\beta)(1+\gamma^u)},
\end{aligned}$$

it then becomes clear that this program is identical to the one in our baseline model, except for the fact that the scale elasticity upstream is now not given by γ^u, but by $\tilde{\gamma}^u > \gamma^u$ (the program also features a rescaled upstream productivity, but that is immaterial). Note that the gap between $\tilde{\gamma}^u$ and γ^u is decreasing in β.

Analogously to equation (A.15), the socially optimal allocation of labor is given by

$$(L^u)^* = \frac{1+\tilde{\gamma}^u}{\tilde{\gamma}^u (1-\alpha)+1} (1-\alpha) L, \quad \text{and} \quad \left(L^d\right)^* = \frac{1}{\tilde{\gamma}^u (1-\alpha)+1} \alpha L.$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$, just as in our baseline model, but the inefficiency is now decreasing in β. Finally, one can also verify that an upstream subsidy equal to $(s^u)^* = \tilde{\gamma}^u / (1+\tilde{\gamma}^u)$ is sufficient to restore efficiency. Because $\tilde{\gamma}^u > \gamma^u$, this subsidy is now higher than in our baseline model, and it is decreasing in β.

II. Multi-Stage Production

We next develop a multi-stage extension of the model. We begin with a simple three-stage production process with a downstream sector, a midstream sector, and an upstream sector. The technologies are given by

$$\begin{aligned}
x^u &= A^u (\ell^u) (L^u)^{\gamma^u} \\
x^m &= A^d (\ell^m)^{\beta} (q^u)^{1-\beta} \left((L^m)^{\beta} (Q^u)^{1-\beta} \right)^{\gamma^m} \\
x^d &= A^d \left(\ell^d\right)^{\alpha} (q^m)^{1-\alpha} \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{\gamma^d},
\end{aligned}$$

Using the fact that, in a decentralized equilibrium, we have

$$\begin{aligned}
P^d Q^d &= wL; \\
Q^d &= A^d \left(\left(L^d\right)^{\alpha} \left(Q^m\right)^{1-\alpha} \right)^{1+\gamma^d}; \\
P^d &= \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^m}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^m\right)^{1-\alpha} \right)^{-\gamma^d}; \\
P^m Q^m &= (1-\alpha) P^d Q^d,
\end{aligned}$$

we immediately obtain

$$L^d = \alpha L.$$

Next, because

$$\begin{aligned}
P^m Q^m &= (1-\alpha) wL; \\
Q^m &= A^m \left(\left(L^m\right)^{\alpha} \left(Q^u\right)^{1-\alpha} \right)^{1+\gamma^m}; \\
P^m &= \frac{1}{A^m} \left(\frac{w}{\beta}\right)^{\beta} \left(\frac{P^u}{1-\beta}\right)^{1-\beta} \left(\left(L^m\right)^{\beta} \left(Q^u\right)^{1-\beta} \right)^{-\gamma^m}; \\
P^u Q^u &= (1-\beta) P^m Q^m,
\end{aligned}$$

we obtain

$$L^m = \beta (1-\alpha) L, \quad \text{and} \quad L^u = (1-\beta)(1-\alpha) L.$$

Now consider the planner problem

$$\begin{aligned}
\max_{L^u, L^m, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^m\right)^{1-\alpha} \right)^{1+\gamma^d} \\
s.t. \quad & Q^m = A^m \left(\left(L^m\right)^{\beta} \left(Q^u\right)^{1-\beta} \right)^{1+\gamma^u} \\
& Q^u = A^u \left(L^u\right)^{1+\gamma^u} \\
& L^u + L^m + L^d = L.
\end{aligned}$$

which delivers

$$\begin{aligned}
\left(L^u\right)^* &= \frac{(1+\gamma^u)(1+\gamma^m)}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} (1-\beta)(1-\alpha) L \\
\left(L^m\right)^* &= \frac{1+\gamma^m}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \beta (1-\alpha) L \\
\left(L^d\right)^* &= \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \alpha L.
\end{aligned}$$

Notice that the gap between the socially optimal and the market allocation of labor is higher the more upstream the stage. Does that mean that subsidies are higher, the more upstream a sector? To answer this question, consider the following key conditions identify a market equilibrium with subsidies:

$$\begin{aligned}
P^d Q^d &= wL - s^m P^m Q^m - s^u P^u Q^u \\
Q^d &= A^d \left(\left(L^d\right)^\alpha \left(Q^m\right)^{1-\alpha} \right)^{1+\gamma^d}; \\
P^d &= \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{(1-s^m) P^m}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^\alpha \left(Q^m\right)^{1-\alpha} \right)^{-\gamma^d}; \\
(1-s^m) P^m Q^m &= (1-\alpha) P^d Q^d \\
Q^m &= A^m \left(\left(L^m\right)^\alpha \left(Q^u\right)^{1-\alpha} \right)^{1+\gamma^m} \\
P^m &= \frac{1}{A^m} \left(\frac{w}{\beta}\right)^\beta \left(\frac{(1-s^u) P^u}{1-\beta}\right)^{1-\beta} \left(\left(L^m\right)^\beta \left(Q^u\right)^{1-\beta} \right)^{-\gamma^m} \\
(1-s^u) P^u Q^u &= (1-\beta) P^m Q^m
\end{aligned}$$

Note that

$$P^d Q^d = wL - \frac{s^m}{1-s^m} (1-\alpha) P^d Q^d - \frac{s^u}{1-s^u} \frac{(1-\beta)}{1-s^m} (1-\alpha) P^d Q^d$$

or

$$P^d Q^d = \frac{wL}{1 + \frac{s^m}{1-s^m} (1-\alpha) + \frac{s^u}{1-s^u} \frac{(1-\beta)}{1-s^m} (1-\alpha)}.$$

Next

$$\begin{aligned}
P^d Q^d &= \left(\frac{wL^d}{\alpha}\right)^\alpha \left(\frac{(1-s^m) P^m Q^m}{1-\alpha}\right)^{1-\alpha} \\
&= \left(\frac{wL^d}{\alpha}\right)^\alpha \left(P^d Q^d\right)^{1-\alpha},
\end{aligned}$$

so

$$\frac{L^d}{L} = \frac{\alpha}{1 + \frac{s^m}{1-s^m} (1-\alpha) + \frac{s^u}{1-s^u} \frac{(1-\beta)}{1-s^m} (1-\alpha)}.$$

Next,

$$\begin{aligned}
P^m Q^m &= \left(\frac{wL^m}{\beta}\right)^\beta \left(\frac{(1-s^u) P^u Q^u}{1-\beta}\right)^{1-\beta} \\
&= \left(\frac{wL^m}{\beta}\right)^\beta \left(P^m Q^m\right)^{1-\beta},
\end{aligned}$$

so

$$P^m Q^m = \frac{(1-\alpha)}{(1-s^m)} P^d Q^d = \frac{wL^m}{\beta}$$

or

$$\frac{L^m}{L} = \frac{\beta \frac{(1-\alpha)}{(1-s^m)}}{1 + \frac{s^m}{1-s^m} (1-\alpha) + \frac{s^u}{1-s^u} \frac{(1-\beta)}{1-s^m} (1-\alpha)}.$$

We thus have that the subsidies s^m and s^u need to satisfy

$$\frac{\alpha}{1 + \frac{s^m}{1-s^m} (1-\alpha) + \frac{s^u}{1-s^u} \frac{(1-\beta)}{1-s^m} (1-\alpha)} = \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \alpha$$

and

$$\frac{\beta \frac{(1-\alpha)}{(1-s^m)}}{1+\frac{s^m}{1-s^m}(1-\alpha)+\frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)} = \frac{1+\gamma^m}{\alpha+(1-\alpha)(1+\gamma^m)(\beta+(1-\beta)(1+\gamma^u))}\beta(1-\alpha),$$

which delivers

$$s^m = \frac{\gamma^m}{1+\gamma^m}; \quad s^u = \frac{\gamma^u}{1+\gamma^u}.$$

As is clear from this expression, subsidies in all sectors producing inputs are positive, but notice that subsidies are higher upstream relative to midstream only if $\gamma^u > \gamma^m$, i.e., only if the scale elasticity is higher upstream than midstream. This contrasts with the results of Liu (2019), who finds that optimal subsidies should necessarily be higher, the more upstream the sector. The reason is that, unlike in Liu's work, we solve for first-best subsidy policy: when the government can only set subsidies in one sector, the size of the subsidy would be higher, the more upstream the sector, because as we have seen above, the gap between the social optimal and market allocation of labor is highest in the upstream sector.

B Open Economy Model: Details on Derivations

B.1 Open Economy Equilibrium with Internal Economies of Scale

In this Appendix, we outline the equilibrium conditions corresponding to the two-country model featuring internal scale economies, product differentiation and monopolistic competition outlined in Section 3.1. We will then work with these equations in Appendix to demonstrate the isomorphism claimed in Proposition 3.

Import tariffs on the downstream sector create a wedge between consumer prices in country i and producer prices in country j. More specifically, given CES preferences, consumer prices in i for goods originating in j are given by:

$$p_{ji}^{d} = \left(1+t_{i}^{d}\right) \frac{\sigma}{\sigma-1} \tau^{d} \frac{1}{A_{j}^{d}} \left(\frac{w_{j}}{\alpha}\right)^{\alpha} \left(\frac{P_{j}^{u}}{1-\alpha}\right)^{1-\alpha} = \frac{1+t_{i}^{d}}{1-v_{j}^{d}} \tilde{p}_{ji}^{d}. \tag{B.1}$$

Similarly, import tariffs on the upstream sector create a wedge between the price paid by final-good producers in i for inputs from j, and the producer price for those inputs obtained by suppliers in country j. In particular, we have

$$p_{ji}^{u} = \left(1+t_{i}^{u}\right) \frac{\theta}{\theta-1} \tau^{u} \frac{w_{j}}{A_{j}^{u}} = \frac{1+t_{i}^{u}}{1-v_{j}^{u}} \tilde{p}_{ji}^{u}. \tag{B.2}$$

In equation (B.1), the price index P_{i}^{u} is given by

$$P_{i}^{u} = \left[\sum_{j\in\{H,F\}} \left(P_{ji}^{u}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{B.3}$$

with

$$P_{ji}^{u} = \left[\int_{0}^{M_{j}^{u}} \left(p_{ji}^{u}\left(\varpi\right)\right)^{1-\theta} d\varpi\right]^{\frac{1}{1-\theta}}. \tag{B.4}$$

When setting $j=i$, the above pricing equations also characterize domestic prices in country j after setting $t_{i}^{d} = t_{i}^{u} = v_{i}^{d} = v_{i}^{u} = 0$ and $\tau^{d} = \tau^{u} = 1$. Note that $p_{ii}^{d} = \tilde{p}_{ii}^{d}$ and $p_{ii}^{u} = \tilde{p}_{ii}^{u}$.

Next, utility maximization implies that when consuming country j varieties, consumers in i allocate to each variety ω a share of spending equal to

$$\frac{p_{ji}^{d}\left(\omega\right) q_{ji}^{d}\left(\omega\right)}{P_{ji}^{d} Q_{ji}^{d}} = \left(\frac{p_{ji}^{d}\left(\omega\right)}{P_{ji}^{d}}\right)^{1-\sigma}, \tag{B.5}$$

of their total spending on country j varieties, where

$$P_{ji}^{d} = \left[\int_{0}^{M_{j}^{d}} \left(p_{ji}^{d}\left(\omega\right)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}}. \tag{B.6}$$

Consumers' (aggregate) spending on Home and Foreign varieties is in turn determined by

$$P_{ji}^{d} Q_{ji}^{d} = \left(\frac{P_{ji}^{d}}{P_{i}^{d}}\right)^{1-\sigma} \left(w_{i} L_{i} + R_{i}\right), \tag{B.7}$$

where P_i^d is the aggregate consumer price index in i

$$P_i^d = \left[\sum_{j \in \{H,F\}} \left(P_{ji}^d \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{B.8}$$

and R_i is tariff revenue, which we have defined in equation (12).

We now turn to profit maximization by downstream producers in country i. First note that free entry implies that firm revenue (net of tariffs) will equal total costs, and that a share of those costs will go to pay labor. As a result, labor compensation by each final-good producer in i is given by

$$w_i \ell_i^d = \alpha \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right). \tag{B.9}$$

Next, when purchasing inputs from upstream producers in country j, final-good producers in country i, will demand an amount of each variety ϖ from country j equal to

$$q_{ji}^u(\varpi) = Q_{ji}^u \left(\omega \right) \left(\frac{p_{ji}^u}{P_{ji}^u} \right)^{-\theta},$$

while aggregate spending on all country j's input varieties is given by

$$P_{ji}^u Q_{ji}^u = (1 - \alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right) \left(\frac{P_{ji}^u}{P_i^u} \right)^{1-\theta} M_i^d. \tag{B.10}$$

Aggregate spending on Home and Foreign intermediate inputs in country i is then given by

$$P_i^u Q_i^u = (1 - \alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right) M_i^d. \tag{B.11}$$

Our final set of equilibrium conditions impose market clearing. First, labor-market clearing in both countries implies that

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u, \tag{B.12}$$

where ℓ_i^d is given in (B.9), and $\ell_i^u = \left(f_i^u + x_i^u \right) / A_i^u$.[4] Second, goods-market clearing imposes

$$q_{ii}^d + \tau^d q_{ij}^d = x_i^d \tag{B.13}$$

and

$$M_i^d q_{ii}^u + M_j^d \tau^u q_{ij}^u = x_i^u. \tag{B.14}$$

Note that free entry upstream and downstream implies that firm revenue is equal to total costs, which delivers

$$x_i^d = (\sigma - 1) f_i^d; \qquad x_i^u = (\theta - 1) f_i^u \tag{B.15}$$

for $i = \{H, F\}$. Firm-level production levels are thus independent of tariff choices, and the only way in which tariffs can affect the allocation of labor across sectors is by changing the measure of firms in each of the two

[4]Naturally, equilibrium also requires trade balance, but this is ensured by the other equilibrium conditions outlined in this section.

sectors. As a result, optimal trade policies will seek to achieve a social-welfare maximizing allocation of labor across sectors, with no concern for the allocation of labor within sectors (across fixed costs of entry versus marginal costs of production).

Despite the simple structure of the model and relatively simple equilibrium conditions, an analysis of how the market equilibrium is affected by input and final-good tariffs set by the Home country is complex, so we begin by considering the special case in which downstream production only uses inputs (and no labor) in production, or $\alpha = 0$.

B.2 Equilibrium of Isomorphic Competitive Economy with External Economies of Scale

In this Appendix we prove the isomorphism claimed in Proposition 3. More specifically, our goal is to show that equilibrium conditions of the decentralized equilibrium of the two-country model in Section 3.1 featuring internal scale economies, product differentiation and monopolistic competition can be reduced to a set of equations identical to equations (16) through (23) applying to the competitive model with external economies of scale developed in this section.

Preferences We begin by noting that given symmetry in final-good production, we can express preferences as

$$\begin{aligned} U_i &= \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[M_i^d (q_{ii}^d)^{\frac{\sigma-1}{\sigma}} + M_j^d (q_{ji}^d)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left(\left(Q_{ii}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{ji}^d\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where

$$Q_{ii}^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} q_{ii}^d; \quad Q_{ji}^d \equiv \left(M_j^d\right)^{\frac{\sigma}{\sigma-1}} q_{ji}^d. \tag{B.16}$$

Starting from (11), we have thus derived (13), which are preferences in the isomorphic economy with two final goods (a Home one and a Foreign one) and external economies of scale.

Labor-Market Clearing Next, remember that ℓ_i^d and ℓ_i^u are the firm-level amounts of labor used downstream and upstream to cover fixed and variable costs. Hence, defining

$$L_i^d \equiv M_i^d \ell_i^d; \quad L_i^u \equiv M_i^u \ell_i^u, \tag{B.17}$$

we have that equation (B.12) in the monopolistic competition model implies equation (16) in the external economies model, or

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u = L_i^u + L_i^d.$$

Upstream Market Clearing and Upstream Endogenous Productivity Next let us define

$$Q_{ii}^u \equiv M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u; \quad Q_{ij}^u \equiv M_j^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ij}^u. \tag{B.18}$$

Given these definitions in (B.18), and given the definition of the input aggregate $Q_i^u(\omega)$ in the monopolistic competition model, that is

$$Q_i^u(\omega) = \left[\sum_{j\in\{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi\right)\right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H,F\},$$

we have that the total usage of inputs by firms in country i is given by

$$\begin{aligned} Q_i^u &= M_i^d Q_i^u(\omega) = \left[M_i^u \left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + M_j^u \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(M_i^d \left(M_j^u\right)^{\frac{\theta}{\theta-1}} \left(q_{ji}^u\right)\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \end{aligned} \tag{B.19}$$

and thus is analogous to a CES aggregator of only two inputs: a Home and a Foreign one, as defined in equation (B.18). These inputs are either produced domestically or are imported.

Now consider the domestic production of those inputs. Let us start from the definition of upstream technology in the monopolistic competition model, that is

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H,F\}.$$

Imposing symmetry and firm-level output in equation (B.15) – i.e., $x_i^u = (\theta - 1) f_i^u$ –, and invoking the definition of L_i^u in (B.17), we have

$$X_i^u \equiv \left(M_i^u\right)^{\frac{\theta}{\theta-1}} x_i^u = \left(\frac{A_i^u}{\theta f_i^u}\right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u \left(L_i^u\right)^{\frac{\theta}{\theta-1}} \tag{B.20}$$

or

$$X_i^u = \hat{A}_i^u F_i^u\left(\ell_i^u\right) = \bar{A}_i^u \left(L_i^u\right)^{1+\gamma^u},$$

where

$$\bar{A}_i^u \equiv \left(\frac{A_i^u}{\theta f_i^u}\right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u,$$

and

$$\gamma^u \equiv 1/(\theta - 1).$$

Because this domestic production X_i^u is sold domestically or exported, we have

$$\bar{A}_i^u \left(L_i^u\right)^{1+\gamma^u} = Q_{ii}^u + Q_{ij}^u,$$

which corresponds exactly to equation (17) in the external economies model.

Downstream Market Clearing and Downstream Endogenous Productivity We can proceed analogously for final-good production. We begin with the definition of technology in the downstream sector in the monopolistic competition model:

$$f_i^d + x_i^d(\omega) = A_i^d(\ell_i^d(\omega))^{\alpha} Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H, F\}.$$

Imposing symmetry and (B.15), we obtain

$$M_i^d = \frac{A_i^d}{\sigma f_i^d}(M_i^d \ell_i^d(\omega))^{\alpha} \left(M_i^d Q_i^u\right)^{1-\alpha}$$

or

$$X_i^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^{\alpha} \left(\left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}}. \tag{B.21}$$

where

$$\bar{A}_i^d \equiv \left(\frac{A_i^d}{\sigma f_i^d}\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f_i^d.$$

This aggregate output X_i^d is sold domestically or exported, and thus

$$\bar{A}_i^d \left(\left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d} = Q_{ii}^d + Q_{ij}^d,$$

where

$$\gamma^d \equiv 1/(\sigma - 1).$$

In sum, starting from the monopolistic competition model, we have derived equation (17) in the external economies model.

Trade Balance Consider next the trade balance condition. Starting from the monopolistic competition economy, we have

$$\frac{p_{ji}^d}{1+t_i^d} M_j^d q_{ji}^d + \frac{p_{ji}^u}{1+t_i^u} M_i^d M_j^u q_{ji}^u = \frac{\tilde{p}_{ij}^d}{1-v_i^d} M_i^d q_{ij}^d + \frac{\tilde{p}_{ij}^u}{1-v_i^u} M_j^d M_i^u q_{ij}^u, \tag{B.22}$$

which equates the import revenue in i paid to exporters in j with export revenue collected from j by producers in i.

Now from equations (B.5) and (B.6), notice that we have

$$\frac{p_{ji}^d(\omega)\, q_{ji}^d(\omega)}{P_{ji}^d Q_{ji}^d} = \left(\frac{p_{ji}^d(\omega)}{P_{ji}^d}\right)^{1-\sigma},$$

and

$$P_{ji}^d = \left[\int_0^{M_j^d} \left(p_{ji}^d(\omega)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

so given symmetry, we have

$$P_{ji}^d = \left(M_j^d\right)^{\frac{1}{1-\sigma}} p_{ji}^d \tag{B.23}$$

and

$$P_{ji}^{d} Q_{ji}^{d} = \left(M_{j}^{d}\right)^{\frac{-1}{\sigma-1}} p_{ji}^{d} \times \left(M_{i}^{d}\right)^{\frac{\sigma}{\sigma-1}} q_{ii}^{d} = M_{j}^{d} p_{ji}^{d} q_{ii}^{d}.$$

Similarly, for inputs

$$P_{ji}^{u} Q_{ji}^{u} = \left(M_{j}^{u}\right)^{\frac{1}{1-\theta}} p_{ji}^{u} \times M_{i}^{d} \left(M_{i}^{u}\right)^{\frac{\theta}{\theta-1}} q_{ji}^{u} = p_{ji}^{u} M_{i}^{d} M_{j}^{u} q_{ji}^{u}.$$

This implies that we can write total imports in the trade balance condition (B.22) as

$$\frac{P_{ji}^{d}}{1+t_{i}^{d}} Q_{ji}^{d} + \frac{P_{ji}^{u}}{1+t_{i}^{u}} Q_{ji}^{u} = \bar{P}_{ji}^{d} Q_{ji}^{d} + \bar{P}_{ji}^{u} Q_{ji}^{u},$$

which corresponds to the left-hand-side of the trade balance condition (19) for the economy with external economies of scale after noting that $\bar{P}_{ji}^{d}$ and $\bar{P}_{ji}^{u}$ are the prices collected by country j (or Foreign) exporters (not those paid by domestic or country i consumers).

Now consider revenue from exporting final goods. Notice that, regardless of whether the Foreign government imposes import tariffs or not, we have that export revenue is

$$\frac{\tilde{p}_{ij}^{d}}{1-v_{i}^{d}} M_{i}^{d} q_{ij}^{d} + \frac{\tilde{p}_{ij}^{u}}{1-v_{i}^{u}} M_{j}^{d} M_{i}^{u} q_{ij}^{u}$$

Prices paid by country j are $\tilde{p}_{ij}^{d} / \left(1-v_{i}^{d}\right)$ and $\tilde{p}_{ij}^{u} / \left(1-v_{i}^{u}\right)$, so following analogous steps, the right-hand-side of (19) becomes

$$\frac{\tilde{P}_{ij}^{d}}{1-v_{i}^{d}} Q_{ij}^{d} + \frac{\tilde{P}_{ij}^{u}}{1-v_{i}^{u}} Q_{ij}^{u} = \bar{P}_{ij}^{d} Q_{ij}^{d} + \bar{P}_{ij}^{u} Q_{ij}^{u},$$

where $\bar{P}_{ij}^{d}$ and $\bar{P}_{ij}^{u}$ are the prices paid by country j (or Foreign) importers (not those paid collected by country i exporters).

Note: In the main text, we denote $\bar{P}_{ji}^{d}$, $\bar{P}_{ji}^{u}$, $\bar{P}_{ij}^{d}$, and $\bar{P}_{ij}^{u}$ as simply P_{ji}^{d}, P_{ji}^{u}, P_{ij}^{d}, and P_{ij}^{u}. We do so not to clutter the notation, but these are distinct from the price indices applying to the monopolistic competition model, which are always built based on prices paid by consumers, regardless of their country.

Optimality Conditions We have so far shown that the four 'resource' constraints (16) through (19) of our isomorphic economy can be derived from our baseline model with monopolistic competition and internal economies of scale. We next turn to an analogous derivation for the optimality conditions (20) through (23).

Given our functional forms for utility and technology, these optimality conditions in the model with external economies of scale are given by

$$\left(\frac{Q_{ii}^{d}}{Q_{ji}^{d}}\right)^{-\frac{1}{\sigma}} = \frac{\left(1-v_{i}^{d}\right)}{\left(1+t_{i}^{d}\right)} \frac{\bar{P}_{ij}^{d}}{\bar{P}_{ji}^{d}}; \tag{B.24}$$

$$\left(\frac{Q_{ii}^{u}}{Q_{ji}^{u}}\right)^{-\frac{1}{\theta}} = \frac{\left(1-v_{i}^{u}\right)}{\left(1+t_{i}^{u}\right)} \frac{\bar{P}_{ij}^{u}}{\bar{P}_{ji}^{u}}; \tag{B.25}$$

$$\left(1-\alpha\right) \hat{A}_{i}^{d} \left(L_{i}^{d}\right)^{\alpha} \left(\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^{u}} \frac{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1-v_{i}^{u}\right)}{\left(1-v_{i}^{d}\right)} \frac{\bar{P}_{ij}^{u}}{\bar{P}_{ij}^{d}}; \tag{B.26}$$

$$\left(1-\alpha\right) \hat{A}_{i}^{u} \frac{1}{Q_{ii}^{u}} \frac{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}} = \alpha \frac{1}{L_{i}^{d}}. \tag{B.27}$$

Optimal in Final-Good Consumption Let us begin with the first one, equating the marginal rate of substitution in final-good consumption to relative prices. Given equation (B.7) in the model with monopolistic competition, we have

$$\frac{Q_{ii}^d}{Q_{ji}^d} = \left(\frac{P_{ii}^d}{P_{ji}^d}\right)^{-\sigma},$$

where Q_{ji}^d, Q_{ji}^d, P_{ji}^d and P_{ji}^d are defined in (B.18) and (B.23). Thus, we have

$$\left(\frac{Q_{ii}^d}{Q_{ji}^d}\right)^{-\frac{1}{\sigma}} = \frac{P_{ii}^d}{P_{ji}^d} = \frac{\left(1-v_i^d\right)\bar{P}_{ij}^d}{\left(1+t_i^d\right)\bar{P}_{ji}^d},$$

where $P_{ii}^d = \left(1-v_i^d\right)\bar{P}_{ij}^d$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}_{ij}^d$ is the price paid by consumers in j for final goods from j). We have thus derived equation (B.24), which corresponds to (20) in the external economies of scale model.

Optimal in Input Consumption The derivation of equation (B.25), equating the marginal rate of substitution in input consumption to relative prices, is completely analogous. In particular, from equation (B.10) in the model with monopolistic competition, we have

$$\frac{Q_{ii}^u}{Q_{ji}^u} = \left(\frac{P_{ii}^u}{P_{ji}^u}\right)^{-\theta},$$

where Q_{ji}^u, Q_{ji}^u, P_{ji}^u and P_{ji}^u are defined in (B.18) and (B.23). Thus, we have

$$\left(\frac{Q_{ii}^u}{Q_{ji}^u}\right)^{-\frac{1}{\theta}} = \frac{P_{ii}^u}{P_{ji}^u} = \frac{\left(1-v_i^u\right)\bar{P}_{ij}^u}{\left(1+t_i^u\right)\bar{P}_{ji}^u},$$

where $P_{ii}^u = \left(1-v_i^d\right)\bar{P}_{ij}^u$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}_{ij}^u$ is the price paid by consumers in j for final goods from j). We have thus derived equation (B.25), which corresponds to (21) in the external economies of scale model.

Optimality Domestic Input Allocation We next move to the third optimality condition (22), which equates the benefits of exporting domestic intermediate inputs with the benefits of using those additional domestic inputs to produce an additional amount of the final good that is in turn exported.

We begin with equation (B.11), and note that aggregate input use in country i in the monopolistic competition model is given by

$$P_i^u Q_i^u = (1-\alpha)\left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d q_{ij}^d\right) M_i^d. \tag{B.28}$$

To reiterate this, note from (B.2) that $\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d = \tau^d p_{ii}^d$, and plugging in equation (B.13), we obtain

$$P_i^u Q_i^u = (1-\alpha)\, p_{ii}^d\left(q_{ii}^d + \tau^d q_{ij}^d\right) M_i^d = (1-\alpha)\, p_{ii}^d x_i^d M_i^d. \tag{B.29}$$

Next invoke equation (B.10) applied to $P_{ii}^u Q_{ii}^u$ to obtain (after plugging in (B.28)):

$$P_i^u Q_i^u = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}. \tag{B.30}$$

Combining (B.29) and (B.30), we obtain:

$$(1-\alpha)\, p_{ii}^d x_i^d M_i^d = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1},$$

which we decompose as

$$(1-\alpha) \times \left(M_i^d\right)^{\frac{-1}{\sigma-1}} p_{ii}^d \times \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}, \tag{B.31}$$

Now remember from equation (B.21) derived above that

$$\left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^{\alpha} \left(\left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}},$$

and also from (B.23) that $\left(M_i^d\right)^{\frac{-1}{\sigma-1}} p_{ii}^d = P_{ii}^d$, so we can write (B.31) as

$$P_{ii}^d (1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} = P_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}.$$

Now invoke (B.10)

$$\frac{Q_{ii}^u}{Q_i^u} = \left(\frac{P_{ii}^u}{P_i^u}\right)^{-\theta},$$

to obtain

$$P_{ii}^d (1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} = P_{ii}^u \left(\frac{Q_{ii}^u}{Q_i^u}\right)^{-\frac{\theta-1}{\theta}},$$

which given the definition of Q_i^u in (B.1) delivers

$$P_{ii}^d (1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}} = P_{ii}^u.$$

The final step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^d = \left(1 - v_i^d\right) \bar{P}_{ij}^d$ and $P_{ii}^u = \left(1 - v_i^d\right) \bar{P}_{ij}^u$, where remember that $\bar{P}_{ij}^d$ and $\bar{P}_{ij}^u$ are the prices paid by country j residents. In sum, we have derived equation (B.26), or

$$(1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1 - v_i^d\right) \bar{P}_{ij}^u}{\left(1 - v_i^d\right) \bar{P}_{ij}^d}.$$

Optimal Labor Market Allocation We finally tackle the fourth optimality condition, associated with the optimal allocation of labor across sectors. We begin with the firm-level monopolistic competition model, equating the wage paid in both sectors. Because of free entry, total revenue upstream must equal total wage payments, while in the downstream sector, wage payments are a share α of total revenue, as indicated in equation (B.9), or

$$\frac{\alpha\left(\tilde{p}_{ii}^{d}q_{ii}^{d}+\frac{1-v_{i}^{d}}{1+t_{j}^{d}}\tilde{p}_{ij}^{d}q_{ij}^{d}\right)}{\ell_{i}^{d}}=\frac{\tilde{p}_{ii}^{u}M_{i}^{d}q_{ii}^{u}+\frac{1-v_{i}^{u}}{1+t_{j}^{u}}\tilde{p}_{ij}^{u}M_{j}^{d}q_{ij}^{u}}{\ell_{i}^{u}}.$$

Now noting that from (B.2), we have $\frac{1-v_{i}^{d}}{1+t_{j}^{d}}\tilde{p}_{ij}^{d}=\tau^{d}p_{ii}^{d}$ (and analogously $\frac{1-v_{i}^{u}}{1+t_{j}^{u}}\tilde{p}_{ij}^{u}=\tau^{u}p_{ii}^{u}$), and plugging in equations (B.13) and (B.14), we have

$$\frac{\alpha p_{ii}^{d}x_{i}^{d}}{\ell_{i}^{d}}=\frac{p_{ii}^{u}x_{i}^{u}}{\ell_{i}^{u}}. \tag{B.32}$$

Next, invoke the price index definitions – see equation B.23) – as well as the definitions $L_{i}^{d}=M_{i}^{d}\ell_{i}^{d}$ and $L_{i}^{u}=M_{i}^{u}\ell_{i}^{u}$, to write (B.32) as

$$\alpha P_{ii}^{d}\frac{x_{i}^{d}\left(M_{i}^{d}\right)^{\frac{\sigma}{\sigma-1}}}{L_{i}^{d}}=P_{ii}^{u}\frac{\left(M_{i}^{u}\right)^{\frac{\theta}{\theta-1}}x_{i}^{u}}{L_{i}^{u}}.$$

Next, plugging (B.20) and (B.21), delivers

$$\frac{\alpha P_{ii}^{d}}{L_{i}^{d}}\hat{A}_{i}^{d}\left(L_{i}^{d}\right)^{\alpha}\left(\left(L_{i}^{d}\right)^{\alpha}\left(\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}=P_{ii}^{u}\hat{A}_{i}^{u},$$

where remember that $\hat{A}_{i}^{d}$ and $\hat{A}_{i}^{u}$ are defined in equations (14) and (15) in the main text.

The next step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^{d}=\left(1-v_{i}^{d}\right)\bar{P}_{ij}^{d}$ and $P_{ii}^{u}=\left(1-v_{i}^{d}\right)\bar{P}_{ij}^{u}$, where remember that $\bar{P}_{ij}^{d}$ and $\bar{P}_{ij}^{u}$ are the prices paid by country j residents, so we have

$$\frac{\alpha}{L_{i}^{d}}\hat{A}_{i}^{d}\left(L_{i}^{d}\right)^{\alpha}\left(\left(L_{i}^{d}\right)^{\alpha}\left(\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}=\hat{A}_{i}^{u}\frac{\left(1-v_{i}^{d}\right)}{\left(1-v_{i}^{d}\right)}\frac{\bar{P}_{ij}^{u}}{\bar{P}_{ij}^{d}}.$$

The final step is to plug optimality condition (B.26) and cancel terms to obtain

$$\frac{\alpha}{L_{i}^{d}}=\left(1-\alpha\right)\hat{A}_{i}^{u}\frac{1}{Q_{ii}^{u}}\frac{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}},$$

which corresponds to the last optimality condition (B.26).

This completes the proof of the isomorphism claimed in Proposition 3.

C Optimal Trade Policy for a Small Open Economy with No Domestic Distortions: Derivations

C.1 First-Best Policies

We begin by characterizing the solution to the program

$$
\begin{aligned}
\max \quad & U\left(Q^d_{HH}, Q^d_{FH}\right) \\
s.t. \quad & \hat{A}^u_H L_H = Q^u_{HH} + Q^u_{HF} \\
& \hat{A}^d_H F^d\left(Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\
& P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}},
\end{aligned}
$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H \left(L_H\right)^{\gamma^u},$$

respectively. We also note that

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and that

$$F^d\left(Q^u_{HH}, Q^u_{FH}\right) = \left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$
\begin{aligned}
& U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u\left[\bar{A}^u_H \left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] + \mu_d\left[\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
& + \mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right].
\end{aligned}
$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, and Q^u_{HF} are as follows:

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && \text{(C.1)} \\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} && \text{(C.2)} \\
\mu_d &= \mu_{TB} \frac{\sigma-1}{\sigma} P^d_{HF} && \text{(C.3)} \\
\mu_u &= \mu_d \left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) && \text{(C.4)} \\
\mu_{TB} P^u_{FH} &= \mu_d \left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right) && \text{(C.5)} \\
\mu_u &= \mu_{TB} \frac{\theta-1}{\theta} P^u_{HF} && \text{(C.6)}
\end{aligned}
$$

Dividing equation (25) by equation (C.2), and plugging in (C.3), we obtain:

$$\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{\frac{\sigma-1}{\sigma} P^d_{HF}}{P^d_{FH}},$$

which corresponds to the first optimality condition (25) in the main text.

Next, we divide equation (C.4) by equation (C.5), and plugging in (C.6), delivers

$$\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} = \frac{\frac{\theta-1}{\theta} P^u_{HF}}{P^u_{FH}},$$

which corresponds to the second optimality condition (26) in the main text.

Finally, combining equation (C.4) with the ratio of equations (C.3) and (C.6) produces

$$\left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}} \frac{P^u_{HF}}{P^d_{HF}},$$

which corresponds to the third optimality condition (27) in the main text.

C.2 Generalizations

As demonstrated in the derivations in the above Appendix C.1, we have made no use of the properties of the functions $U\left(Q^d_{HH}, Q^d_{FH}\right)$ and $F^d\left(Q^u_{HH}, Q^u_{FH}\right)$. In particular, we could assume that

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma_H-1}{\sigma_H}} + \left(Q^d_{FH}\right)^{\frac{\sigma_H-1}{\sigma_H}}\right)^{\frac{\sigma_H}{\sigma_H-1}}$$

and that

$$F^d\left(Q^u_{HH}, Q^u_{FH}\right) = \left(\left(Q^u_{HH}\right)^{\frac{\theta_H-1}{\theta_H}} + \left(Q^u_{FH}\right)^{\frac{\theta_H-1}{\theta_H}}\right)^{\frac{\theta_H}{\theta_H-1}},$$

with potentially $\sigma_H \neq \sigma$ and $\theta_H \neq \theta$. It is clear from the derivations in Section 4.1 that the first-best trade policies will continue to satisfy

$$\begin{aligned}
1+t^d_H &= \left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t^u_H &= 1+\bar{T}; \\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1-v^u_H &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

The only significant difference in this case is that if we want to invoke our isomorphism to claim that these policies also implement the first-best in the Krugman vertical economy with internal economies of scale, then we necessarily need to impose $\gamma^d = 1/\left(\sigma_H - 1\right)$, and thus the level of the tariff escalation is closely related to the degree of differentiation in the final-good sector. This is not particularly surprising, since love-for-variety effects will be more powerful, the lower the degree of substitutability across final goods.

C.3 Alternative First-Best Implementations

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes domestic production subsidies, domestic consumption subsidies, or domestic production/consumption subsidies that only apply to domestic transactions.

C.3.1 Discriminatory Domestic Subsidies

We first consider the case in which the Home government has access to discriminatory domestic subsidies s^d_{HH} and s^u_{HH} that apply only to purchases of final goods and of intermediate inputs involving only Home residents. The inclusion of these instruments alters the decentralized market equilibrium conditions (20), (21) and (22) as follows:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} &= \left(1-s^d_{HH}\right)\frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} &= \left(1-s^u_{HH}\right)\frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) &= \left(1-s^u_{HH}\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (25), (26), and (27), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
\left(1+t^d_H\right)\left(1-s^d_{HH}\right) &= \left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1+t^u_H &= 1+\bar{T};\\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
\left(1-s^u_{HH}\right)\left(1-v^u_H\right) &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1+t^d_H\right)\left(1-s^d_{HH}\right)$ matters), while an upstream discriminatory subsidy is a perfect substitute for the upstream export tax (only the product $\left(1-s^u_{HH}\right)\left(1-v^u_H\right)$ matters). A straightforward implication of this result is that, whenever $1+\gamma^d=\sigma/\left(\sigma-1\right)$, as imposed by our isomorphism, the first-best can be attained by setting $\left(1+t^d_H\right)\left(1-s^d_{HH}\right)=\sigma/\left(\sigma-1\right)$ and $\left(1-s^u_{HH}\right)\left(1-v^u_H\right)=\left(\theta-1\right)/\theta$. Thus, the first-best can be achieved with only discriminatory subsidies, or with a combination of a subsidy in one sector and a trade instrument in the other sector. When only domestic instruments are used, we necessarily have $s^d_{HH}=1/\sigma$ and $s^u_{HH}=1/\theta$. Whether or not the resulting first-best policies entail tariff escalation depends on the level of the downstream domestic subsidy, since $\left(1+t^d_H\right)/\left(1+t^u_H\right)=\left(1+\gamma^d\right)/\left(1-s^d_{HH}\right)$.

C.3.2 Production Subsidies

We next consider the case of production subsidies s^d_H and s^u_H that apply to Home production of final goods and intermediate inputs, regardless of where those goods are sold (domestically or exported). The inclusion of these instruments alters the decentralized market equilibrium conditions (20), (21) and (22) as follows:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) &= \left(1-s^d_H\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (25), (26), and (27), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
1+t^d_H &= \left(1-s^d_H\right)\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1+t^u_H &= 1+\bar{T};\\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1-s^d_H\right)\left(1+\bar{T}\right);\\
1-v^u_H &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

Notice that, as long as $s^d_H > 0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s^d_H = \gamma^d/\left(1+\gamma^d\right)$, the first-best can be achieved with this production subsidy and two export taxes ($1-v^d_H = (\sigma-1)/\sigma$ and $1-v^u_H = (\theta-1)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s^d_H = (1+\gamma^u)/\gamma^u = (\theta-1)/\theta$, the first-best can be achieved with this production subsidy and two import tariffs ($t^d_H = 1/(\sigma-1)$ and $t^u_H = 1/(\theta-1)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1+t^d_H}{1+t^u_H} = \left(1-s^d_H\right)\left(1+\gamma^d\right).$$

C.3.3 Consumption Subsidies

We finally consider the case of consumption subsidies s^d_H and s^u_H that apply to Home consumption of final goods and of intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments alters the decentralized market equilibrium conditions (20), (21) and (22) as follows:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) &= \left(1-s^u_H\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

These equations are completely analogous to those applying to the case of production subsidies, with s^u_H replacing s^d_H, so the conclusions that arise from it are also analogous.

C.4 Second-Best Import Tariffs

In this Appendix we characterize the second-best import tariffs when the government only has access to import tariffs upstream and downstream.

A. Second-Best Import Tariffs with Scale Economies

As mentioned in the main text, the second-best optimal allocation will seek to solve the same problem laid out in Section 4.1 expanded to include the additional constraint:

$$\hat{A}^d_H F^d_{Q^u_{HH}}(Q^u_{HH}, Q^u_{FH}) = \frac{P^d_{HF}}{P^u_{HF}} = \frac{(Q^u_{HF})^{-\frac{1}{\theta}} P^u_{FF} (Q^u_{FF})^{\frac{1}{\theta}}}{(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}.$$

More specifically, the planner problem is now

$$\begin{aligned}
\max \quad & U\left(Q^d_{HH}, Q^d_{FH}\right) \\
s.t. \quad & \hat{A}^u_H L_H = Q^u_{HH} + Q^u_{HF} \\
& \hat{A}^d_H F^d(Q^u_{HH}, Q^u_{FH}) = Q^d_{HH} + Q^d_{HF} \\
& P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}(Q^u_{HF})^{-\frac{1}{\theta}} P^u_{FF}(Q^u_{FF})^{\frac{1}{\theta}} \\
& \hat{A}^d_H F^d_{Q^u_{HH}}(Q^u_{HH}, Q^u_{FH}) = \frac{(Q^u_{HF})^{-\frac{1}{\theta}} P^u_{FF}(Q^u_{FF})^{\frac{1}{\theta}}}{(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d(Q^u_{HH}, Q^u_{FH})\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H (L_H)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$\begin{aligned}
& U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u \left[\bar{A}^u_H (L_H)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] + \mu_d \left[\bar{A}^d_H \left(F^d(Q^u_{HH}, Q^u_{FH})\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
& + \mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}(Q^u_{HF})^{-\frac{1}{\theta}} P^u_{FF}(Q^u_{FF})^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right] \\
& + \mu_{SB}\left[\hat{A}^d_H F^d_{Q^u_{HH}}(Q^u_{HH}, Q^u_{FH}) - \frac{(Q^u_{HF})^{-\frac{1}{\theta}} P^u_{FF}(Q^u_{FF})^{\frac{1}{\theta}}}{(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}\right]
\end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, and Q^u_{HF} are as follows:

$$
\begin{aligned}
U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_d && \text{(C.7)}\\
U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_{TB} P_{FH}^d && \text{(C.8)}\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma}P_{HF}^d - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d} && \text{(C.9)}\\
\mu_u &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\\
&\quad +\mu_{SB}\bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\\
&\quad \times\left[\gamma^d\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F^d\left(Q_{HH}^u, Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}\right] && \text{(C.10)}\\
\mu_{TB}P_{FH}^u &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\\
&\quad +\mu_{SB}\bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\\
&\quad \times\left[\gamma^d\frac{F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F^d\left(Q_{HH}^u, Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}\right] && \text{(C.11)}\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta}P_{HF}^u + \mu_{SB}\frac{1}{\theta}\frac{1}{Q_{HF}^u}\frac{P_{HF}^u}{P_{HF}^d} && \text{(C.12)}
\end{aligned}
$$

In these derivations, note that we have used

$$
\begin{aligned}
\frac{\partial\left(\bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)}{\partial Q_{HH}^u} &= \bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\\
&\quad\times\left[\gamma^d\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F^d\left(Q_{HH}^u, Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial\left(\bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)}{\partial Q_{FH}^u} &= \bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)\\
&\quad\times\left[\gamma^d\frac{F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F^d\left(Q_{HH}^u, Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}\right].
\end{aligned}
$$

From equations (C.7), (C.8), and (C.9), we obtain:

$$
\frac{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} = \frac{P_{FH}^d}{P_{HF}^d}\left(\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}\right).
$$

Because in a competitive equilibrium with import tariffs we have

$$
\frac{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} = \left(1+t_H^d\right)\frac{P_{FH}^d}{P_{HF}^d}, \tag{C.13}
$$

we can establish that

$$1+t_H^d=\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}. \tag{C.14}$$

Also, note from equations (C.7) and (C.8), as well as (C.13), that

$$1+t_H^d=\frac{\mu_{TB}P_{HF}^d}{\mu_d}. \tag{C.15}$$

In a competitive equilibrium with import tariffs, we also have that

$$\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\frac{P_{HF}^u}{\left(1+t_H^u\right)P_{FH}^u}. \tag{C.16}$$

Furthermore, the last constraint in the planner problem can be written as:

$$\bar{A}_H^d\left(F^d\left(Q_{HH}^u,Q_{FH}^u\right)\right)^{\gamma^d}F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)=\frac{P_{HF}^u}{P_{HF}^d}. \tag{C.17}$$

Now combine equations (C.11), (C.15), (C.16), and (C.17) to obtain

$$\frac{1+t_H^d}{1+t_H^u}=1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\left[\gamma^d\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F^d\left(Q_{HH}^u,Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}\right]. \tag{C.18}$$

We next work with equations (C.10) and plug in (C.11) and (C.17) to obtain

$$\mu_{TB}P_{FH}^u\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\mu_u+\mu_{SB}\frac{P_{HF}^u}{P_{HF}^d}\left[\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}\right]$$

And plugging μ_u from equation (C.12), we get

$$\frac{\mu_{TB}}{\mu_{SB}\frac{P_{HF}^u}{P_{HF}^d}}=\frac{\frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}}{P_{FH}^u\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{\theta-1}{\theta}P_{HF}^u}$$

Invoking equation (C.16) we can simplify this last expression further to

$$\frac{\mu_{TB}}{\mu_{SB}}P_{HF}^d=\frac{\frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}}{\frac{1}{\left(1+t_H^u\right)}-\frac{\theta-1}{\theta}} \tag{C.19}$$

The three equations (C.14), (C.18), and (C.19) are sufficient to characterize the properties of second-best import tariffs. In particular, these equations can be reduced to

$$1+t_H^d = \frac{\sigma}{\sigma-1}+\left[\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right)\right]\frac{A}{C} \tag{C.20}$$

$$\frac{1+t_H^d}{1+t_H^u} = 1+\gamma^d+\left[\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right)\right]\frac{B}{C}, \tag{C.21}$$

where

$$
\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}>0;\\
B &= \gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)};\\
C &= \frac{1}{\theta}\frac{1}{Q^u_{HF}}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}-\frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}.
\end{aligned}
$$

Using

$$
F^d\left(Q^u_{HH},Q^u_{FH}\right)=\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}+\left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},
$$

it is easy to verify that

$$
\begin{aligned}
B &= \gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}=\left(\gamma^d+\frac{1}{\theta}\right)\frac{1}{Q^u_{HH}}\frac{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}}+\left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}}>0\\
C &= \frac{1}{\theta}\frac{1}{Q^u_{HF}}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}-\frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}=\frac{1}{\theta}\left(\frac{1}{Q^u_{HH}}+\frac{1}{Q^u_{HF}}\right)>0.
\end{aligned}
$$

Now note that, manipulating (C.20) and (C.21), we obtain

$$
\begin{aligned}
1+t^d_H &= \frac{\sigma}{\sigma-1}+\left[\frac{1+t^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+t^d_H\right)\right]\frac{A}{C}\\
\frac{1+t^d_H}{1+t^u_H} &= 1+\gamma^d+\left[\frac{1+t^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+t^d_H\right)\right]\frac{B}{C},
\end{aligned}
$$

Solving this system delivers

$$
1+t^d_H=\frac{\sigma}{\sigma-1}\frac{1-\frac{B}{C}+\frac{1+\gamma^d}{\frac{\sigma}{\sigma-1}}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}
$$

and

$$
\frac{1+t^d_H}{1+t^u_H}=\left(1+\gamma^d\right)\frac{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{\theta-1}{\theta}\frac{\frac{\sigma}{\sigma-1}}{1+\gamma^d}\frac{B}{C}}{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{B}{C}}.
$$

Noting that $1+\gamma^d=\frac{\sigma}{\sigma-1}$ in our isomorphism, immediately implies

$$
1+t^d_H>\frac{\sigma}{\sigma-1}
$$

and

$$
\frac{1+t^d_H}{1+t^u_H}>1+\gamma^d=\frac{\sigma}{\sigma-1}.
$$

This proves Proposition 6.

B. Second-Best Import Tariffs with No Scale Economies

Given the above derivations, it is straightforward to prove Proposition 7. Simply set $\gamma^d = 0$ in the system (C.20) and (C.21), and obtain

$$1 + t_H^d = \frac{\sigma}{\sigma - 1} \frac{1 - \frac{B}{C} + \frac{\sigma-1}{\sigma}\frac{A}{C}}{1 - \frac{B}{C} + \frac{\theta-1}{\theta}\frac{A}{C}}$$

and

$$\frac{1 + t_H^d}{1 + t_H^u} = \frac{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}\frac{B}{C}}{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{B}{C}}.$$

From the second equation, is clear that $\frac{1+t_H^d}{1+t_H^u} > 1$ if and only if $\frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1} < 1$, or $\sigma > \theta$. Furthermore, when $\theta = \sigma$, we have

$$1 + t_H^d = 1 + t_H^u = \frac{\sigma}{\sigma - 1} = \frac{\theta}{\theta - 1}.$$

D Optimal Trade Policy for a Small Open Economy with Domestic Distortions: Derivations

D.1 First-Best Policies with an Upstream Production Subsidy

We begin by characterizing the solution to the program

$$\max \quad U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned} s.t. \quad & L^u_H + L^d_H = L_H \\ & \hat{A}^u_H\left(L^u_H\right)L^u_H = Q^u_{HH} + Q^u_{HF} \\ & \hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\ & P^d_{FH}Q^d_{FH} + P^u_{FH}Q^u_{FH} = Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}}P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H\left(L_H\right)^{\gamma^u},$$

respectively. We also note that

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and that

$$F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(L^d_i\right)^{\alpha}\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$\begin{aligned} & U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_L\left[L_H - L^u_H - L^d_H\right] + \mu_u\left[\bar{A}^u_H\left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] \\ & + \mu_d\left[\bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\ & + \mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}}P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}}P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH}Q^d_{FH} - P^u_{FH}Q^u_{FH}\right]. \end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}, L^d_H, and L^u_H

are as follows:

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && \text{(D.1)}\\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} && \text{(D.2)}\\
\mu_d &= \mu_{TB} \frac{\sigma-1}{\sigma} P^d_{HF} && \text{(D.3)}\\
\mu_u &= \mu_d \left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) && \text{(D.4)}\\
\mu_{TB} P^u_{FH} &= \mu_d \left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) && \text{(D.5)}\\
\mu_u &= \mu_{TB} \frac{\theta-1}{\theta} P^u_{HF} && \text{(D.6)}\\
\mu_L &= \mu_u \left(1+\gamma^u\right) \bar{A}^u_H \left(L_H\right)^{\gamma^u} && \text{(D.7)}\\
\mu_L &= \mu_d \left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) && \text{(D.8)}
\end{aligned}
$$

Dividing equation (D.1) by equation (D.2), and plugging in (D.3), we obtain:

$$
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} = \frac{\frac{\sigma-1}{\sigma} P^d_{HF}}{P^d_{FH}},
$$

which corresponds to the first optimality condition (25) in the main text.

Next, we divide equation (D.4) by equation (D.5), and plugging in (D.6), delivers

$$
\frac{F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} = \frac{\frac{\theta-1}{\theta} P^u_{HF}}{P^u_{FH}},
$$

which corresponds to the second optimality condition (26) in the main text.

Next, combining equation (D.4) with the ratio of equations (D.3) and (D.6) produces

$$
\left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}} \frac{P^u_{HF}}{P^d_{HF}},
$$

which corresponds to the third optimality condition (27) in the main text.

Finally, from equations (D.7) by equation (D.8), and plugging in (D.4), we obtain

$$
F^d_{L^d_H}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(1+\gamma^u\right) \bar{A}^u_H \left(L_H\right)^{\gamma^u} F^d_{Q^u_{HH}}\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right),
$$

which corresponds to equation (33) in the main text, after noting that $\bar{A}^u_H \left(L_H\right)^{\gamma^u} = \hat{A}^u_H$.

D.2 First-Best Policies with Alternative Instruments

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes instruments other than domestic upstream production subsidies and trade taxes.

D.2.1 Discriminatory Domestic Subsidies

Consider first the case in which the Home government has access to discriminatory domestic subsidies s^d_{HH} and s^u_{HH} that apply only to purchases of final goods and of intermediate inputs involving only Home residents.

The inclusion of these instruments alters the decentralized market equilibrium conditions (20), (21), (22), and (23) as follows:

$$\begin{aligned}
\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} &= \left(1 - s_{HH}^d\right) \frac{\left(1 - v_H^d\right)}{\left(1 + t_H^d\right)} \frac{P_{HF}^d}{P_{FH}^d}; \\
\frac{F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)} &= \left(1 - s_{HH}^u\right) \frac{\left(1 - v_H^u\right)}{\left(1 + t_H^u\right)} \frac{P_{HF}^u}{P_{FH}^u}; \\
\hat{A}_H^d F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \left(1 - s_{HH}^u\right) \frac{\left(1 - v_H^u\right)}{\left(1 - v_H^d\right)} \frac{P_{HF}^u}{P_{HF}^d}; \\
F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \frac{1}{1 - s_{HH}^u} \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u.\right)
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (25), (26), (27), and (33), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right) &= \left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
1 + t_H^u &= 1 + \bar{T}; \\
1 - v_H^d &= \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
\left(1 - s_{HH}^u\right)\left(1 - v_H^u\right) &= \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right); \\
\frac{1}{1 - s_{HH}^u} &= \frac{\theta}{\theta - 1}.
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right)$ matters). By contrast, an export tax is *no longer* a perfect substitute when $\alpha > 0$, then the first best requires an upstream subsidy More specifically, because the first-best calls for $s_{HH}^u = 1/\theta$, we must necessarily have

$$1 - v_H^u = 1 + t_H^u = 1 + \bar{T}.$$

A straightforward implication of this result is that, whenever $1 + \gamma^d = \sigma / (\sigma - 1)$, as imposed by our isomorphism, the first-best can be attained by setting $\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right) = \sigma / (\sigma - 1)$, $1 - s_{HH}^u = (\theta - 1)/\theta$, and $v_H^u = v_H^d = t_H^u = 0$. Thus, the first-best can be achieved with only two discriminatory subsidies, or with a combination of an upstream subsidy and a downstream import tariff. When only domestic instruments are used, we necessarily have $s_{HH}^d = 1/\sigma$ and $s_{HH}^u = 1/\theta$. If the downstream subsidy is not used, then $1 + t_H^d = 1 + \gamma^d$, and thus $\left(1 + t_H^d\right)/\left(1 + t_H^u\right) = 1 + \gamma^d$ as well.

D.2.2 Production Subsidies

We next consider the case in which the Home government uses a nondiscriminatory downstream production subsidy s_H^d in addition to a nondiscriminatory upstream production subsidy, as in our baseline implementation. The inclusion of this instrument does not affect the market equilibrium condition (23), while it alters the decentralized market equilibrium conditions (20), (21), (22) in a manner analogous to the case $\alpha = 0$, that is:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)} \frac{P^d_{HF}}{P^d_{FH}}; \\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)} \frac{P^u_{HF}}{P^u_{FH}}; \\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) &= \left(1-s^d_H\right) \frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)} \frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (25), (26), and (27), it is clear that the first-best can be achieved by setting

$$\begin{aligned}
1+t^d_H &= \left(1-s^d_H\right)\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t^u_H &= 1+\bar{T}; \\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1-s^d_H\right)\left(1+\bar{T}\right); \\
1-v^u_H &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

Notice that, as long as $s^d_H > 0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s^d_H = \gamma^d/\left(1+\gamma^d\right)$, the first-best can be achieved with this production subsidy, the upstream production subsidy at a level $s^u_H = \gamma^u/\left(1+\gamma^u\right)$ and two export taxes ($1-v^d_H = (\sigma-1)/\sigma$ and $1-v^u_H = (\theta-1)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s^d_H = (1+\gamma^u)/\gamma^u = (\theta-1)/\theta$, the first-best can be achieved with this production subsidy and two import tariffs ($t^d_H = 1/(\sigma-1)$ and $t^u_H = 1/(\theta-1)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1+t^d_H}{1+t^u_H} = \left(1-s^d_H\right)\left(1+\gamma^d\right).$$

When only upstream production subsidies and trade taxes are used, the first-best policies continues to feature a tariff escalation wedge equal to $1+\gamma^d = \sigma/(\sigma-1)$.

D.2.3 Consumption Subsidies

We finally consider the case of consumption subsidies s^d_H and s^u_H that apply to Home consumption of final goods and intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments does not affect the market equilibrium condition (23), as long as s^u_H is set at $s^u_H = 1/\theta$. Furthermore, the decentralized market equilibrium conditions (20), (21) and (22) become:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)} \frac{P^d_{HF}}{P^d_{FH}}; \\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)} \frac{P^u_{HF}}{P^u_{FH}}; \\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) &= \left(1-s^u_H\right) \frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)} \frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

These equations are completely analogous to those applying to the case of production subsidies, with s_H^u replacing s_H^d, but note that we now necessarily have $s_H^u = 1/\theta$. As a result, replacing $1+\gamma^d = \sigma/(\sigma-1)$, we obtain

$$\begin{aligned}
1+t_H^d &= \frac{\theta-1}{\theta}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

In such a case, it is clear that the relative size of $1+t_H^d$ and $1+t_H^u$ depends on $\frac{\theta-1}{\theta}\left(1+\gamma^d\right) = \frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}$, and thus on the relative size of σ and θ.

D.3 Second-Best Policies

In this Appendix we characterize the second-best import tariffs for the general case $\alpha \geq 0$ when the government only has access to import tariffs upstream and downstream. We first derive the key equations characterizing tariff levels and tariff escalation, and we later explore special cases.

A. Second-Best Import Tariffs with Scale Economies: Key Equations

The second-best optimal allocation will seek to solve the same problem laid out in Section 4.1 expanded to include the additional constraints:

$$\hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{P_{HF}^d}{P_{HF}^u} = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}$$

and

$$F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right).$$

Second-Best Planner Problem and First-Order Conditions More specifically, the planner sets $\left\{L_H^u, L_H^d, Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u\right\}$ to

$$\max \quad U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned}
s.t. \quad & L_H^u + L_H^d = L_H \\
& \hat{A}_H^u\left(L_H^u\right) L_H^u = Q_{HH}^u + Q_{HF}^u \\
& \hat{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right) F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = Q_{HH}^d + Q_{HF}^d \\
& P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u = Q_{HF}^d \left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} \\
& \hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}} \\
& F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H \left(L^u_H\right)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$\begin{aligned}
&U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u \left[\bar{A}^u_H \left(L^u_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] + \mu_d \left[\bar{A}^d_H \left(F^d \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
&+\mu_{TB} \left[Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF} \left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF} \left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right] \\
&+\mu_{SB} \left[\hat{A}^d_H F^d_{Q^u_{HH}} \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) - \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF} \left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}\right] \\
&+\mu_{LC} \left[\frac{F^d_{L^d_H} \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}} \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} - \hat{A}^u \left(L^u_H\right)\right]
\end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}, L^d_H, and L^u_H are as follows:

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_d && \text{(D.9)}\\
U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) &= \mu_{TB} P^d_{FH} && \text{(D.10)}\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma} P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && \text{(D.11)}\\
\mu_u &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot) &&\\
&\quad +\mu_{SB}\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot) &&\\
&\quad \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}(\cdot)}{F^d(\cdot)} + \frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] &&\\
&\quad +\mu_{LC}\left[\frac{F^d_{L^d_H,Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)} - \frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] && \text{(D.12)}\\
\mu_{TB} P^u_{FH} &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{FH}}(\cdot) &&\\
&\quad +\mu_{SB}\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot)\times\left[\gamma^d\frac{F^d_{Q^u_{FH}}(\cdot)}{F^d(\cdot)} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] &&\\
&\quad +\mu_{LC}\left[\frac{F^d_{L^d_H,Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)} - \frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] && \text{(D.13)}\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta} P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && \text{(D.14)}\\
\mu_L &= \mu_u\left(1+\gamma^u\right)\bar{A}^u_H\left(L^u_H\right)^{\gamma^u} - \mu_{LC}\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1} && \text{(D.15)}\\
\mu_L &= \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{L^d_H}(\cdot) &&\\
&\quad +\mu_{SB}\hat{A}^d_H\left[\gamma^d\frac{F^d_{L^d_H}(\cdot)}{F^d(\cdot)} F^d_{Q^u_{HH}}(\cdot) + F^d_{Q^u_{HH},L^d_H}(\cdot)\right] &&\\
&\quad +\mu_{LC}\left[\frac{F^d_{L^d_H,L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)} - \frac{F^d_{L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\frac{F^d_{Q^u_{HH},L^d_H}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right] && \text{(D.16)}
\end{aligned}
$$

In these derivations, note that we have used

$$
\begin{aligned}
\frac{\partial\left(\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot)\right)}{\partial Q^u_{HH}} &= \bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot)\\
&\quad\times\left[\gamma^d\frac{F^d_{Q^u_{HH}}(\cdot)}{F^d(\cdot)} + \frac{F^d_{Q^u_{HH},Q^u_{HH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial\left(\bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot)\right)}{\partial Q^u_{FH}} &= \bar{A}^d_H\left(F^d(\cdot)\right)^{\gamma^d} F^d_{Q^u_{HH}}(\cdot)\\
&\quad\times\left[\gamma^d\frac{F^d_{Q^u_{FH}}(\cdot)}{F(\cdot)} + \frac{F^d_{Q^u_{HH},Q^u_{FH}}(\cdot)}{F^d_{Q^u_{HH}}(\cdot)}\right].
\end{aligned}
$$

Useful Expressions with Our Functional Forms Remember that technology is given by

$$F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \left(L_H^d\right)^{\alpha}\left(\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},$$

and define

$$\pi_{HH}^u \equiv \frac{\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^u\right)^{\frac{\theta-1}{\theta}}}$$

and

$$X_H^d \equiv \bar{A}_H^d\left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{1+\gamma^d}.$$

We next note that:

$$\begin{aligned}
F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d} \\
F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{Q_{HH}^u}\pi_{HH}^u \\
F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{Q_{FH}^u}\left(1-\pi_{HH}^u\right) \\
F_{L_H^d,L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha(\alpha-1)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d L_H^d} \\
F_{Q_{HH}^u,Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{Q_{HH}^u}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[-\alpha\pi_{HH}^u - \frac{1}{\theta}\left(1-\pi_{HH}^u\right)\right] \\
F_{Q_{HH}^u,Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\frac{1-\pi_{HH}^u}{Q_{FH}^u}\frac{\pi_{HH}^u}{Q_{HH}^u}\left(\frac{1}{\theta}-\alpha\right) \\
F_{L_H^d,Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha(1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d}\frac{\pi_{HH}^u}{Q_{HH}^u} \\
F_{L_H^d,Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha(1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d}\frac{1-\pi_{HH}^u}{Q_{FH}^u}
\end{aligned}$$

First-Order Conditions with Functional Forms We can now plug some of the above expressions into our first-order conditions

$$
\begin{aligned}
U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right) &= \mu_d && \text{(D.17)}\\
U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right) &= \mu_{TB}P^d_{FH} && \text{(D.18)}\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF}-\mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && \text{(D.19)}\\
\mu_u &= \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}+\mu_{SB}(1-\alpha)\frac{X^d_H}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\gamma^d(1-\alpha)\pi^u_{HH}-\alpha\pi^u_{HH}-\frac{1}{\theta}\pi^u_{FH}\right]\\
&\quad+\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{(1-\alpha)}\left[1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right] && \text{(D.20)}\\
\mu_{TB}P^u_{FH} &= \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{1-\pi^u_{HH}}{Q^u_{FH}}+\mu_{SB}(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]\\
&\quad+\mu_{LC}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\frac{Q^u_{HH}}{\pi^u_{HH}} && \text{(D.21)}\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF}+\mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} && \text{(D.22)}\\
\mu_L &= \mu_u\left(1+\gamma^u\right)\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}-\mu_{LC}\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1} && \text{(D.23)}\\
\mu_L &= \mu_d\left(1+\gamma^d\right)\frac{\alpha X^d_H}{L^d_H}+\mu_{SB}\left(1+\gamma^d\right)\frac{\alpha(1-\alpha)X^d_H}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}}-\mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L^d_HL^d_H}\frac{Q^u_{HH}}{\pi^u_{HH}} && \text{(D.24)}
\end{aligned}
$$

Manipulating the First-Order Conditions From equations (D.17), (D.18), and (D.19), we obtain:

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}=\frac{P^d_{FH}}{P^d_{HF}}\left(\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}}\right).$$

Because in a competitive equilibrium with tariffs we have

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}=\left(1+t^d_H\right)\frac{P^d_{FH}}{P^d_{HF}}, \tag{D.25}$$

we can establish that

$$1+t^d_H=\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}}. \tag{D.26}$$

Also, note from equations (D.17), (D.18), and (D.25) that we have

$$1+t^d_H=\frac{\mu_{TB}P^d_{HF}}{\mu_d}, \tag{D.27}$$

and in a competitive equilibrium with import tariffs (but no export taxes)

$$\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}=\frac{P^u_{HF}}{\left(1+t^u_H\right)P^u_{FH}}. \tag{D.28}$$

Furthermore, the penultimate constraint in the initial optimization can be written as:

$$\frac{X^d_H}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)=\frac{P^u_{HF}}{P^d_{HF}}, \tag{D.29}$$

and the last one as

$$\frac{\alpha}{1-\alpha}\frac{Q^u_{HH}}{\pi^u_{HH}L^d_H}=\frac{X^u_H}{L^u_H}. \tag{D.30}$$

Now combine equations (D.21), (D.27), (D.28), (D.29), and (D.30)

$$\frac{1+t^d_H}{1+t^u_H}=1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\gamma^d\left(1-\alpha\right)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{P^d_{HF}}{P^u_{HF}}. \tag{D.31}$$

We next plug equation (D.21) into equation (D.20) to obtain

$$\mu_{TB}P^u_{FH}\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{Q^u_{FH}}{1-\pi^u_{HH}}=\mu_u+\mu_{SB}\left(1-\alpha\right)\frac{X^d_H}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{1}{\theta}-\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}$$

Next, plugging μ_u from equation (D.22), and invoking equation (D.28) and (D.29), we obtain

$$\mu_{TB}P^u_{HF}\left[\frac{1}{1+t^u_H}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}\left(1-\alpha\right)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{(1-\alpha)}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}$$

Next, invoking equation (D.28) and (D.29), we can simplify this to

$$\mu_{TB}P^u_{HF}\left[\frac{1}{1+t^u_H}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}X^d_H\left(1-\alpha\right)\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}.$$

And, plugging in (D.27). this delivers

$$\frac{1+t^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+t^d_H\right)=\frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}\frac{P^d_{HF}}{P^u_{HF}}. \tag{D.32}$$

We finally seek to solve for μ_{LC} as a function of μ_{SB}. We begin with equation (D.15) and (D.24)

$$\frac{\mu_u}{\mu_d}\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}=\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\frac{\alpha X^d_H}{L^d_H}+\frac{\mu_{SB}}{\mu_d}\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\frac{\alpha\left(1-\alpha\right)X^d_H}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}}+\frac{\mu_{LC}}{\mu_d}\frac{1}{1+\gamma^u}\left[\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1}-\frac{\alpha}{1-\alpha}\frac{1}{L^d_HL^d_H}\frac{Q^u_{HH}}{\pi^u_{HH}}\right].$$

Next, plug equation (D.12) and using (D.30), we obtain

$$\begin{aligned}&\frac{\left(1+\gamma^d\right)\gamma^u}{\left(1+\gamma^u\right)}X^d_H+\frac{\mu_{SB}}{\mu_d}\left(1-\alpha\right)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi^u_{HH}\right)}{\pi^u_{HH}}\right)\right]\\ =\;&\frac{\mu_{LC}}{\mu_d}\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^uL^d_H}{L^u_H}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right],\end{aligned}$$

which we can express as

$$\frac{\mu_{LC}}{\mu_d}=\frac{\frac{\left(1+\gamma^d\right)\gamma^u}{\left(1+\gamma^u\right)}X^d_H+\frac{\mu_{SB}}{\mu_d}\left(1-\alpha\right)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi^u_{HH}\right)}{\pi^u_{HH}}\right)\right]}{\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^uL^d_H}{L^u_H}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}. \tag{D.33}$$

Recap of Key Equations

$$
\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d} \\
\frac{1+t_H^d}{1+t_H^u} &= 1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L_H^d}\frac{P_{HF}^d}{P_{HF}^u} \\
\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right) &= \frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right]-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{1}{L_H^d}\frac{1}{\theta}\frac{1}{\pi_{HH}^u}\frac{P_{HF}^d}{P_{HF}^u} \\
\frac{\mu_{LC}}{\mu_d} &= \frac{\frac{(1+\gamma^d)\gamma^u}{(1+\gamma^u)}X_H^d+\frac{\mu_{SB}}{\mu_d}(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{(1+\gamma^d)}{(1+\gamma^u)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^u L_H^d}{L_H^u}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}
\end{aligned}
$$

We have not been successful in proving any general results, so let us study some special cases.

B. Second-Best Import Tariffs with No Scale Economies in Either Sector

Given the above derivations, it is straightforward to prove that Proposition 7 applies even when $\alpha > 0$. Simply set $\gamma^d = \gamma^u = 0$ in equations (D.31), (D.32)and (D.33). First note, equation (D.33) becomes

$$
\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]},
$$

and plugging (D.30),

$$
\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}(1-\alpha)^2 X_H^d L_H^d\left(\frac{\pi_{HH}^u}{Q_{HH}^u}\right)^2\frac{1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}}{1+\frac{\alpha}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}}.
$$

Plugging this expression for $\frac{\mu_{LC}}{\mu_d}$ into (D.31), delivers

$$
\frac{1+t_H^d}{1+t_H^u}=1+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\frac{\alpha+\pi_{HH}^u(1-\alpha)}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right]
$$

And finally, plugging $\frac{\mu_{LC}}{\mu_d}$ into equation (D.32) delivers

$$
\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right)=\frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\frac{(1-\alpha)\theta\pi_{HH}^u}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right].
$$

In sum, we can write the system as

$$
\begin{aligned}
1+t_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}A \\
\frac{1+t_H^d}{1+t_H^u} &= 1+\frac{\mu_{SB}}{\mu_d}B \\
\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} > 0 \\
B &= \frac{\pi^u_{HH}}{Q^u_{HH}}(1-\alpha)\left[\frac{\alpha+\pi^u_{HH}(1-\alpha)}{\alpha+\pi^u_{HH}(\theta-\alpha)}\right] > 0 \\
C &= \frac{1}{\theta}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\frac{(1-\alpha)\,\theta\pi^u_{HH}}{\alpha+\pi^u_{HH}(\theta-\alpha)}\right] > 0
\end{aligned}
$$

So we have

$$
\begin{aligned}
1+t^d_H &= \frac{\sigma}{\sigma-1}+\left[\frac{1+t^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+t^d_H\right)\right]\frac{A}{C} \\
\frac{1+t^d_H}{1+t^u_H} &= 1+\left(\frac{1+t^d_H}{1+t^u_H}-\left(1+t^d_H\right)\frac{\theta-1}{\theta}\right)\frac{B}{C}
\end{aligned}
$$

When solving this system, we obtain

$$
\frac{1+t^d_H}{1+t^u_H}=\frac{1-\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta}\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}},
$$

which is higher or lower than 1 depending on the relative size of σ and θ. More specifically, when $\sigma > \theta$, $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta} < 1$, and we have tariff escalation. But when $\sigma < \theta$, then $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta} > 1$, and we have tariff de-escalation.

C. Second-Best Import Tariffs with No Scale Economies Upstream ($\gamma^u = 0$)

We next study the case in which $\gamma^d > 0$ but $\gamma^u = 0$. In that case, equation (D.33) reduces to

$$
\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)\right]}{\bar{A}^u_H\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]},
$$

and we can write

$$
\begin{aligned}
1+t^d_H &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}A \\
\frac{1+t^d_H}{1+t^u_H} &= 1+\gamma^d+\frac{\mu_{SB}}{\mu_d}B \\
\frac{1+t^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+t^d_H\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}
$$

with

$$
\begin{aligned}
B &= \frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{X_H^d\left[\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}\alpha\frac{\theta-1}{\theta}\frac{1}{L_H^d}\frac{P_{HF}^d}{P_{HF}^u}\right] \\
&= \frac{\pi_{HH}^u}{Q_{HH}^u}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\left[\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}\frac{\theta-1}{\theta}\right] \\
&= \frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\frac{\pi_{HH}^u(\theta-1)+\alpha\sigma+\pi_{HH}^u\sigma(1-\alpha)}{(\sigma-1)(\alpha(1-\pi_{HH}^u)+\pi_{HH}^u\theta)}>0.
\end{aligned}
$$

and

$$
C=\frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\frac{1}{\theta}\left[1-\frac{\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}\right]>0.
$$

Given $B>0$ and $C>0$, it is then straightforward to use the same steps as in the proof of the $\alpha=0$ case in Appendix C.4 to show that

$$
1+t_H^d>\frac{\sigma}{\sigma-1}
$$

and

$$
\frac{1+t_H^d}{1+t_H^u}>1+\gamma^d.
$$

We thus provide a simple analytic solution that shows tariff escalation is optimal when upstream goods are produced under constant returns to scale.

E Quantitative Analysis

E.1 Numerical Simulations of Second-Best Tariff Escalation

In this Appendix, we describe how we solve numerically for second-best trade policies in the small, open economy (SOE), and report the results of solving this problem for various parameter values. First, we perform an extensive grid search over various values of the key parameters σ, θ and α. Second, we describe results of a second exercise in which we evaluate tariff escalation for different values of A_d, A_u, τ_d and τ_u. Table E.1 provides the list of values we consider for each parameter.

Table E.1: List of Parameters in Grid Search Exercise

Parameter	List of values
Elast. Subs. Downstream, σ	$\{2, 3, 4, \dots, 8\}$
Elast. Subs. Upstream, θ	$\{2, 3, 4, \dots, 8\}$
Labor share Downstream, α	$\{0, 0.1, 0.2, \dots, 0.9\}$
Foreign Prod. Downstream, A_d	$\{0.16, 0.24, 0.32, 0.49, 0.65\}$
Foreign Prod. Upstream, A_u	$\{0.07, 0.14, 0.21, 0.28\}$
Iceberg cost Downstream, τ_d	$\{1.19, 1.78, 2.37, 3.56, 4.75\}$
Iceberg cost Upstream, τ_u	$\{1.02, 1.52, 2.03, 3.05, 4.07\}$

Notes: Each row presents the list of values considered for each parameter during the grid search exercise. For the two elasticities of substitution and the labor share we cover the entire range of reasonable values in the literature. For iceberg costs and the two productivities we start from their calibrated value, table 1 in section 6.1, and consider a 25% and 50% decrease, and a 50% and 100% increase.

E.1.1 Solving for optimal import tariff

Solving for optimal tariffs in this second-best setting requires providing values for import prices and export demand shifters, both of which are exogenously given in the SOE, and for productivities in both sectors. To recover the value of these parameters, we use the fact that the LOE approximates the SOE assumption from Home's perspective when its population is low relative to the population of the rest of the world. Therefore, import prices and export demand shifters in the SOE can be constructed from the equilibrium of the LOE under this limit according to,[5]

$$\begin{aligned}
P^d_{soe,FH} &= \left(M^d_F\right)^{\frac{1}{1-\sigma}} p^d_{FH} \\
P^u_{soe,FH} &= \left(M^u_F\right)^{\frac{1}{1-\theta}} p^u_{FH} \\
P^d_{soe,HF} &= \left(\frac{1}{\tau_d}\right)^{\frac{\sigma}{\sigma-1}} P^d_{HF} C^{\frac{1}{\sigma}}_{HF} \\
P^u_{soe,HF} &= \left(\frac{1}{\tau_u}\right)^{\frac{\theta}{\theta-1}} P^u_{HF} Q^{\frac{1}{\theta}}_{HF}.
\end{aligned}$$

[5]Note the export demand shifters are corrected by iceberg costs because of how we write the feasibility constraints in the SOE.

Similarly, productivity levels in the two sectors can be constructed from the equilibrium of the LOE and the isomorphism in Proposition 3,

$$A^u_{soe} = \left(\frac{A^u_H}{f^\theta_H}\right)^{\frac{\theta}{\theta-1}} (\theta - 1)\, f^u_H, \qquad A^d_{soe} = \left(\frac{A^d_H}{f^d_H \sigma}\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1)\, f^d_H.$$

Given these values, it is straightforward to compute numerically the optimal import tariff of the problem described in Appendix D.3. In practice, however, we solve for the optimal import tariff in the LOE when $L_H/L_F = 0.01$ for each combination of the parameters in Table E.1 because this method is more numerically stable than the corresponding one in the SOE. Table E.2 shows that this approximation works well for the calibrated values of Section 6.1 with $L_H/L_F = 0.01$.

Table E.2: Optimal taxes in the large and small open-economy

	$1+t^d$	$1+t^u$	$1-v^u$	$1-s^u$
LOE - First-Best	0.249		0.200	0.200
SOE - First-Best	0.25		0.2	0.2
LOE - Second-Best	0.328	0.184		
SOE - Second-Best	0.332	0.191		

Notes: Table compares the first- and second-best policy in the LOE and SOE. We compute the optimal tariffs using the estimated parameters in section 6.1 with $L_H/L_F = 0.01$.

E.1.2 Grid over σ, θ and α

We first solve for optimal trade policy for values of σ and θ ranging from 2 to 8, and for values of α ranging from 0 to 0.9, as described in Table E.1. We fix the values A_d, A_u, τ_d and τ_u to the values we estimate in Section 6.1. Overall, we explore $7 \times 7 \times 10 = 490$ configurations of parameters. We can successfully solve the model for 453 of these 490 cases. Most problematic cases are associated with very low values of σ, for which we find high tariff escalation values when we can solve the model.

Statistics tariff ratio wedge

The next three tables report statistics related to the tariff escalation wedge, the mean, the median, the standard deviation, the minimum and the maximum. The second column of Table E.3 provides values of these statistics for the 453 cases for which we have a solution. On average, 'gross' downstream tariffs are 25 percent higher than 'gross' upstream tariffs, and the medians show a similar divergence (16 percent). Some cases feature tariff escalation levels as high as 2.31, while tariff de-escalation remains modest even in the most extreme cases (minimum of 0.75).

The third and fourth column of Table E.3 recalculates the statistic but looking at the cases for which the tariff ratio is above and below 1, respectively. It is interesting to note that when our model predicts tariff de-escalation, it does so with fairly moderate levels (median of 0.97).

Table E.3: Statistics of the tariff escalation wedge: all cases

	All cases	With escalation	With de-escalation
Mean	1.25	1.28	0.95
Median	1.16	1.18	0.97
Standard Deviation	0.28	0.28	0.006
Minimum value	0.75	1.001	0.75
Maximum value	2.31	2.31	0.99
N	453	412	41

Notes: Table reports statistics for the tariff escalation wedge for optimal import tariffs computed in the grid search exercise.

E.1.3 Tariff escalation and parameter space

We now present results about the parameter combinations that generate de-escalated tariffs. Table E.4 reports the fraction of cases for which tariffs are de-escalated for a given parameter, for each possible value that this parameter can take. For example, we have 70 combinations with $\sigma = 4$, with 1 percent of these cases featuring de-escalated tariffs.[6]

Table E.4: Tariff De-escalation across the parameter space, I

$\sigma =$	2	3	4	5	6	7	8			
Share:	0.00	0.00	0.01	0.07	0.11	0.17	0.23			
$\theta =$	2	3	4	5	6	7	8			
Share:	0.13	0.22	0.17	0.09	0.05	0.00	0.00			
$\alpha =$	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
Share:	0.00	0.00	0.00	0.00	0.00	0.02	0.06	0.13	0.36	0.55

Notes: Table reports the share of cases in each cell for which optimal tariffs are de-escalated, i.e $\left(1+t_H^d\right)/\left(1+t_H^u\right) < 1$.

Table E.5 presents the fraction of cases with de-escalated tariffs for the ten values of α, split into cases with $\sigma > \theta$, $\sigma < \theta$, and $\sigma = \theta$. In line with the intuition in the draft, optimal tariffs are more likely to be de-escalated when the returns to scale upstream are larger than those downstream, and when the downstream labor share is high.

[6] From the 70 combinations with $\sigma = 4$, we cannot solve for the equilibrium for 2 cases. The 1 percent reported in the table is the share out of the 68 cases for which we have a solution.

Table E.5: Tariff De-escalation across the parameter space, II

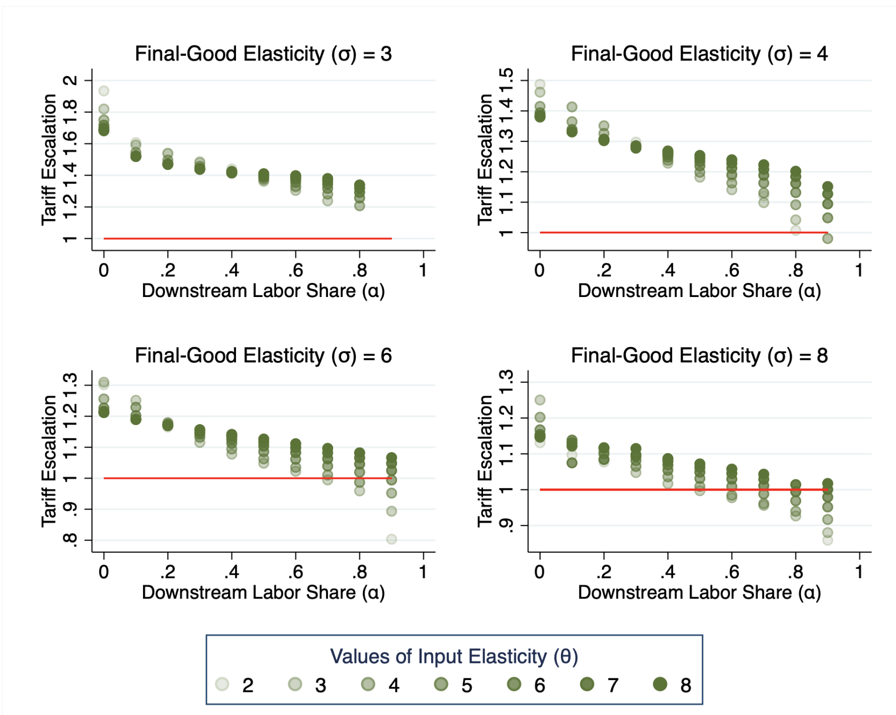
	Values of α									
	0.00	0.10	0.20	0.30	0.40	0.50	0.60	0.70	0.80	0.90
$\sigma > \theta$	0.00	0.00	0.00	0.00	0.00	0.05	0.14	0.30	0.70	0.94
$\sigma < \theta$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
$\sigma = \theta$	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.20

Notes: Table presents the share of cases in each column for which the optimal tariff are de-escalated.

E.1.4 Second-best tariff escalation and the labor share

We replicate Figure 2 for different values of the elasticity of substitution across downstream goods.

Figure E.1: Second-Best Tariff Escalation and the Labor Share



Notes: Figure plots the ratio of optimal downstream to upstream tariffs (i.e., tariff escalation) as a function of the downstream labor share (α) and upstream elasticity of substitution (θ) for different values of the elasticity of substitution downstream (σ).

E.2 Grid over A_d, A_u, τ_d and τ_u

We now perform a similar analysis, but simulate the model for various values of the parameters A_d, A_u, τ_d and τ_u. We start from their calibrated value in Table 1 in Section 6.1, and consider a 25 percent and a 50 percent decrease, and a 50 percent and a 100 percent increase for each parameter. We fix $\sigma = \theta = 5$ and $\alpha = 0.55$, as in our baseline quantitative analysis. The grid has 625 different combinations in total, and we can solve for 575 cases. The following tables provide descriptive statistics for the tariff escalation wedge .

Table E.6: Statistics of the tariff escalation wedge: all cases

	All cases	With escalation	With de-escalation
Mean	1.11	1.12	0.91
Median	1.12	1.12	0.91
Standard Deviation	0.04	0.01	0.00
Minimum value	0.91	1.10	0.91
Maximum value	1.13	1.13	0.91
N	575	552	23

Notes: Table reports statistics for the tariff escalation wedge for the optimal import tariff computed in the grid search exercise.

Tariff escalation and parameter space

Finally, we present results illustrating the combinations of parameters that generate de-escalated tariffs. Table E.7 reports the fraction of cases for which tariffs are de-escalated for a given parameter.

Table E.7: Tariff De-escalation across the parameter space, I

A_d	0.16	0.24	0.32	0.49	0.65
Share:	0.20	0.00	0.00	0.00	0.00
A_u	0.07	0.11	0.14	0.21	0.28
Share:	0.00	0.00	0.00	0.00	0.20
τ_d	1.19	1.78	2.37	3.56	4.75
Share:	0.04	0.04	0.04	0.04	0.04
τ_u	1.02	1.52	2.03	3.05	4.07
Freq:	0.04	0.04	0.04	0.04	0.04

Notes: Table reports the share of cases in each cell for which optimal tariff are de-escalated, i.e $\left(1+t_H^d\right) / \left(1+t_H^u\right) < 1$.

E.3 Robustness for Calibrated Parameters

Table E.8: Calibrated Parameters - Robustness

	τ^d	τ^u	A^d_{RoW}	A^u_{RoW}
$\theta = 4.43$ and $\sigma = 6.44$	1.787	2.2986	0.289	0.114
$\theta = 2.35$ and $\sigma = 3.08$	5.021	8.536	0.248	0.102
$\theta = 8.52$ and $\sigma = 8.41$	1.508	1.446	0.301	0.121
$\theta = 2.5$ and $\sigma = 4$	3.007	6.878	0.260	0.103
$\theta = 5.5$ and $\sigma = 4$	3.007	1.877	0.281	0.116
$\alpha = 0.75$	2.249	2.040	0.196	0.114
$\alpha = 0.25$	2.239	2.042	0.119	0.124
$\alpha = 0$	2.375	2.073	0.598	0.128

Notes: This table reports the re-calibrated parameters used in our robustness exercise in Table 2.

F Data Appendix

F.1 Data Construction for Figure 5

US Tariff Data.

- We use US import tariff data at the 8-digit level from the US Harmonized Tariff Schedule (HTS) available at https://dataweb.usitc.gov/tariff/annual. We use the most-favored-nation (MFN) ad valorem tariff rate whenever possible. In approximately 25% of the cases, the MFN ad valorem rate is not available and instead a "specific" tariff rate is applied such as "68 cents/head", "1 cents/kg", "0.9 cents each" etc. In these cases we perform an imputation by calculating an ad valorem equivalent tariff rate using unit values obtained from the US Census Bureau.
- In a next step we use the imputed ad valorem tariff rate to calculate applied MFN ad valorem tariff rates for all goods, taking trade agreements between the US and other countries into account. That is, we calculate the applied MFN ad valorem tariff rate as an import weighted average of the MFN ad-valorem rate and the tariff rate that is paid by countries that are members of a trade agreement.[7] US import data for the year 2015 come from the US Census Bureau.
- Data on tariffs imposed in February and March 2018 on almost all countries (washers; solar panels; iron and steel; aluminum) come from Fajgelbaum et al. (2020) and all subsequent tariffs imposed on imports from China throughout 2018 and 2019 from Chad Bown (available here).

ROW Tariff Data.

- We use tariff data for 115 countries plus the European Union at the 6-digit HS code level from the WTO Tariff Download Facility available at http://tariffdata.wto.org/default.aspx. We use the most-favored-nation (MFN) ad valorem tariff rate which constitutes the simple average duty of all products within a 6-digit HS code classification.
- We use data on retaliatory tariffs imposed by China throughout 2018 and 2019 from Chad Bown (available here). Data on retaliatory tariffs imposed by the European Union, Canada, Mexico, India and Turkey stem from Li (2018). Using data on these tariff waves we adjust the MFN applied tariff rates taking 2015 US export value weighted averages with US export data coming from the US Census Bureau.

Intermediate and Final Goods Classification

- We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC). The cross-walk between HTS10 codes and end-use categories is available here. We classify goods as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods (including capital goods) start with BEC code 41, 521, 112, 321, 522, 61, 62, 63. All other goods have no classification.

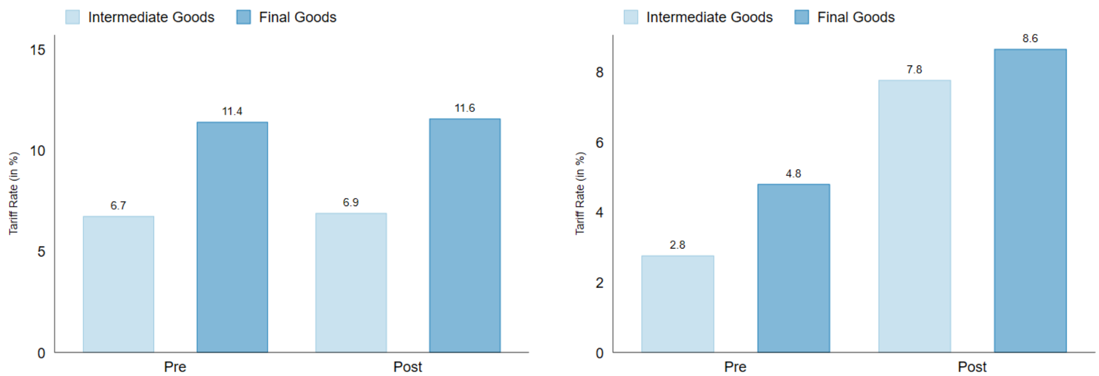
[7] We account for the following trade agreements: Generalized System of Preferences (GSP, 41 countries), The Agreement on Trade in Civil Aircraft (32 countries), NAFTA (3 countries), Caribbean Basin Initiative (CBI, 17 countries), African Growth and Opportunity Act (AGOA, 40 countries), Caribbean Basin Trade Partnership Act (CBTPA, 8 countries), Dominican Republic-Central America FTA (6 countries) and the Agreement on Trade in Pharmaceutical Products (7 countries).

Tariff Escalation Unweighted

- As alternative to Figure 5 which shows trade-weighted tariff rates, Figure F.1 displays an unweighted version of the tariff increase on intermediate and final goods by the ROW on imports from the US throughout the trade war and vice versa.

Figure F.1: Comparison of ROW and US Input & Final-Good Tariffs (Unweighted)

(a) ROW average tariffs on US **(b)** US average tariffs on ROW



Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC. Goods are classified as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods start with BEC code 41, 521, 112, 321, 522, 61, 62, 63 (including capital goods). All other goods have no classification.

F.2 Elasticity Estimation

Below we explain the estimation of the elasticities of substitution in the upstream and downstream sectors using three different approaches: the trade elasticity approach, the sectoral markup approach, and the scale elasticity approach. We present results for all three approaches and demonstrate how they differ.

Sectoral Markup Approach Our first elasticity estimation approach relies on sectoral markups. Information on firm-level markups allows us to derive elasticities in a straightforward manner since equations (A.1) and (A.2) illustrate that $markup = \frac{elasticity}{elasticity-1}$. We thus compile data for this exercise as follows:

We obtain upstream/downstream sector classifications using WIOD. We use 2014 sales of the US to the US and RoW to calculate the share of total sales per sector that goes to final consumers. We then classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median. This yields a dataset which shows upstream and downstream classifications for 87 sectors at the 2-digit NACE level (European industry classification). This 2-digit NACE data we combine with a NACE-NAICS concordance file that maps 4-digit NACE (we only use the first 2 digits) to 6-digit NAICS. If there are multiple NACE 2-digit codes for a NAICS 6-digit code, we choose the NACE 2-digit code that has larger total US sales. This yields a final dataset that shows upstream and downstream classifications for 1,175 different NAICS 6-digit codes. We combine these data with data kindly provided by Baqaee and Farhi (2020) (BF) based on 6-digit NAICS codes. The BF data list markups and sales for 31,683 different firms from 1978 – 2018. They provide three different types of markups calculated based on a user cost, a production function, or an accounting profits method. We select their data between

2012 and 2017 and focus on the markups calculated using the production function estimation approach. We further exclude firms that have markups smaller than 1 (14% of all firm-year observations).

Table F.1: Elasticities

	mean	sd	min	p5	p25	p50	p75	p95	max	count
Upstream	4.43	4.26	1.10	1.15	1.60	2.75	5.04	16.50	16.50	11045
Downstream	6.44	6.05	1.29	1.46	2.44	4.03	7.49	22.24	22.24	14773

Notes: The table shows weighted mean elasticities for upstream and downstream sectors between 2012 and 2017 across all firms in the WIOD that have markups greater than 1. Elasticities stem from the production function estimation approach. Weights represent the share of firm sales in total sales. We winsorize elasticities and sales at the 5-95th percentile by sector.

We then calculate firm-level elasticities as $elasticity = \frac{markup}{markup-1}$ and winsorize elasticities and sales at the 5-95th percentile by sector. Finally, we calculate weighted mean elasticities for upstream and downstream sectors across all firms where weights represent the share of firm sales in total sales. Table F.1 presents elasticities for upstream and downstream sectors pooling all years from 2012 to 2017.

Trade Elasticity Approach In our second elasticity estimation approach, we estimate elasticities in the upstream and downstream sectors by measuring the response of imports in the upstream and downstream sectors to changes in import tariffs. More specifically, we calculate the changes in US import values in both sectors during the US-China trade war (January 2018 to December 2019) that raised US import tariffs on upstream goods by 4.1 percentage points and downstream goods by 4.4 percentage points. We obtain data on import values at the country-HTS10-month level from the US Census Bureau's Application Programming Interface (API). Data on US import tariffs are constructed as described in Section F.1.

We regress 12-month log changes in import values on 12-month log changes in tariff rates via the following regression specification:

$$\Delta ln(v_{ijt}) = \alpha_j + \tau_{it} + \beta \Delta ln(1 + Tariff_{ijt}) + \omega_{ijt}, \tag{F.1}$$

where i indicates foreign countries, j denotes products, and t corresponds to time; α_j is a product fixed effect; τ_{it} is a country-time fixed effect; and ω_{ijt} is a stochastic error. We denote import values by v_{ijt}. We estimate equation F.1 separately for intermediate and final goods using both log differences and the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$.[8] The results are presented in Table F.2.

Column 1 (3) suggests that a one percent increase in tariffs on intermediate (final) goods is associated with a 1.05 (1.81) percent decrease in import value. However, since tariffs can lead to zero imports, which will be dropped from the regression, columns 2 and 4 perform the same regression this time using the inverse hyperbolic sine instead of the log change. This adjustments leads to greater trade elasticities for both types of goods. A one percent increase in tariffs on intermediate (final) goods is associated with a 2.35 (3.08) percent decrease in import value. Note that the estimates from this specification correspond to an elasticity of substitution between intermediate (final) goods of 2.35 (2.08).

[8]Note that regression coefficients based on the hyperbolic sine transformation are sensitive to the scale of the import values. This is, results vary depending on whether import values are measured in thousands, millions, etc. Following Amiti et al. (2019), we measure import values in single US dollars.

Table F.2: Impact of US Tariffs on Import Values

	Intermediate Goods		Final Goods	
	(1) Log Change Import Value $\Delta ln(v_{ijt})$	(2) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$	(3) Log Change Import Value $\Delta ln(v_{ijt})$	(4) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$
log change tariff				
$\Delta ln(1 + Tariff_{ijt})$	-1.05***	-2.35***	-1.81***	-3.08***
	(0.07)	(0.44)	(0.08)	(0.35)
N	1302744	2220920	1253577	2251844
R2	0.027	0.048	0.022	0.045

Notes: Observations are at the country-HTS10-month level for the period January 2018 to December 2019. Since the specification is in 12-month changes, the data includes observations from January 2017 onwards. Robust standard errors in parentheses. Variables are in twelve-month log change. All columns include product-level and country-time fixed effects. The dependent variables are the log change and the change in the inverse hyperbolic sine of US import values of intermediate and final goods, respectively. We use the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$. $*p < 0.05$, $**p < 0.01$, $***p < 0.001$.

Scale Elasticity Approach Our final elasticity estimation approach exploits the isomorphism of our model to a model with external economies of scale. As discussed in Section 2, these models are isomorphic provided that the following restrictions between the external economies of scale parameters and the elasticities of substitution across varieties hold: $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$. Data on γ^u and γ^d thus allow us to easily derive information on elasticities.

Data on scale elasticities comes from Bartelme et al. (2019). The authors provide 2SLS estimates on scale elasticities for 15 manufacturing industries presented in Table F.3. We classify these industries into upstream and downstream industries following the same procedure as in the *Sectoral Markup Approach* and then calculate the average scale elasticity in those sectors.

Table F.3: Scale Elasticities

Industry	NACE Rev. 2	WIOD class.	Scale elast.
Food products, beverages and tobacco	10, 11, 12	downstream	0.16
Textiles	13, 14, 15	downstream	0.12
Wood and products of wood and cork	16	upstream	0.11
Paper products and printing	17, 18	upstream	0.11
Coke and refined petroleum products	19	upstream	0.07
Chemicals and pharmaceutical products	20, 21	upstream	0.2
Rubber and plastic products	22	upstream	0.25
Other non-metallic mineral products	23	upstream	0.13
Basic metals	24	upstream	0.11
Fabricated metal products	25	upstream	0.13
Computer, electronic and optical products	26	downstream	0.13
Electrical equipment	27	upstream	0.09
Machinery and equipment, nec	28	downstream	0.09
Motor vehicles, trailers and semi-trailers	29	downstream	0.15
Other transport equipment	30	downstream	0.16

Notes: Industries and 2SLS scale elasticities stem from Bartelme et al. (2019). Upstream and downstream classifications stem from WIOD where we classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median.

For the upstream sector we obtain an average scale elasticity of 0.133 and for the downstream sector an average scale elasticity of 0.135. Exploiting the isomorphism between this setup and our framework with monopolistic competition and free entry, we convert these to $\theta = 8.52$ and $\sigma = 8.41$.

F.3 Share of Inputs in the Downstream Sector

As in the “Sectoral Markup Approach," we classify sectors into upstream and downstream depending on whether the share of total sales to final consumers is below or above the median across all sectors. From the WIOD in 2014 we calculate the share of inputs in the downstream sector as the ratio of intermediate inputs to sales in the downstream sectors leading to an estimate of $1 - \alpha = 0.45$.