Trade Policy and Global Sourcing:
A Rationale for Tariff Escalation

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Online Appendix (Not for Publication)

A Closed-Economy Model: Details on Derivations

A.1 Equilibrium

Given the CES assumptions built into our framework and the lack of strategic interactions, firms in both sectors charge a constant markup over their marginal cost, which delivers

$$p^u = \frac{\theta}{\theta - 1} \frac{w}{A^u} \tag{A.1}$$

and

$$p^d = \frac{\sigma}{\sigma - 1} \frac{1}{A^d} \frac{w^\alpha (P^u)^{1-\alpha}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}, \tag{A.2}$$

where P^u is the price index of intermediate inputs associated with Q^u, or

$$P^u = \left(\int_0^{M^u} p^u(\varpi)^{1-\theta} d\varpi \right)^{\frac{1}{1-\theta}}.$$

Firms make zero profits due to free entry, which pins down firm size according to:

$$x^u = (\theta - 1) f^u, \qquad x^d = (\sigma - 1) f^d. \tag{A.3}$$

Naturally, in equilibrium we must have $x^d = q^d$ and $x^u = M^d q^u$. The total measure of firms in the economy can be determined as follows. First, note that from the household's demand for downstream goods we have

$$M^d p^d q^d = wL, \tag{A.4}$$

which plugging in (A.2) and (A.3) delivers

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((1-\alpha) \frac{\theta - 1}{\theta} A^u \right)^{1-\alpha} (M^u)^{\frac{1-\alpha}{\theta - 1}} L. \tag{A.5}$$

Next, note that labor-market clearing imposes

$$L = M^u \frac{(f^u + x^u)}{A^u} + M^d \frac{\alpha p^d x^d}{w}. \tag{A.6}$$

Plugging in equations (A.3) and (A.4), we can solve for the total measure of firms in the upstream sector:

$$M^u = \frac{(1-\alpha) A^u L}{f^u \theta}. \tag{A.7}$$

Then, equations (A.5) and (6) determine the measure of firms in the downstream sector:

$$M^d = \frac{\alpha^\alpha A^d}{f^d \sigma} \left((\theta - 1) f^u \right)^{1-\alpha} \left(\frac{(1-\alpha) A^u}{f^u \theta} \right)^{\frac{(1-\alpha)\theta}{\theta - 1}} (L)^{\frac{\theta - \alpha}{\theta - 1}}. \tag{A.8}$$

Finally, aggregate welfare is simply given by $U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} q^d$, where M^d is given in (7) and $q^d = x^d$ in (A.3).

When $\alpha \to 1$, we obtain

$$U = \left(\frac{A^d}{f^d \sigma} L \right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f^d,$$

which is the standard formula in Krugman (1980).[1] Welfare is increasing in market size with an elasticity equal to $\frac{\sigma}{\sigma-1} > 1$, reflecting the variety gains associated with living in an economy that provides a larger number of final-good varieties.

Relative to this "Krugman" benchmark, in the presence of an active upstream sector (i.e., $\alpha < 1$), our model continues to feature scale effects, and in fact the elasticity of welfare to market size is larger than in the model with only a final-good sector. To see this, we can write welfare as

$$U = \left(\frac{(\sigma-1)A^d/\sigma}{((\sigma-1)f^d)^{\frac{1}{\sigma}}} \left(\frac{(\theta-1)\, A^u/\theta}{((\sigma-1)f^u)^{1/\theta}} \right)^{\frac{(1-\alpha)\theta}{\theta-1}} (L)^{\frac{\theta-\alpha}{\theta-1}} \right)^{\frac{\sigma}{\sigma-1}} \xi_\alpha, \tag{A.9}$$

where ξ_α is a function of only α and θ. Note that $\frac{\theta-\alpha}{\theta-1} \geq 1$, and thus this framework features larger scale effects than our model without an input sector.

To gain a better understanding of the role of imperfect competition and increasing returns to scale on welfare in our closed economy, we next turn to characterizing the social optimum in our model, and explore conditions under which the above market equilibrium is efficient.

A.2 Social Planner Problem

The social planner maximizes the objective in (1), choosing M^d, M^u, ℓ^d, ℓ^u, x^d and x^u subject to feasibility, or:

$$\begin{aligned}
\max_{M^d, M^u, \ell^d, \ell^u, x^d, x^u} \quad & U = \left(M^d\right)^{\frac{\sigma}{\sigma-1}} x^d \\
s.t. \quad & L = \ell^u M^u + \ell^d M^d \\
& f^u + x^u = A^u \ell^u \\
& f^d + x^d = A^d \left(\ell^d\right)^\alpha \left((M^u)^{\frac{\theta}{\theta-1}} \frac{x^u}{M^d} \right)^{1-\alpha}.
\end{aligned}$$

Working with the first-order conditions of this problem, we find that

$$(x^u)^* = (\theta - 1) f^u \tag{A.10}$$

and

$$(M^u)^* = \frac{\theta}{\theta-\alpha} \frac{(1-\alpha)A^u L}{\theta f^u} \tag{A.11}$$

in the upstream sector, and

$$\left(x^d\right)^* = (\sigma - 1) f^d \tag{A.12}$$

[1] A small and immaterial point of departure from Krugman (1980) is the fact that we have modeled the productivity terms A^d and A^u as shaping both the marginal and fixed costs of production. As a result, firm size is independent of these productivity parameters, but these parameters affect welfare directly.

and

$$\left(M^{d}\right)^{*}=\left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha}\left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{\alpha^{\alpha}A^{d}}{\sigma f^{d}}\left((\theta-1)\,f^{u}\right)^{1-\alpha}\left(\frac{(1-\alpha)A^{u}}{\theta f^{u}}\right)^{\frac{(1-\alpha)\theta}{\theta-1}}(L)^{\frac{\theta-\alpha}{\theta-1}} \tag{A.13}$$

in the downstream sector. Comparing equations (A.10)-(A.13) to the corresponding ones in the market equilibrium, we conclude that:[2]

Proposition 1. In the decentralized equilibrium, firm-level output is at its socially optimal level in both sectors, but the market equilibrium features too little entry into both the downstream and upstream sectors unless $\alpha = 1$ (so the upstream sector is shut down) or $\alpha = 0$ (so the downstream sector does not use labor directly in production).

Why is the market equilibrium typically inefficient? At first glance, it may appear that the only source of inefficiency is the markup charged by upstream producers, which distorts the choice between labor and the bundle of input varieties for downstream firms. More specifically, this upstream markup makes inputs relatively more expensive and, in response, downstream firms substitute towards labor. At the same time, that markup also incentivizes entry upstream, which generates a variety-productivity effect downstream. To disentangle these two opposing forces, it is useful to compare the market allocation of labor to the social planner's optimal allocation.

Combining equations (2), (A.3), and (A.7), the aggregate decentralized market allocation of labor to the upstream sector is given by

$$M^{u}\ell^{u}=(1-\alpha)L,$$

while from equations (A.10) and (A.11), the social planner would allocate a share of labor to that sector equal to

$$M^{u}\ell^{u}=\frac{\theta}{\theta-\alpha}(1-\alpha)L>(1-\alpha)L.$$

Thus, the market equilibrium is inefficient, in the sense that it allocates too little labor to the upstream sector. It might seem intuitive that this inefficiency is associated with upstream markups leading to a double-marginalization inefficiency. However, note that the market allocation of labor to the upstream sector is actually *independent* of the degree of input substitutability (θ), and thus, of the level of upstream markups. In other words, lower input substitutability – and thus higher markups – do *not* depress the market allocation of labor to the upstream sector; instead, they increase the social-welfare maximizing allocation of labor to that sector. This fact does not necessarily rule out the relevance of a double marginalization inefficiency, but it does suggest that the market inefficiency may alternatively be interpreted as reflecting that, in the market equilibrium, upstream firms do not internalize the fact that their entry generates positive spillovers for firms in the downstream sector, with the size of this spillover decreasing in the degree of input substitutability θ.[3]

When $\alpha = 1$ or $\alpha = 0$, all labor is allocated to either the downstream sector (when $\alpha = 1$) or to the upstream sector (when $\alpha = 0$), and because firm-level output is always efficient, there is no scope for a market inefficiency in those two cases.

[2]Notice that for $\theta > 1$, $\left(\frac{\theta-1}{\theta-\alpha}\right)^{\alpha}\left(\frac{\theta}{\theta-\alpha}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \geq 1$, with equality when α is either 0 or 1.

[3]This can be verified from the fourth constraint of the social planner problem above, which indicates that downstream productivity is proportional to $(M^{u})^{\frac{\theta(1-\alpha)}{\theta-1}}$.

A.3 Optimal Policy

Suppose we endow a government with the ability to provide subsidies (or charge taxes) on the purchases of final goods or intermediate inputs. Denote these taxes by s^d and s^u in the downstream and upstream sectors, respectively. We assume that subsidy proceeds are extracted from households (or tax revenue is rebated to households) in a lump-sum manner.

Once we introduce taxes, price indexes become:

$$P^u = (M^u)^{\frac{1}{1-\theta}} (1-s^u)p^u, \qquad P^d = \left(M^d\right)^{\frac{1}{1-\sigma}} (1-s^d)p^d$$

and household disposable income becomes,

$$I = wL - M^d s^d p^d x^d - M^u s^u p^u x^u.$$

It is straightforward to show that taxes and subsidies do not alter the equilibrium firm size, which is still pinned down by free entry at the (efficient) levels given in (A.3). Turning to the determination of the measure of firms in each sector, we first invoke households' demand for downstream goods combined with goods-market clearing and household total income to obtain

$$M^d = \frac{wL - s^u M^u p^u x^u}{p^d x^d}.$$

Next, labor market clearing ensures that equation (A.6) still holds. The equilibrium measure of firms, given subsidies s^d and s^u, is then:

$$M^u = \frac{1}{1-\alpha s^u} \frac{(1-\alpha)A^u L}{\theta f^u}$$

$$M^d = (1-s^u)^\alpha \left(\frac{1}{1-\alpha s^u}\right)^{\frac{\theta-\alpha}{\theta-1}} \frac{\alpha^\alpha A^d}{\sigma f^d} \left[\frac{(1-\alpha)A^u}{\theta f^u}\right]^{(1-\alpha)\frac{\theta}{\theta-1}} \left((\theta-1)\, f^u\right)^{1-\alpha} L^{\frac{\theta-\alpha}{\theta-1}}.$$

Notice that downstream subsidies s^d have no impact on the market allocation. Because they are a *redundant* instrument, we can safely set them to 0. From the above expressions, it is then clear that:

Proposition 2. The social planner can restore efficiency in the market equilibrium by subsidizing upstream production at a rate $(s^u)^* = 1/\theta$.

Notice that the subsidy corresponds to the reciprocal of the elasticity of substitution across inputs. As a result, this subsidy encourages the entry of upstream suppliers especially when the inputs they produce are relatively less substitutable. There are two potential (and non-exclusive) explanations for this result. First, the lower is θ, the larger is the market power of and thus the markup charged by input suppliers, and thus the larger the subsidy required to undo this double marginalization inefficiency. Second, the lower is θ, the larger are the variety gains associated with upstream entry on the productivity of downstream firms, so to the extent that those gains are not internalized by input suppliers, again the larger is the required subsidy upstream.

A.4 Double Marginalization versus External Effects

We next dig a little bit deeper into the source of the market inefficiency. More specifically, we show that our vertical Krugman economy is isomorphic to a competitive vertical economy with external economies of scale. In this variant of our model, it is clear that the market inefficiency is due only to upstream suppliers failing to internalize the positive productivity effects of their entry on downstream firms (since there are no markups), and an upstream subsidy is again sufficient to restore efficiency.

The vertical economy with external economies of scale features consumers that spend their income on a single homogeneous final good. On the production side, this final good is produced combining labor and a homogeneous intermediate input, which is in turn produced with labor. The homogeneous intermediate input and final good are produced according to the technologies

$$\begin{aligned} x^u &= A^u \ell^u \left(L^u\right)^{\gamma^u} \\ x^d &= A^d \left(\ell^d\right)^{\alpha} \left(q^u\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{\gamma^d}, \end{aligned}$$

where L^u and L^d are the aggregate allocations of labor to the upstream and downstream sector, Q^u is total production upstream, and γ^u and γ^d measure the magnitude of external economies of scale.

Individual firms are symmetric, competitive, and infinitessimal, so they take the aggregates as given and price at marginal cost. The resulting prices for the upstream and downstream sector are given by

$$P^u = \frac{w}{A^u} \left(L^u\right)^{-\gamma^u}$$

and

$$P^d = \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$, $Q^u = A^u \left(L^u\right)^{1+\gamma^u}$ and $Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d}$, it is straightforward to infer that the equilibrium allocation of labor across sectors is given by

$$L^u = (1-\alpha) L, \quad \text{and} \quad L^d = \alpha L,$$

just as in our "Krugman" vertical economy with internal economies of scale. In addition, one can also show that whenever $\gamma^u = 1/(\theta-1)$ and $\gamma^d = 1/(\sigma-1)$, with an appropriate choice of the technological parameters A^d and A^u, the equilibria of the two models are fully isomorphic, not just in terms of the allocation of labor across sectors, but also in terms of prices and welfare.

As in the case of internal economies of scale, this market equilibrium can easily be shown to be inefficient. In particular, setting up the planner problem,

$$\begin{aligned} \max_{L^u, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha}\right)^{1+\gamma^d} \\ s.t. \quad & Q^u = A^u \left(L^u\right)^{1+\gamma^u} \\ & L^u + L^d = L, \end{aligned} \tag{A.14}$$

it is straightforward to see that this delivers

$$\left(L^u\right)^* = \frac{1+\gamma^u}{\gamma^u (1-\alpha)+1} (1-\alpha) L, \quad \text{and} \quad \left(L^d\right)^* = \frac{1}{\gamma^u (1-\alpha)+1} \alpha L. \tag{A.15}$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$. Finally, one can also verify that an upstream subsidy equal to $\left(s^u\right)^* = \gamma^u / \left(1+\gamma^u\right)$ is sufficient to restore efficiency. While it is perhaps surprising that the planner need not make any correction for the external economies in the downstream sector, this result is due to the fact that all of that sector's output is sold to consumers. By contrast, the fact that inputs are all sold to firms means that their underprovision requires a subsidy so that it does not distort final-good producers' purchases of inputs versus labor.

In sum, we have shown that a model with external economies of scales is isomorphic to our model with internal economies of scale as long as $\gamma^u = 1/(\theta - 1)$, and that the rationale for the use of upstream subsidies to restore efficiency can be tied to a love-for-variety productivity effect, rather than it being necessarily driven by a double marginalization inefficiency.

A.5 Extensions

In this Appendix, we briefly develop two extensions of our closed-economy model, both featuring a more complex input sector.

I. Roundabout Production Upstream

We first allow the upstream sector to use not only labor in production, but also the same bundle of inputs Q^u used in the final-good sector. More specifically, and focusing on the isomorphic economy with perfect competition, homogeneous goods, and external economies of scale developed in section A.4, we now assume

$$\begin{aligned} x^u &= A^u (\ell^u)^\beta (q^u)^{1-\beta} \left((L^u)^\beta (Q^u)^{1-\beta} \right)^{\gamma^u} \\ x^d &= A^d \left(\ell^d\right)^\alpha (q^u)^{1-\alpha} \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{\gamma^d}, \end{aligned}$$

where β governs the labor instensity of production upstream. It is clear from the second of these expressions that firms in the downstream sector will spend a fraction of its costs on the upstream sector, or

$$P^u q^u = (1-\alpha) P^d x^d.$$

Noting that, due to symmetry, $x^u = Q^u$ and $x^d = Q^d$, and that the decentralized market prices for the downstream sector is given by

$$P^d = \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{-\gamma^d}.$$

Invoking $P^d Q^d = wL$ and $Q^d = A^d \left(\left(L^d\right)^\alpha (Q^u)^{1-\alpha} \right)^{1+\gamma^d}$ we thus obtain

$$\frac{1}{A^d} \left(\frac{w}{\alpha}\right)^\alpha \left(\frac{P^u Q^u}{1-\alpha}\right)^{1-\alpha} A^d \left(L^d\right)^\alpha = wL$$

or

$$\left(\frac{w}{\alpha}\right)^\alpha (wL)^{1-\alpha} \left(L^d\right)^\alpha = wL,$$

from which it is immediate that

$$L^u = (1-\alpha) L, \quad \text{and} \quad L^d = \alpha L,$$

just as in our baseline model

We next consider the planner problem,

$$\begin{aligned} \max_{L^u, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} \left(Q^u\right)^{1-\alpha} \right)^{1+\gamma^d} \\ s.t. \quad & Q^u = A^u \left(\left(L^u\right)^{\beta} \left(Q^u\right)^{1-\beta} \right)^{1+\gamma^u} \\ & L^u + L^d = L. \end{aligned}$$

Noting that the second constraint can be written as

$$Q^u = \tilde{A}^u \left(L^u\right)^{1+\tilde{\gamma}^u},$$

where

$$\begin{aligned} \tilde{A}^u &= \left(A^u\right)^{\frac{1}{1-(1-\beta)(1+\gamma^u)}} \\ \tilde{\gamma}^u &= \frac{\gamma^u}{1-(1-\beta)\left(1+\gamma^u\right)}, \end{aligned}$$

it then becomes clear that this program is identical to the one in our baseline model, except for the fact that the scale elasticity upstream is now not given by γ^u, but by $\tilde{\gamma}^u > \gamma^u$ (the program also features a rescaled upstream productivity, but that is immaterial). Note that the gap between $\tilde{\gamma}^u$ and γ^u is decreasing in β.

Analogously to equation (A.15), the socially optimal allocation of labor is given by

$$\left(L^u\right)^* = \frac{1+\tilde{\gamma}^u}{\tilde{\gamma}^u\left(1-\alpha\right)+1}\left(1-\alpha\right)L, \quad \text{and} \quad \left(L^d\right)^* = \frac{1}{\tilde{\gamma}^u\left(1-\alpha\right)+1}\alpha L.$$

Clearly, the market equilibrium features too little labor allocated to the upstream sector whenever $\gamma^u > 0$ and $0 < \alpha < 1$, just as in our baseline model, but the inefficiency is now decreasing in β. Finally, one can also verify that an upstream subsidy equal to $\left(s^u\right)^* = \tilde{\gamma}^u / \left(1+\tilde{\gamma}^u\right)$ is sufficient to restore efficiency. Because $\tilde{\gamma}^u > \gamma^u$, this subsidy is now higher than in our baseline model, and it is decreasing in β.

II. Multi-Stage Production

We next develop a multi-stage extension of the model. We begin with a simple three-stage production process with a downstream sector, a midstream sector, and an upstream sector. The technologies are given by

$$\begin{aligned} x^u &= A^u \left(\ell^u\right) \left(L^u\right)^{\gamma^u} \\ x^m &= A^d \left(\ell^m\right)^{\beta} \left(q^u\right)^{1-\beta} \left(\left(L^m\right)^{\beta} \left(Q^u\right)^{1-\beta} \right)^{\gamma^m} \\ x^d &= A^d \left(\ell^d\right)^{\alpha} \left(q^m\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} \left(Q^m\right)^{1-\alpha} \right)^{\gamma^d}, \end{aligned}$$

Using the fact that, in a decentralized equilibrium, we have

$$\begin{aligned} P^d Q^d &= wL; \\ Q^d &= A^d \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{1+\gamma^d}; \\ P^d &= \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{P^m}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{-\gamma^d}; \\ P^m Q^m &= (1-\alpha) P^d Q^d, \end{aligned}$$

we immediately obtain

$$L^d = \alpha L.$$

Next, because

$$\begin{aligned} P^m Q^m &= (1-\alpha) wL; \\ Q^m &= A^m \left((L^m)^{\alpha} (Q^u)^{1-\alpha} \right)^{1+\gamma^m}; \\ P^m &= \frac{1}{A^m} \left(\frac{w}{\beta}\right)^{\beta} \left(\frac{P^u}{1-\beta}\right)^{1-\beta} \left((L^m)^{\beta} (Q^u)^{1-\beta} \right)^{-\gamma^m}; \\ P^u Q^u &= (1-\beta) P^m Q^m, \end{aligned}$$

we obtain

$$L^m = \beta (1-\alpha) L, \quad \text{and} \quad L^u = (1-\beta)(1-\alpha) L.$$

Now consider the planner problem

$$\begin{aligned} \max_{L^u, L^m, L^d} \quad & Q^d = A^d \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{1+\gamma^d} \\ s.t. \quad & Q^m = A^m \left((L^m)^{\beta} (Q^u)^{1-\beta} \right)^{1+\gamma^u} \\ & Q^u = A^u (L^u)^{1+\gamma^u} \\ & L^u + L^m + L^d = L. \end{aligned}$$

which delivers

$$\begin{aligned} (L^u)^* &= \frac{(1+\gamma^u)(1+\gamma^m)}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} (1-\beta)(1-\alpha) L \\ (L^m)^* &= \frac{1+\gamma^m}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \beta (1-\alpha) L \\ \left(L^d\right)^* &= \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))} \alpha L. \end{aligned}$$

Notice that the gap between the socially optimal and the market allocation of labor is higher the more upstream the stage. Does that mean that subsidies are higher, the more upstream a sector?

Consider the following key conditions identify a market equilibrium with subsidies:

$$\begin{aligned}
P^d Q^d &= wL - s^m P^m Q^m - s^u P^u Q^u \\
Q^d &= A^d \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{1+\gamma^d}; \\
P^d &= \frac{1}{A^d} \left(\frac{w}{\alpha}\right)^{\alpha} \left(\frac{(1-s^m) P^m}{1-\alpha}\right)^{1-\alpha} \left(\left(L^d\right)^{\alpha} (Q^m)^{1-\alpha} \right)^{-\gamma^d}; \\
(1-s^m) P^m Q^m &= (1-\alpha) P^d Q^d \\
Q^m &= A^m \left((L^m)^{\alpha} (Q^u)^{1-\alpha} \right)^{1+\gamma^m} \\
P^m &= \frac{1}{A^m} \left(\frac{w}{\beta}\right)^{\beta} \left(\frac{(1-s^u) P^u}{1-\beta}\right)^{1-\beta} \left((L^m)^{\beta} (Q^u)^{1-\beta} \right)^{-\gamma^m} \\
(1-s^u) P^u Q^u &= (1-\beta) P^m Q^m
\end{aligned}$$

Note that

$$P^d Q^d = wL - \frac{s^m}{1-s^m}(1-\alpha) P^d Q^d - \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha) P^d Q^d$$

or

$$P^d Q^d = \frac{wL}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}.$$

Next

$$\begin{aligned}
P^d Q^d &= \left(\frac{wL^d}{\alpha}\right)^{\alpha} \left(\frac{(1-s^m) P^m Q^m}{1-\alpha}\right)^{1-\alpha} \\
&= \left(\frac{wL^d}{\alpha}\right)^{\alpha} \left(P^d Q^d\right)^{1-\alpha},
\end{aligned}$$

so

$$\frac{L^d}{L} = \frac{\alpha}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}.$$

Next,

$$\begin{aligned}
P^m Q^m &= \left(\frac{wL^m}{\beta}\right)^{\beta} \left(\frac{(1-s^u) P^u Q^u}{1-\beta}\right)^{1-\beta} \\
&= \left(\frac{wL^m}{\beta}\right)^{\beta} (P^m Q^m)^{1-\beta},
\end{aligned}$$

so

$$P^m Q^m = \frac{(1-\alpha)}{(1-s^m)} P^d Q^d = \frac{wL^m}{\beta}$$

or

$$\frac{L^m}{L} = \frac{\beta \frac{(1-\alpha)}{(1-s^m)}}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)}.$$

We thus have that the subsidies s^m and s^u need to satisfy

$$\frac{\alpha}{1 + \frac{s^m}{1-s^m}(1-\alpha) + \frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}(1-\alpha)} = \frac{1}{\alpha + (1-\alpha)(1+\gamma^m)(\beta + (1-\beta)(1+\gamma^u))}\alpha$$

and

$$\frac{\beta\frac{(1-\alpha)}{(1-s^m)}}{1+\frac{s^m}{1-s^m}\left(1-\alpha\right)+\frac{s^u}{1-s^u}\frac{(1-\beta)}{1-s^m}\left(1-\alpha\right)}=\frac{1+\gamma^m}{\alpha+\left(1-\alpha\right)\left(1+\gamma^m\right)\left(\beta+\left(1-\beta\right)\left(1+\gamma^u\right)\right)}\beta\left(1-\alpha\right),$$

which delivers

$$s^m=\frac{\gamma^m}{1+\gamma^m};\quad s^u=\frac{\gamma^u}{1+\gamma^u}.$$

As is clear from this expression, subsidies in all sectors producing inputs are positive, but notice that subsidies are higher upstream relative to midstream only if $\gamma^u > \gamma^m$, i.e., only if the scale elasticity is higher upstream than midstream. This contrasts with the results of Liu (2019), who finds that optimal subsidies should necessarily be higher, the more upstream the sector. The reason is that, unlike in Liu's work, we solve for first-best subsidy policy: when the government can only set subsidies in one sector, the size of the subsidy would be higher, the more upstream the sector, because as we have seen above, the gap between the social optimal and market allocation of labor is highest in the upstream sector.

B Open Economy Model: Details on Derivations

B.1 Open Economy Equilibrium with Internal Economies of Scale

In this Appendix, we outline the equilibrium conditions corresponding to the two-country model featuring internal scale economies, product differentiation and monopolistic competition outlined in section 3.1. We will then work with these equations in Appendix to demonstrate the isomorphism claimed in Proposition 3.

Import tariffs on the downstream sector create a wedge between consumer prices in country i and producer prices in country j. More specifically, given CES preferences, consumer prices in i for goods originating in j are given by:

$$p_{ji}^{d} = \left(1+t_{i}^{d}\right)\frac{\sigma}{\sigma-1}\tau^{d}\frac{1}{A_{j}^{d}}\left(\frac{w_{j}}{\alpha}\right)^{\alpha}\left(\frac{P_{j}^{u}}{1-\alpha}\right)^{1-\alpha} = \frac{1+t_{i}^{d}}{1-v_{j}^{d}}\tilde{p}_{ji}^{d}. \tag{B.1}$$

Similarly, import tariffs on the upstream sector create a wedge between the price paid by final-good producers in i for inputs from j, and the producer price for those inputs obtained by suppliers in country j. In particular, we have

$$p_{ji}^{u} = \left(1+t_{i}^{u}\right)\frac{\theta}{\theta-1}\tau^{u}\frac{w_{j}}{A_{j}^{u}} = \frac{1+t_{i}^{u}}{1-v_{j}^{u}}\tilde{p}_{ji}^{u}. \tag{B.2}$$

In equation (B.1), the price index P_{i}^{u} is given by

$$P_{i}^{u} = \left[\sum_{j\in\{H,F\}}\left(P_{ji}^{u}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}, \tag{B.3}$$

with

$$P_{ji}^{u} = \left[\int_{0}^{M_{j}^{u}}\left(p_{ji}^{u}\left(\varpi\right)\right)^{1-\theta}d\varpi\right]^{\frac{1}{1-\theta}}. \tag{B.4}$$

When setting $j=i$, the above pricing equations also characterize domestic prices in country j after setting $t_{i}^{d}=t_{i}^{u}=v_{i}^{d}=v_{i}^{u}=0$ and $\tau^{d}=\tau^{u}=1$. Note that $p_{ii}^{d}=\tilde{p}_{ii}^{d}$ and $p_{ii}^{u}=\tilde{p}_{ii}^{u}$.

Next, utility maximization implies that when consuming country j varieties, consumers in i allocate to each variety ω a share of spending equal to

$$\frac{p_{ji}^{d}\left(\omega\right)q_{ji}^{d}\left(\omega\right)}{P_{ji}^{d}Q_{ji}^{d}} = \left(\frac{p_{ji}^{d}\left(\omega\right)}{P_{ji}^{d}}\right)^{1-\sigma}, \tag{B.5}$$

of their total spending on country j varieties, where

$$P_{ji}^{d} = \left[\int_{0}^{M_{j}^{d}}\left(p_{ji}^{d}\left(\omega\right)\right)^{1-\sigma}d\omega\right]^{\frac{1}{1-\sigma}}. \tag{B.6}$$

Consumers' (aggregate) spending on Home and Foreign varieties is in turn determined by

$$P_{ji}^{d}Q_{ji}^{d} = \left(\frac{P_{ji}^{d}}{P_{i}^{d}}\right)^{1-\sigma}\left(w_{i}L_{i}+R_{i}\right), \tag{B.7}$$

where P_i^d is the aggregate consumer price index in i

$$P_i^d = \left[\sum_{j \in \{H,F\}} \left(P_{ji}^d \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}, \tag{B.8}$$

and R_i is tariff revenue, which we have defined in equation (10).

We now turn to profit maximization by downstream producers in country i. First note that free entry implies that firm revenue (net of tariffs) will equal total costs, and that a share of those costs will go to pay labor. As a result, labor compensation by each final-good producer in i is given by

$$w_i \ell_i^d = \alpha \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right). \tag{B.9}$$

Next, when purchasing inputs from upstream producers in country j, final-good producers in country i, will demand an amount of each variety ϖ from country j equal to

$$q_{ji}^u(\varpi) = Q_{ji}^u(\omega) \left(\frac{p_{ji}^u}{P_{ji}^u} \right)^{-\theta},$$

while aggregate spending on all country j's input varieties is given by

$$P_{ji}^u Q_{ji}^u = (1-\alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right) \left(\frac{P_{ji}^u}{P_i^u} \right)^{1-\theta} M_i^d. \tag{B.10}$$

Aggregate spending on Home and Foreign intermediate inputs in country i is then given by

$$P_i^u Q_i^u = (1-\alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d \right) M_i^d. \tag{B.11}$$

Our final set of equilibrium conditions impose market clearing. First, labor-market clearing in both countries implies that

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u, \tag{B.12}$$

where ℓ_i^d is given in (B.9), and $\ell_i^u = (f_i^u + x_i^u) / A_i^u$.[4] Second, goods-market clearing imposes

$$q_{ii}^d + \tau^d q_{ij}^d = x_i^d \tag{B.13}$$

and

$$M_i^d q_{ii}^u + M_j^d \tau^u q_{ij}^u = x_i^u. \tag{B.14}$$

Note that free entry upstream and downstream implies that firm revenue is equal to total costs, which delivers

$$x_i^d = (\sigma - 1) f_i^d; \qquad x_i^u = (\theta - 1) f_i^u \tag{B.15}$$

for $i = \{H, F\}$. Firm-level production levels are thus independent of tariff choices, and the only way in which tariffs can affect the allocation of labor across sectors is by changing the measure of firms in each of the two

[4] Naturally, equilibrium also requires trade balance, but this is ensured by the other equilibrium conditions outlined in this section.

sectors. As a result, optimal trade policies will seek to achieve a social-welfare maximizing allocation of labor across sectors, with no concern for the allocation of labor within sectors (across fixed costs of entry versus marginal costs of production).

Despite the simple structure of the model and relatively simple equilibrium conditions, an analysis of how the market equilibrium is affected by input and final-good tariffs set by the Home country is complex, so we begin by considering the special case in which downstream production only uses inputs (and no labor) in production, or $\alpha = 0$.

B.2 Equilibrium of Isomorphic Competitive Economy with External Economies of Scale

In this Appendix we prove the isomorphism claimed in Proposition 3. More specifically, our goal is to show that equilibrium conditions of the decentralized equilibrium of the two-country model in section 3.1 featuring internal scale economies, product differentiation and monopolistic competition can be reduced to a set of equations identical to equations (14) through (21) applying to the competitive model with external economies of scale developed in this section.

Preferences We begin by noting that given symmetry in final-good production, we can express preferences as

$$\begin{aligned} U_i &= \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^d} q_{ji}^d(\omega)^{\frac{\sigma-1}{\sigma}} d\omega \right) \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left[M_i^d (q_{ii}^d)^{\frac{\sigma-1}{\sigma}} + M_j^d (q_{ji}^d)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} \\ &= \left(\left(Q_{ii}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{ji}^d\right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \end{aligned}$$

where

$$Q_{ii}^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} q_{ii}^d; \quad Q_{ji}^d \equiv \left(M_j^d\right)^{\frac{\sigma}{\sigma-1}} q_{ji}^d. \tag{B.16}$$

Starting from (9), we have thus derived (11), which are preferences in the isomorphic economy with two final goods (a Home one and a Foreign one) and external economies of scale.

Labor-Market Clearing Next, remember that ℓ_i^d and ℓ_i^u are the firm-level amounts of labor used downstream and upstream to cover fixed and variable costs. Hence, defining

$$L_i^d \equiv M_i^d \ell_i^d; \quad L_i^u \equiv M_i^u \ell_i^u, \tag{B.17}$$

we have that equation (B.12) in the monopolistic competition model implies equation (14) in the external economies model, or

$$L_i = M_i^d \ell_i^d + M_i^u \ell_i^u = L_i^u + L_i^d.$$

Upstream Market Clearing and Upstream Endogenous Productivity Next let us define

$$Q_{ii}^u \equiv M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u; \quad Q_{ij}^u \equiv M_j^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ij}^u. \tag{B.18}$$

Given these definitions in (B.18), and given the definition of the input aggregate $Q_i^u(\omega)$ in the monopolistic competition model, that is

$$Q_i^u(\omega) = \left[\sum_{j \in \{H,F\}} \left(\int_0^{M_j^u} q_{ji}^u(\varpi)^{\frac{\theta-1}{\theta}} d\varpi \right) \right]^{\frac{\theta}{\theta-1}}, \qquad \theta > 1, \quad i \in \{H, F\},$$

we have that the total usage of inputs by firms in country i is given by

$$\begin{aligned} Q_i^u &= M_i^d Q_i^u(\omega) = \left[M_i^u \left(q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + M_j^u \left(q_{ji}^u\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(M_i^d \left(M_i^u\right)^{\frac{\theta}{\theta-1}} q_{ii}^u \right)^{\frac{\theta-1}{\theta}} + \left(M_i^d \left(M_j^u\right)^{\frac{\theta}{\theta-1}} \left(q_{ji}^u\right) \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \\ &= \left[\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \end{aligned} \tag{B.19}$$

and thus is analogous to a CES aggregator of only two inputs: a Home and a Foreign one, as defined in equation (B.18). These inputs are either produced domestically or are imported.

Now consider the domestic production of those inputs. Let us start from the definition of upstream technology in the monopolistic competition model, that is

$$f_i^u + x_i^u(\varpi) = A_i^u \ell_i^u(\varpi), \qquad \varpi \in [0, M_i^u], \quad i \in \{H, F\}.$$

Imposing symmetry and firm-level output in equation (B.15) – i.e., $x_i^u = (\theta - 1) f_i^u$ –, and invoking the definition of L_i^u in (B.17), we have

$$X_i^u \equiv \left(M_i^u\right)^{\frac{\theta}{\theta-1}} x_i^u = \left(\frac{A_i^u}{\theta f_i^u} \right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u \left(L_i^u\right)^{\frac{\theta}{\theta-1}} \tag{B.20}$$

or

$$X_i^u = \hat{A}_i^u F_i^u \left(\ell_i^u\right) = \bar{A}_i^u \left(L_i^u\right)^{1+\gamma^u},$$

where

$$\bar{A}_i^u \equiv \left(\frac{A_i^u}{\theta f_i^u} \right)^{\frac{\theta}{\theta-1}} (\theta - 1) f_i^u,$$

and

$$\gamma^u \equiv 1/\left(\theta - 1\right).$$

Because this domestic production X_i^u is sold domestically or exported, we have

$$\bar{A}_i^u \left(L_i^u\right)^{1+\gamma^u} = Q_{ii}^u + Q_{ij}^u,$$

which corresponds exactly to equation (15) in the external economies model.

Downstream Market Clearing and Downstream Endogenous Productivity We can proceed analogously for final-good production. We begin with the definition of technology in the downstream sector in the monopolistic competition model:

$$f_i^d + x_i^d(\omega) = A_i^d(\ell_i^d(\omega))^\alpha Q_i^u(\omega)^{1-\alpha}, \qquad \omega \in [0, M_i^d], \quad \alpha \in [0,1], \quad i \in \{H, F\}.$$

Imposing symmetry and (B.15), we obtain

$$M_i^d = \frac{A_i^d}{\sigma f_i^d}(M_i^d \ell_i^d(\omega))^\alpha \left(M_i^d Q_i^u\right)^{1-\alpha}$$

or

$$X_i^d \equiv \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^\alpha \left(\left(L_i^d\right)^\alpha \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}}. \tag{B.21}$$

where

$$\bar{A}_i^d \equiv \left(\frac{A_i^d}{\sigma f_i^d}\right)^{\frac{\sigma}{\sigma-1}} (\sigma - 1) f_i^d.$$

This aggregate output X_i^d is sold domestically or exported, and thus

$$\bar{A}_i^d \left(\left(L_i^d\right)^\alpha \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\right)^{\gamma^d} = Q_{ii}^d + Q_{ij}^d,$$

where

$$\gamma^d \equiv 1/(\sigma - 1).$$

In sum, starting from the monopolistic competition model, we have derived equation (15) in the external economies model.

Trade Balance Consider next the trade balance condition. Starting from the monopolistic competition economy, we have

$$\frac{p_{ji}^d}{1+t_i^d} M_j^d q_{ji}^d + \frac{p_{ji}^u}{1+t_i^u} M_i^d M_j^u q_{ji}^u = \frac{\tilde{p}_{ij}^d}{1-v_i^d} M_i^d q_{ij}^d + \frac{\tilde{p}_{ij}^u}{1-v_i^u} M_j^d M_i^u q_{ij}^u, \tag{B.22}$$

which equates the import revenue in i paid to exporters in j with export revenue collected from j by producers in i.

Now from equations (B.5) and (B.6), notice that we have

$$\frac{p_{ji}^d(\omega)\, q_{ji}^d(\omega)}{P_{ji}^d Q_{ji}^d} = \left(\frac{p_{ji}^d(\omega)}{P_{ji}^d}\right)^{1-\sigma},$$

and

$$P_{ji}^d = \left[\int_0^{M_j^d} \left(p_{ji}^d(\omega)\right)^{1-\sigma} d\omega\right]^{\frac{1}{1-\sigma}},$$

so given symmetry, we have

$$P_{ji}^d = \left(M_j^d\right)^{\frac{1}{1-\sigma}} p_{ji}^d \tag{B.23}$$

and

$$P^d_{ji}Q^d_{ji} = \left(M^d_j\right)^{\frac{-1}{\sigma-1}} p^d_{ji} \times \left(M^d_i\right)^{\frac{\sigma}{\sigma-1}} q^d_{ii} = M^d_j p^d_{ji} q^d_{ii}.$$

Similarly, for inputs

$$P^u_{ji}Q^u_{ji} = \left(M^u_j\right)^{\frac{1}{1-\theta}} p^u_{ji} \times M^d_i \left(M^u_i\right)^{\frac{\theta}{\theta-1}} q^u_{ji} = p^u_{ji} M^d_i M^u_j q^u_{ji}.$$

This implies that we can write total imports in the trade balance condition (B.22) as

$$\frac{P^d_{ji}}{1+t^d_i}Q^d_{ji} + \frac{P^u_{ji}}{1+t^u_i}Q^u_{ji} = \bar{P}^d_{ji}Q^d_{ji} + \bar{P}^u_{ji}Q^u_{ji},$$

which corresponds to the left-hand-side of the trade balance condition (17) for the economy with external economies of scale after noting that $\bar{P}^d_{ji}$ and $\bar{P}^u_{ji}$ are the prices collected by country j (or Foreign) exporters (not those paid by domestic or country i consumers).

Now consider revenue from exporting final goods. Notice that, regardless of whether the Foreign government imposes import tariffs or not, we have that export revenue is

$$\frac{\tilde{p}^d_{ij}}{1-v^d_i}M^d_i q^d_{ij} + \frac{\tilde{p}^u_{ij}}{1-v^u_i}M^d_j M^u_i q^u_{ij}$$

Prices paid by country j are $\tilde{p}^d_{ij}/\left(1-v^d_i\right)$ and $\tilde{p}^u_{ij}/\left(1-v^u_i\right)$, so following analogous steps, the right-hand-side of (17) becomes

$$\frac{\tilde{P}^d_{ij}}{1-v^d_i}Q^d_{ij} + \frac{\tilde{P}^u_{ij}}{1-v^u_i}Q^u_{ij} = \bar{P}^d_{ij}Q^d_{ij} + \bar{P}^u_{ij}Q^u_{ij},$$

where $\bar{P}^d_{ij}$ and $\bar{P}^u_{ij}$ are the prices paid by country j (or Foreign) importers (not those paid collected by country i exporters).

Note: In the main text, we denote $\bar{P}^d_{ji}$, $\bar{P}^u_{ji}$, $\bar{P}^d_{ij}$, and $\bar{P}^u_{ij}$ as simply P^d_{ji}, P^u_{ji}, P^d_{ij}, and P^u_{ij}. We do so not to clutter the notation, but these are distinct from the price indices applying to the monopolistic competition model, which are always built based on prices paid by consumers, regardless of their country.

Optimality Conditions We have so far shown that the four 'resource' constraints (14) through (17) of our isomorphic economy can be derived from our baseline model with monopolistic competition and internal economies of scale. We next turn to an analogous derivation for the optimality conditions (18) through (21).

Given our functional forms for utility and technology, these optimality conditions in the model with external economies of scale are given by

$$\left(\frac{Q^d_{ii}}{Q^d_{ji}}\right)^{-\frac{1}{\sigma}} = \frac{\left(1-v^d_i\right)}{\left(1+\tau^d_i\right)}\frac{\bar{P}^d_{ij}}{\bar{P}^d_{ji}}; \quad \text{(B.24)}$$

$$\left(\frac{Q^u_{ii}}{Q^u_{ji}}\right)^{-\frac{1}{\theta}} = \frac{\left(1-v^u_i\right)}{\left(1+\tau^u_i\right)}\frac{\bar{P}^u_{ij}}{\bar{P}^u_{ji}}; \quad \text{(B.25)}$$

$$(1-\alpha)\hat{A}^d_i\left(L^d_i\right)^{\alpha}\left(\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}}\frac{1}{Q^u_{ii}}\frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1-v^u_i\right)}{\left(1-v^d_i\right)}\frac{\bar{P}^u_{ij}}{\bar{P}^d_{ij}}; \quad \text{(B.26)}$$

$$(1-\alpha)\hat{A}^u_i\frac{1}{Q^u_{ii}}\frac{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}}}{\left(Q^u_{ii}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{ji}\right)^{\frac{\theta-1}{\theta}}} = \alpha\frac{1}{L^d_i}. \quad \text{(B.27)}$$

Optimal in Final-Good Consumption Let us begin with the first one, equating the marginal rate of substitution in final-good consumption to relative prices. Given equation (B.7) in the model with monopolistic competition, we have

$$\frac{Q_{ii}^d}{Q_{ji}^d} = \left(\frac{P_{ii}^d}{P_{ji}^d}\right)^{-\sigma},$$

where Q_{ji}^d, Q_{ji}^d, P_{ji}^d and P_{ji}^d are defined in (B.18) and (B.23). Thus, we have

$$\left(\frac{Q_{ii}^d}{Q_{ji}^d}\right)^{-\frac{1}{\sigma}} = \frac{P_{ii}^d}{P_{ji}^d} = \frac{\left(1 - v_i^d\right) \bar{P}_{ij}^d}{\left(1 + \tau_i^d\right) \bar{P}_{ji}^d},$$

where $P_{ii}^d = \left(1 - v_i^d\right) \bar{P}_{ij}^d$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}_{ij}^d$ is the price paid by consumers in j for final goods from j). We have thus derived equation (B.24), which corresponds to (18) in the external economies of scale model.

Optimal in Input Consumption The derivation of equation (B.25), equating the marginal rate of substitution in input consumption to relative prices, is completely analogous. In particular, from equation (B.10) in the model with monopolistic competition, we have

$$\frac{Q_{ii}^u}{Q_{ji}^u} = \left(\frac{P_{ii}^u}{P_{ji}^u}\right)^{-\theta},$$

where Q_{ji}^u, Q_{ji}^u, P_{ji}^u and P_{ji}^u are defined in (B.18) and (B.23). Thus, we have

$$\left(\frac{Q_{ii}^u}{Q_{ji}^u}\right)^{-\frac{1}{\theta}} = \frac{P_{ii}^u}{P_{ji}^u} = \frac{\left(1 - v_i^u\right) \bar{P}_{ij}^u}{\left(1 + \tau_i^u\right) \bar{P}_{ji}^u},$$

where $P_{ii}^u = \left(1 - v_i^d\right) \bar{P}_{ij}^u$ because of the indifference between selling domestically or exporting to country j (remember that, in the external economies of scale model, $\bar{P}_{ij}^u$ is the price paid by consumers in j for final goods from j). We have thus derived equation (B.25), which corresponds to (19) in the external economies of scale model.

Optimality Domestic Input Allocation We next move to the third optimality condition (20), which equates the benefits of exporting domestic intermediate inputs with the benefits of using those additional domestic inputs to produce an additional amount of the final good that is in turn exported.

We begin with equation (B.11), and note that aggregate input use in country i in the monopolistic competition model is given by

$$P_i^u Q_i^u = (1 - \alpha) \left(\tilde{p}_{ii}^d q_{ii}^d + \frac{1 - v_i^d}{1 + t_j^d} \tilde{p}_{ij}^d q_{ij}^d\right) M_i^d. \tag{B.28}$$

To reiterate this, note from (B.2) that $\frac{1-v_i^d}{1+t_j^d}\tilde{p}_{ij}^d = \tau^d p_{ii}^d$, and plugging in equation (B.13), we obtain

$$P_i^u Q_i^u = (1 - \alpha)\, p_{ii}^d \left(q_{ii}^d + \tau^d q_{ij}^d\right) M_i^d = (1 - \alpha)\, p_{ii}^d x_i^d M_i^d. \tag{B.29}$$

Next invoke equation (B.10) applied to $P_{ii}^u Q_{ii}^u$ to obtain (after plugging in (B.28)):

$$P_i^u Q_i^u = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}. \tag{B.30}$$

Combining (B.29) and (B.30), we obtain:

$$(1-\alpha)\, p_{ii}^d x_i^d M_i^d = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1},$$

which we decompose as

$$(1-\alpha) \times \left(M_i^d\right)^{\frac{-1}{\sigma-1}} p_{ii}^d \times \left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = P_{ii}^u Q_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}, \tag{B.31}$$

Now remember from equation (B.21) derived above that

$$\left(M_i^d\right)^{\frac{\sigma}{\sigma-1}} x_i^d = \bar{A}_i^d \left[\left(L_i^d\right)^{\alpha} \left(\left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}\right]^{\frac{\sigma}{\sigma-1}},$$

and also from (B.23) that $\left(M_i^d\right)^{\frac{-1}{\sigma-1}} p_{ii}^d = P_{ii}^d$, so we can write (B.31) as

$$P_{ii}^d (1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} = P_{ii}^u \left(\frac{P_{ii}^u}{P_i^u}\right)^{\theta-1}.$$

Now invoke (B.10)

$$\frac{Q_{ii}^u}{Q_i^u} = \left(\frac{P_{ii}^u}{P_i^u}\right)^{-\theta},$$

to obtain

$$P_{ii}^d (1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} = P_{ii}^u \left(\frac{Q_{ii}^u}{Q_i^u}\right)^{-\frac{\theta-1}{\theta}},$$

which given the definition of Q_i^u in (B.1) delivers

$$P_{ii}^d (1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}} = P_{ii}^u.$$

The final step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^d = \left(1-v_i^d\right) \bar{P}_{ij}^d$ and $P_{ii}^u = \left(1-v_i^d\right) \bar{P}_{ij}^u$, where remember that $\bar{P}_{ij}^d$ and $\bar{P}_{ij}^u$ are the prices paid by country j residents. In sum, we have derived equation (B.26), or

$$(1-\alpha)\, \hat{A}_i^d \left(L_i^d\right)^{\alpha} \left(\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}} \frac{1}{Q_{ii}^u} \frac{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{ji}^u\right)^{\frac{\theta-1}{\theta}}} = \frac{\left(1-v_i^d\right) \bar{P}_{ij}^u}{\left(1-v_i^d\right) \bar{P}_{ij}^d}.$$

Optimal Labor Market Allocation We finally tackle the fourth optimality condition, associated with the optimal allocation of labor across sectors. We begin with the firm-level monopolistic competition model, equating the wage paid in both sectors. Because of free entry, total revenue upstream must equal total wage payments, while in the downstream sector, wage payments are a share α of total revenue, as indicated in equation (B.9), or

$$\frac{\alpha\left(\tilde{p}_{ii}^{d}q_{ii}^{d}+\frac{1-v_{i}^{d}}{1+t_{j}^{d}}\tilde{p}_{ij}^{d}q_{ij}^{d}\right)}{\ell_{i}^{d}}=\frac{\tilde{p}_{ii}^{u}M_{i}^{d}q_{ii}^{u}+\frac{1-v_{i}^{u}}{1+t_{j}^{u}}\tilde{p}_{ij}^{u}M_{j}^{d}q_{ij}^{u}}{\ell_{i}^{u}}.$$

Now noting that from (B.2), we have $\frac{1-v_{i}^{d}}{1+t_{j}^{d}}\tilde{p}_{ij}^{d}=\tau^{d}p_{ii}^{d}$ (and analogously $\frac{1-v_{i}^{u}}{1+t_{j}^{u}}\tilde{p}_{ij}^{u}=\tau^{u}p_{ii}^{u}$), and plugging in equations (B.13) and (B.14), we have

$$\frac{\alpha p_{ii}^{d}x_{i}^{d}}{\ell_{i}^{d}}=\frac{p_{ii}^{u}x_{i}^{u}}{\ell_{i}^{u}}. \tag{B.32}$$

Next, invoke the price index definitions – see equation B.23) – as well as the definitions $L_{i}^{d}=M_{i}^{d}\ell_{i}^{d}$ and $L_{i}^{u}=M_{i}^{u}\ell_{i}^{u}$, to write (B.32) as

$$\alpha P_{ii}^{d}\frac{x_{i}^{d}\left(M_{i}^{d}\right)^{\frac{\sigma}{\sigma-1}}}{L_{i}^{d}}=P_{ii}^{u}\frac{\left(M_{i}^{u}\right)^{\frac{\theta}{\theta-1}}x_{i}^{u}}{L_{i}^{u}}.$$

Next, plugging (B.20) and (B.21), delivers

$$\frac{\alpha P_{ii}^{d}}{L_{i}^{d}}\hat{A}_{i}^{d}\left(L_{i}^{d}\right)^{\alpha}\left(\left(L_{i}^{d}\right)^{\alpha}\left(\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}=P_{ii}^{u}\hat{A}_{i}^{u},$$

where remember that $\hat{A}_{i}^{d}$ and $\hat{A}_{i}^{u}$ are defined in equations (12) and (13) in the main text.

The next step is to note, as we did above, that indifference between selling domestically and exporting, delivers $P_{ii}^{d}=\left(1-v_{i}^{d}\right)\bar{P}_{ij}^{d}$ and $P_{ii}^{u}=\left(1-v_{i}^{d}\right)\bar{P}_{ij}^{u}$, where remember that $\bar{P}_{ij}^{d}$ and $\bar{P}_{ij}^{u}$ are the prices paid by country j residents, so we have

$$\frac{\alpha}{L_{i}^{d}}\hat{A}_{i}^{d}\left(L_{i}^{d}\right)^{\alpha}\left(\left(L_{i}^{d}\right)^{\alpha}\left(\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}}\right)^{1-\alpha}=\hat{A}_{i}^{u}\frac{\left(1-v_{i}^{d}\right)}{\left(1-v_{i}^{d}\right)}\frac{\bar{P}_{ij}^{u}}{\bar{P}_{ij}^{d}}.$$

The final step is to plug optimality condition (B.26) and cancel terms to obtain

$$\frac{\alpha}{L_{i}^{d}}=\left(1-\alpha\right)\hat{A}_{i}^{u}\frac{1}{Q_{ii}^{u}}\frac{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{ii}^{u}\right)^{\frac{\theta-1}{\theta}}+\left(Q_{ji}^{u}\right)^{\frac{\theta-1}{\theta}}},$$

which corresponds to the last optimality condition (B.26).

This completes the proof of the isomorphism claimed in Proposition 3.

C Optimal Trade Policy for a Small Open Economy with No Domestic Distortions: Derivations

C.1 First-Best Policies

We begin by characterizing the solution to the program

$$\begin{aligned} \max \quad & U\left(Q^d_{HH}, Q^d_{FH}\right) \\ s.t. \quad & \hat{A}^u_H L_H = Q^u_{HH} + Q^u_{HF} \\ & \hat{A}^d_H F^d\left(Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\ & P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}, \end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H \left(L_H\right)^{\gamma^u},$$

respectively. We also note that

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and that

$$F^d\left(Q^u_{HH}, Q^u_{FH}\right) = \left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$\begin{aligned} & U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u\left[\bar{A}^u_H \left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] + \mu_d\left[\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\ & + \mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right]. \end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, and Q^u_{HF} are as follows:

$$U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right) = \mu_d \tag{C.1}$$

$$U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right) = \mu_{TB} P^d_{FH} \tag{C.2}$$

$$\mu_d = \mu_{TB} \frac{\sigma-1}{\sigma} P^d_{HF} \tag{C.3}$$

$$\mu_u = \mu_d\left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) \tag{C.4}$$

$$\mu_{TB} P^u_{FH} = \mu_d\left(1+\gamma^d\right) \bar{A}^d_H \left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d} F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right) \tag{C.5}$$

$$\mu_u = \mu_{TB} \frac{\theta-1}{\theta} P^u_{HF} \tag{C.6}$$

Dividing equation (23) by equation (C.2), and plugging in (C.3), we obtain obtain:

$$\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} = \frac{\frac{\sigma-1}{\sigma} P_{HF}^d}{P_{FH}^d},$$

which corresponds to the first optimality condition (23) in the main text.

Next, we divide equation (C.4) by equation (C.5), and plugging in (C.6), delivers

$$\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right)} = \frac{\frac{\theta-1}{\theta} P_{HF}^u}{P_{FH}^u},$$

which corresponds to the second optimality condition (24) in the main text.

Finally, combining equation (C.4) with the ratio of equations (C.3) and (C.6) produces

$$\left(1+\gamma^d\right) \bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}} \frac{P_{HF}^u}{P_{HF}^d},$$

which corresponds to the third optimality condition (25) in the main text.

C.2 Generalizations

As demonstrated in the derivations in the above Appendix C.1, we have made no use of the properties of the functions $U\left(Q_{HH}^d, Q_{FH}^d\right)$ and $F^d\left(Q_{HH}^u, Q_{FH}^u\right)$. In particular, we could assume that

$$U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma_H-1}{\sigma_H}} + \left(Q_{FH}^d\right)^{\frac{\sigma_H-1}{\sigma_H}}\right)^{\frac{\sigma_H}{\sigma_H-1}}$$

and that

$$F^d\left(Q_{HH}^u, Q_{FH}^u\right) = \left(\left(Q_{HH}^u\right)^{\frac{\theta_H-1}{\theta_H}} + \left(Q_{FH}^u\right)^{\frac{\theta_H-1}{\theta_H}}\right)^{\frac{\theta_H}{\theta_H-1}},$$

with potentially $\sigma_H \neq \sigma$ and $\theta_H \neq \theta$. It is clear from the derivations in section 4.1 that the first-best trade policies will continue to satisfy

$$\begin{aligned}
1+t_H^d &= \left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

The only significant difference in this case is that if we want to invoke our isomorphism to claim that these policies also implement the first best in the Krugman vertical economy with internal economies of scale, then we necessarily need to impose $\gamma^d = 1/\left(\sigma_H - 1\right)$, and thus the level of the tariff escalation is closely related to the degree of differentiation in the final-good sector. This is not particularly surprising, since love-for-variety effects will be more powerful, the lower the degree of substitutability across final goods.

C.3 Alternative First-Best Implementations

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes domestic production subsidies, domestic consumption subsidies, or domestic production/consumption subsidies that only apply to domestic transactions.

C.3.1 Discriminatory Domestic Subsidies

We first consider the case in which the Home government has access to discriminatory domestic subsidies s^d_{HH} and s^u_{HH} that apply only to purchases of final goods and of intermediate inputs involving only Home residents. The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19) and (20) as follows:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} &= \left(1-s^d_{HH}\right)\frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} &= \left(1-s^u_{HH}\right)\frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) &= \left(1-s^u_{HH}\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), and (25), it is clear that the first best can be achieved by setting

$$\begin{aligned}
\left(1+t^d_H\right)\left(1-s^d_{HH}\right) &= \left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
1+t^u_H &= 1+\bar{T};\\
1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\
\left(1-s^u_{HH}\right)\left(1-v^u_H\right) &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1+t^d_H\right)\left(1-s^d_{HH}\right)$ matters), while an upstream discriminatory subsidy is a perfect substitute for the upstream export tax (only the product $\left(1-s^u_{HH}\right)\left(1-v^u_H\right)$ matters). A straightforward implication of this result is that, whenever $1+\gamma^d=\sigma/\left(\sigma-1\right)$, as imposed by our isomorphism, the first best can be attained by setting $\left(1+t^d_H\right)\left(1-s^d_{HH}\right)=\sigma/\left(\sigma-1\right)$ and $\left(1-s^u_{HH}\right)\left(1-v^u_H\right)=\left(\theta-1\right)/\theta$. Thus, the first best can be achieved with only discriminatory subsidies, or with a combination of a subsidy in one sector and a trade instrument in the other sector. When only domestic instruments are used, we necessarily have $s^d_{HH}=1/\sigma$ and $s^u_{HH}=1/\theta$. Whether or not the resulting first-best policies entail tariff escalation depends on the level of the downstream domestic subsidy, since $\left(1+t^d_H\right)/\left(1+t^u_H\right)=\left(1+\gamma^d\right)/\left(1-s^d_{HH}\right)$.

C.3.2 Production Subsidies

We next consider the case of production subsidies s^d_H and s^u_H that apply to Home production of final goods and intermediate inputs, regardless of where those goods are sold (domestically or exported). The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19) and (20) as follows:

$$\begin{aligned}\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\ \frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\ \hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) &= \left(1-s^d_H\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), and (25), it is clear that the first best can be achieved by setting

$$\begin{aligned}1+t^d_H &= \left(1-s^d_H\right)\left(1+\gamma^d\right)\left(1+\bar{T}\right);\\ 1+t^u_H &= 1+\bar{T};\\ 1-v^d_H &= \frac{\sigma-1}{\sigma}\left(1+\gamma^d\right)\left(1-s^d_H\right)\left(1+\bar{T}\right);\\ 1-v^u_H &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).\end{aligned}$$

Notice that, as long as $s^d_H > 0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s^d_H = \gamma^d/\left(1+\gamma^d\right)$, the first best can be achieved with this production subsidy and two export taxes ($1-v^d_H = (\sigma-1)/\sigma$ and $1-v^u_H = (\theta-1)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s^d_H = (1+\gamma^u)/\gamma^u = (\theta-1)/\theta$, the first best can be achieved with this production subsidy and two import tariffs ($t^d_H = 1/(\sigma-1)$ and $t^u_H = 1/(\theta-1)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1+t^d_H}{1+t^u_H} = \left(1-s^d_H\right)\left(1+\gamma^d\right).$$

C.3.3 Consumption Subsidies

We finally consider the case of consumption subsidies s^d_H and s^u_H that apply to Home consumption of final goods and of intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19) and (20) as follows:

$$\begin{aligned}\frac{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)} &= \frac{\left(1-v^d_H\right)}{\left(1+t^d_H\right)}\frac{P^d_{HF}}{P^d_{FH}};\\ \frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} &= \frac{\left(1-v^u_H\right)}{\left(1+t^u_H\right)}\frac{P^u_{HF}}{P^u_{FH}};\\ \hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) &= \left(1-s^u_H\right)\frac{\left(1-v^u_H\right)}{\left(1-v^d_H\right)}\frac{P^u_{HF}}{P^d_{HF}}.\end{aligned}$$

These equations are completely analogous to those applying to the case of prouduction subsidies, with s^u_H replacing s^d_H, so the conclusions that arise from it are also analogous.

C.4 Second-Best Import Tariffs

In this Appendix we characterize the second-best import tariffs when the government only has access to import tariffs upstream and downstream.

A. Second-Best Import Tariffs with Scale Economies

As mentioned in the main text, the second-best optimal allocation will seek to solve the same problem laid out in section 4.1 expanded to include the additional constraint:

$$\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{P^d_{HF}}{P^u_{HF}} = \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}.$$

More specifically, the planner problem is now

$$\begin{aligned}
\max \quad & U\left(Q^d_{HH}, Q^d_{FH}\right) \\
s.t. \quad & \hat{A}^u_H L_H = Q^u_{HH} + Q^u_{HF} \\
& \hat{A}^d_H F^d\left(Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\
& P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} \\
& \hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) = \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H\left(L_H\right)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$\begin{aligned}
& U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u\left[\bar{A}^u_H\left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] + \mu_d\left[\bar{A}^d_H\left(F^d\left(Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
& + \mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right] \\
& + \mu_{SB}\left[\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) - \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{\left(Q^d_{HF}\right)^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}\right]
\end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, and Q^u_{HF} are as follows:

$$U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_d \tag{C.7}$$

$$U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_{TB}P^d_{FH} \tag{C.8}$$

$$\mu_d = \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF} - \mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \tag{C.9}$$

$$\begin{aligned}\mu_u =\ & \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\ & +\mu_{SB}\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\ & \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right]\end{aligned} \tag{C.10}$$

$$\begin{aligned}\mu_{TB}P^u_{FH} =\ & \mu_d\left(1+\gamma^d\right)\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)\\ & +\mu_{SB}\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)\\ & \times\left[\gamma^d\frac{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right]\end{aligned} \tag{C.11}$$

$$\mu_u = \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF} + \mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \tag{C.12}$$

In these derivations, note that we have used

$$\begin{aligned}\frac{\partial\left(\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\right)}{\partial Q^u_{HH}} =\ & \bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\ & \times\left[\gamma^d\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right]\end{aligned}$$

and

$$\begin{aligned}\frac{\partial\left(\bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\right)}{\partial Q^u_{FH}} =\ & \bar{A}^d_H\left(F^d\left(Q^u_{HH},Q^u_{FH}\right)\right)^{\gamma^d}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)\\ & \times\left[\gamma^d\frac{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}+\frac{F^d_{Q^u_{HH},Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}\right].\end{aligned}$$

From equations (C.7), (C.8), and (C.9), we obtain:

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)} = \frac{P^d_{FH}}{P^d_{HF}}\left(\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}}\right).$$

Because in a competitive equilibrium with import tariffs we have

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)} = \left(1+t^d_H\right)\frac{P^d_{FH}}{P^d_{HF}}, \tag{C.13}$$

we can establish that

$$1+t_H^d=\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{HF}^d}\frac{P_{HF}^u}{P_{HF}^d}. \tag{C.14}$$

Also, note from equations (C.7) and (C.8), as well as (C.13), that

$$1+t_H^d=\frac{\mu_{TB}P_{HF}^d}{\mu_d}. \tag{C.15}$$

In a competitive equilibrium with import tariffs, we also have that

$$\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\frac{P_{HF}^u}{\left(1+t_H^u\right)P_{FH}^u}. \tag{C.16}$$

Furthermore, the last constraint in the planner problem can be written as:

$$\bar{A}_H^d\left(F^d\left(Q_{HH}^u,Q_{FH}^u\right)\right)^{\gamma^d}F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)=\frac{P_{HF}^u}{P_{HF}^d}. \tag{C.17}$$

Now combine equations (C.11), (C.15), (C.16), and (C.17) to obtain

$$\frac{1+t_H^d}{1+t_H^u}=1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\left[\gamma^d\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F^d\left(Q_{HH}^u,Q_{FH}^u\right)}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}\right]. \tag{C.18}$$

We next work with equations (C.10) and plug in (C.11) and (C.17) to obtain

$$\mu_{TB}P_{FH}^u\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}=\mu_u+\mu_{SB}\frac{P_{HF}^u}{P_{HF}^d}\left[\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}\right]$$

And plugging μ_u from equation (C.12), we get

$$\frac{\mu_{TB}}{\mu_{SB}\frac{P_{HF}^u}{P_{HF}^d}}=\frac{\frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}}{P_{FH}^u\frac{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{\theta-1}{\theta}P_{HF}^u}$$

Invoking equation (C.16) we can simplify this last expression further to

$$\frac{\mu_{TB}}{\mu_{SB}}P_{HF}^d=\frac{\frac{1}{\theta}\frac{1}{Q_{HF}^u}+\frac{F_{Q_{HH}^u,Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}-\frac{F_{Q_{HH}^u,Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}{F_{Q_{HH}^u}^d\left(Q_{HH}^u,Q_{FH}^u\right)}}{\frac{1}{\left(1+t_H^u\right)}-\frac{\theta-1}{\theta}} \tag{C.19}$$

The three equations (C.14), (C.18), and (C.19) are sufficient to characterize the properties of second-best import tariffs. In particular, these equations can be reduced to

$$1+t_H^d = \frac{\sigma}{\sigma-1}+\left[\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right)\right]\frac{A}{C} \tag{C.20}$$

$$\frac{1+t_H^d}{1+t_H^u} = 1+\gamma^d+\left[\frac{1+t_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+t_H^d\right)\right]\frac{B}{C}, \tag{C.21}$$

where

$$
\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{Q_{HF}^{d}}\frac{P_{HF}^{u}}{P_{HF}^{d}} > 0; \\
B &= \gamma^{d}\frac{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}; \\
C &= \frac{1}{\theta}\frac{1}{Q_{HF}^{u}} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} - \frac{F_{Q_{HH}^{u},Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}.
\end{aligned}
$$

Using

$$
F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right) = \left(\left(Q_{HH}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^{u}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta}{\theta-1}},
$$

it is easy to verify that

$$
\begin{aligned}
B &= \gamma^{d}\frac{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} = \left(\gamma^{d}+\frac{1}{\theta}\right)\frac{1}{Q_{HH}^{u}}\frac{\left(Q_{HH}^{u}\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{HH}^{u}\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^{u}\right)^{\frac{\theta-1}{\theta}}} > 0 \\
C &= \frac{1}{\theta}\frac{1}{Q_{HF}^{u}} + \frac{F_{Q_{HH}^{u},Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{FH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} - \frac{F_{Q_{HH}^{u},Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)}{F_{Q_{HH}^{u}}^{d}\left(Q_{HH}^{u},Q_{FH}^{u}\right)} = \frac{1}{\theta}\left(\frac{1}{Q_{HH}^{u}}+\frac{1}{Q_{HF}^{u}}\right) > 0.
\end{aligned}
$$

Now note that, manipulating (C.20) and (C.21), we obtain

$$
\begin{aligned}
1+t_{H}^{d} &= \frac{\sigma}{\sigma-1} + \left[\frac{1+t_{H}^{d}}{1+t_{H}^{u}} - \frac{\theta-1}{\theta}\left(1+t_{H}^{d}\right)\right]\frac{A}{C} \\
\frac{1+t_{H}^{d}}{1+t_{H}^{u}} &= 1+\gamma^{d} + \left[\frac{1+t_{H}^{d}}{1+t_{H}^{u}} - \frac{\theta-1}{\theta}\left(1+t_{H}^{d}\right)\right]\frac{B}{C},
\end{aligned}
$$

Solving this system delivers

$$
1+t_{H}^{d} = \frac{\sigma}{\sigma-1}\frac{1-\frac{B}{C}+\frac{1+\gamma^{d}}{\frac{\sigma}{\sigma-1}}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}
$$

and

$$
\frac{1+t_{H}^{d}}{1+t_{H}^{u}} = \left(1+\gamma^{d}\right)\frac{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{\theta-1}{\theta}\frac{\frac{\sigma}{\sigma-1}}{1+\gamma^{d}}\frac{B}{C}}{1+\frac{\theta-1}{\theta}\frac{A}{C}-\frac{B}{C}}.
$$

Noting that $1+\gamma^{d} = \frac{\sigma}{\sigma-1}$ in our isomorphism, immediately implies

$$
1+t_{H}^{d} > \frac{\sigma}{\sigma-1}
$$

and

$$
\frac{1+t_{H}^{d}}{1+t_{H}^{u}} > 1+\gamma^{d} = \frac{\sigma}{\sigma-1}.
$$

This proves Proposition 6.

B. Second-Best Import Tariffs with No Scale Economies

Given the above derivations, it is straightforward to prove Proposition 7. Simply set $\gamma^d = 0$ in the system (C.20) and (C.21), and obtain

$$1 + t_H^d = \frac{\sigma}{\sigma - 1} \frac{1 - \frac{B}{C} + \frac{\sigma-1}{\sigma}\frac{A}{C}}{1 - \frac{B}{C} + \frac{\theta-1}{\theta}\frac{A}{C}}$$

and

$$\frac{1 + t_H^d}{1 + t_H^u} = \frac{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}\frac{B}{C}}{1 + \frac{\theta-1}{\theta}\frac{A}{C} - \frac{B}{C}}.$$

From the second equation, is is clear that $\frac{1+t_H^d}{1+t_H^u} > 1$ if and only if $\frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1} < 1$, or $\sigma > \theta$. Furthermore, when $\theta = \sigma$, we have

$$1 + t_H^d = 1 + t_H^u = \frac{\sigma}{\sigma - 1} = \frac{\theta}{\theta - 1}.$$

D Optimal Trade Policy for a Small Open Economy with Domestic Distortions: Derivations

D.1 First-Best Policies with an Upstream Production Subsidy

We begin by characterizing the solution to the program

$$\max \quad U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned}
s.t. \quad & L^u_H + L^d_H = L_H \\
& \hat{A}^u_H\left(L^u_H\right) L^u_H = Q^u_{HH} + Q^u_{HF} \\
& \hat{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right) F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = Q^d_{HH} + Q^d_{HF} \\
& P^d_{FH} Q^d_{FH} + P^u_{FH} Q^u_{FH} = Q^d_{HF} (Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}},
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H\left(L_H\right)^{\gamma^u},$$

respectively. We also note that

$$U\left(Q^d_{HH}, Q^d_{FH}\right) = \left(\left(Q^d_{HH}\right)^{\frac{\sigma-1}{\sigma}} + \left(Q^d_{FH}\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

and that

$$F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) = \left(L^d_i\right)^{\alpha}\left(\left(Q^u_{HH}\right)^{\frac{\theta-1}{\theta}} + \left(Q^u_{FH}\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},$$

but this will prove immaterial for the derivations below.

We first write the Lagrangian of this problem

$$\begin{aligned}
& U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_L\left[L_H - L^u_H - L^d_H\right] + \mu_u\left[\bar{A}^u_H\left(L_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] \\
& +\mu_d\left[\bar{A}^d_H\left(F^d\left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
& +\mu_{TB}\left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF}\left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF}\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF}\left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right].
\end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}, L^d_H, and L^u_H

are as follows:

$$
\begin{aligned}
U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_d && \text{(D.1)}\\
U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_{TB} P_{FH}^d && \text{(D.2)}\\
\mu_d &= \mu_{TB}\frac{\sigma-1}{\sigma} P_{HF}^d && \text{(D.3)}\\
\mu_u &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) && \text{(D.4)}\\
\mu_{TB} P_{FH}^u &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) && \text{(D.5)}\\
\mu_u &= \mu_{TB}\frac{\theta-1}{\theta} P_{HF}^u && \text{(D.6)}\\
\mu_L &= \mu_u\left(1+\gamma^u\right)\bar{A}_H^u\left(L_H\right)^{\gamma^u} && \text{(D.7)}\\
\mu_L &= \mu_d\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) && \text{(D.8)}
\end{aligned}
$$

Dividing equation (D.1) by equation (D.2), and plugging in (D.3), we obtain obtain:

$$\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} = \frac{\frac{\sigma-1}{\sigma} P_{HF}^d}{P_{FH}^d},$$

which corresponds to the first optimality condition (23) in the main text.

Next, we divide equation (D.4) by equation (D.5), and plugging in (D.6), delivers

$$\frac{F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)} = \frac{\frac{\theta-1}{\theta} P_{HF}^u}{P_{FH}^u},$$

which corresponds to the second optimality condition (24) in the main text.

Next, combining equation (D.4) with the ratio of equations (D.3) and (D.6) produces

$$\left(1+\gamma^d\right)\bar{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right)^{\gamma^d} F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \frac{\frac{\theta-1}{\theta}}{\frac{\sigma-1}{\sigma}}\frac{P_{HF}^u}{P_{HF}^d},$$

which corresponds to the third optimality condition (25) in the main text.

Finally, from equations (D.7) by equation (D.8), and plugging in (D.4), we obtain

$$F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \left(1+\gamma^u\right)\bar{A}_H^u\left(L_H\right)^{\gamma^u} F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right),$$

which corresponds to equation (31) in the main text, after noting that $\bar{A}_H^u\left(L_H\right)^{\gamma^u} = \hat{A}_H^u$.

D.2 First-Best Policies with Alternative Instruments

In this Appendix, we explore the structure of first-best policies when the set of available instruments includes instruments other than domestic upstream production subsidies and trade taxes.

D.2.1 Discriminatory Domestic Subsidies

Consider first the case in which the Home government has access to discriminatory domestic subsidies s_{HH}^d and s_{HH}^u that apply only to purchases of final goods and of intermediate inputs involving only Home residents.

The inclusion of these instruments alters the decentralized market equilibrium conditions (18), (19), (20), and (21) as follows:

$$\begin{aligned}
\frac{U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)}{U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right)} &= \left(1 - s_{HH}^d\right)\frac{\left(1 - v_H^d\right)}{\left(1 + t_H^d\right)}\frac{P_{HF}^d}{P_{FH}^d};\\
\frac{F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)} &= \left(1 - s_{HH}^u\right)\frac{\left(1 - v_H^u\right)}{\left(1 + t_H^u\right)}\frac{P_{HF}^u}{P_{FH}^u};\\
\hat{A}_H^d F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \left(1 - s_{HH}^u\right)\frac{\left(1 - v_H^u\right)}{\left(1 - v_H^d\right)}\frac{P_{HF}^u}{P_{HF}^d}\\
F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \frac{1}{1 - s_{HH}^u}\hat{A}^u\left(L_H^u\right)F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), (25), and (31), it is clear that the first best can be achieved by setting

$$\begin{aligned}
\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right) &= \left(1 + \gamma^d\right)\left(1 + \bar{T}\right);\\
1 + t_H^u &= 1 + \bar{T};\\
1 - v_H^d &= \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 + \bar{T}\right);\\
\left(1 - s_{HH}^u\right)\left(1 - v_H^u\right) &= \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right);\\
\frac{1}{1 - s_{HH}^u} &= \frac{\theta - 1}{\theta}.
\end{aligned}$$

These equations illustrate that a downstream discriminatory subsidy is a perfect substitute for the downstream import tariff (only the product $\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right)$ matters), while an upstream discriminatory subsidy is *no longer* a perfect substitute for the upstream export tax, as it was in the case in which $\alpha = 0$. More specifically, because the first-best calls for $s_{HH}^u = 1/\theta$, we must necessarily have

$$1 - v_H^u = 1 + t_H^u = 1 + \bar{T}.$$

A straightforward implication of this result is that, whenever $1 + \gamma^d = \sigma/\left(\sigma - 1\right)$, as imposed by our isomorphism, the first best can be attained by setting $\left(1 + t_H^d\right)\left(1 - s_{HH}^d\right) = \sigma/\left(\sigma - 1\right)$, $1 - s_{HH}^u = \left(\theta - 1\right)/\theta$, and $v_H^u = v_H^d = t_H^u = 0$. Thus, the first best can be achieved with only two discriminatory subsidies, or with a combination of an upstream subsidy and a downstream import tariff. When only domestic instruments are used, we necessarily have $s_{HH}^d = 1/\sigma$ and $s_{HH}^u = 1/\theta$. Whether or not the resulting first-best policies entail tariff escalation depends on the level of the downstream domestic subsidy, since $\left(1 + t_H^d\right)/\left(1 + t_H^u\right) = \left(1 + \gamma^d\right)/\left(1 - s_{HH}^d\right)$.

D.2.2 Production Subsidies

We next consider the case in which the Home government uses a nondiscriminatory downstream production subsidy s_H^d in addition to a nondiscriminatory upstream production subsidy, as in our baseline implementation. The inclusion of this instrument does not affect the market equilibrium condition (21), while it alters the decentralized market equilibrium conditions (18), (19), (20) in a manner analogous to the case $\alpha = 0$, that is:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \frac{\left(1 - v^d_H\right)}{\left(1 + t^d_H\right)} \frac{P^d_{HF}}{P^d_{FH}}; \\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} &= \frac{\left(1 - v^u_H\right)}{\left(1 + t^u_H\right)} \frac{P^u_{HF}}{P^u_{FH}}; \\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) &= \left(1 - s^d_H\right) \frac{\left(1 - v^u_H\right)}{\left(1 - v^d_H\right)} \frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

Comparing these equations to those characterizing the optimal allocations, that is equations (23), (24), and (25), it is clear that the first best can be achieved by setting

$$\begin{aligned}
1 + t^d_H &= \left(1 - s^d_H\right)\left(1 + \gamma^d\right)\left(1 + \bar{T}\right); \\
1 + t^u_H &= 1 + \bar{T}; \\
1 - v^d_H &= \frac{\sigma - 1}{\sigma}\left(1 + \gamma^d\right)\left(1 - s^d_H\right)\left(1 + \bar{T}\right); \\
1 - v^u_H &= \frac{\theta - 1}{\theta}\left(1 + \bar{T}\right).
\end{aligned}$$

Notice that, as long as $s^d_H > 0$, the set of first-best policies will entail this subsidy and at least two additional trade instruments. For instance, when setting $s^d_H = \gamma^d / \left(1 + \gamma^d\right)$, the first best can be achieved with this production subsidy, the upstream production subsidy at a level $s^u_H = \gamma^u / \left(1 + \gamma^u\right)$ and two export taxes ($1 - v^d_H = \left(\sigma - 1\right)/\sigma$ and $1 - v^u_H = \left(\theta - 1\right)/\theta$), while setting all import tariffs to zero. Alternatively, when setting $s^d_H = \left(1 + \gamma^u\right)/\gamma^u = \left(\theta - 1\right)/\theta$, the first best can be achieved with this production subsidy and two import tariffs ($t^d_H = 1/\left(\sigma - 1\right)$ and $t^u_H = 1/\left(\theta - 1\right)$).

Regardless of the actual implementation, the tariff escalation wedge is given by:

$$\frac{1 + t^d_H}{1 + t^u_H} = \left(1 - s^d_H\right)\left(1 + \gamma^d\right).$$

When only upstream production subsidies and trade taxes are used, the first-best policies continues to feature a tariff escalation wedge equal to $1 + \gamma^d = \sigma / \left(\sigma - 1\right)$.

D.2.3 Consumption Subsidies

We finally consider the case of consumption subsidies s^d_H and s^u_H that apply to Home consumption of final goods and of intermediate inputs, regardless of where those goods are purchased (domestically or imported). The inclusion of these instruments does not affect the market equilibrium condition (21), as long as s^u_H is set at $s^u_H = 1/\theta$. Furthermore, the decentralized market equilibrium conditions (18), (19) and (20) become:

$$\begin{aligned}
\frac{U_{Q^d_{HH}}\left(Q^d_{HH}, Q^d_{FH}\right)}{U_{Q^d_{FH}}\left(Q^d_{HH}, Q^d_{FH}\right)} &= \frac{\left(1 - v^d_H\right)}{\left(1 + t^d_H\right)} \frac{P^d_{HF}}{P^d_{FH}}; \\
\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH}, Q^u_{FH}\right)} &= \frac{\left(1 - v^u_H\right)}{\left(1 + t^u_H\right)} \frac{P^u_{HF}}{P^u_{FH}}; \\
\hat{A}^d_H F^d_{Q^u_{HH}}\left(Q^u_{HH}, Q^u_{FH}\right) &= \left(1 - s^u_H\right) \frac{\left(1 - v^u_H\right)}{\left(1 - v^d_H\right)} \frac{P^u_{HF}}{P^d_{HF}}.
\end{aligned}$$

These equations are completely analogous to those applying to the case of prouduction subsidies, with s_H^u replacing s_H^d, but note that we now necessarily have $s_H^u = 1/\theta$. As a result, replacing $1+\gamma^d = \sigma/(\sigma-1)$, we obtain

$$\begin{aligned}
1+t_H^d &= \frac{\theta-1}{\theta}\left(1+\gamma^d\right)\left(1+\bar{T}\right); \\
1+t_H^u &= 1+\bar{T}; \\
1-v_H^d &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right); \\
1-v_H^u &= \frac{\theta-1}{\theta}\left(1+\bar{T}\right).
\end{aligned}$$

In such a case, it is clear that the relative size of $1+t_H^d$ and $1+t_H^u$ depends on $\frac{\theta-1}{\theta}\left(1+\gamma^d\right) = \frac{\theta-1}{\theta}\frac{\sigma}{\sigma-1}$, and thus on the relative size of σ and θ.

D.3 Second-Best Policies

In this Appendix we characterize the second-best import tariffs for the general case $\alpha \geq 0$ when the government only has access to import tariffs upstream and downstream. We first derive the key equations characterizing tariff levels and tariff escalation, and we later explore special cases.

A. Second-Best Import Tariffs with Scale Economies: Key Equations

The second-best optimal allocation will seek to solve the same problem laid out in section 4.1 expanded to include the additional constraints:

$$\hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{P_{HF}^d}{P_{HF}^u} = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}}$$

and

$$F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right).$$

Second-Best Planner Problem and First-Order Conditions More specifically, the planner sets $\left\{L_H^u, L_H^d, Q_{HH}^d, Q_{FH}^d, Q_{HF}^d, Q_{HH}^u, Q_{FH}^u, Q_{HF}^u\right\}$ to

$$\max \quad U\left(Q_{HH}^d, Q_{FH}^d\right) = \left(\left(Q_{HH}^d\right)^{\frac{\sigma-1}{\sigma}} + \left(Q_{FH}^d\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}$$

$$\begin{aligned}
s.t. \quad & L_H^u + L_H^d = L_H \\
& \hat{A}_H^u\left(L_H^u\right) L_H^u = Q_{HH}^u + Q_{HF}^u \\
& \hat{A}_H^d\left(F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\right) F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = Q_{HH}^d + Q_{HF}^d \\
& P_{FH}^d Q_{FH}^d + P_{FH}^u Q_{FH}^u = Q_{HF}^d \left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}} + Q_{HF}^u \left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}} \\
& \hat{A}_H^d F_{Q_{HH}^u}^d\left(Q_{HH}^u, Q_{FH}^u\right) = \frac{\left(Q_{HF}^u\right)^{-\frac{1}{\theta}} P_{FF}^u \left(Q_{FF}^u\right)^{\frac{1}{\theta}}}{\left(Q_{HF}^d\right)^{-\frac{1}{\sigma}} P_{FF}^d \left(Q_{FF}^d\right)^{\frac{1}{\sigma}}} \\
& F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \hat{A}^u\left(L_H^u\right) F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)
\end{aligned}$$

where $\hat{A}^u_H$ and $\hat{A}^d_H$ are given by

$$\hat{A}^d_H = \bar{A}^d_H \left(F^d \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{\gamma^d}$$

and

$$\hat{A}^u_H = \bar{A}^u_H \left(L^u_H\right)^{\gamma^u},$$

respectively.

We first write the Lagrangian of this problem

$$\begin{aligned}
&U\left(Q^d_{HH}, Q^d_{FH}\right) + \mu_u \left[\bar{A}^u_H \left(L^u_H\right)^{1+\gamma^u} - Q^u_{HH} - Q^u_{HF}\right] + \mu_d \left[\bar{A}^d_H \left(F^d \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)\right)^{1+\gamma^d} - Q^d_{HH} - Q^d_{HF}\right] \\
&+\mu_{TB} \left[Q^d_{HF}(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}} + Q^u_{HF} \left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF} \left(Q^u_{FF}\right)^{\frac{1}{\theta}} - P^d_{FH} Q^d_{FH} - P^u_{FH} Q^u_{FH}\right] \\
&+\mu_{SB} \left[\hat{A}^d_H F^d_{Q^u_{HH}} \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right) - \frac{\left(Q^u_{HF}\right)^{-\frac{1}{\theta}} P^u_{FF} \left(Q^u_{FF}\right)^{\frac{1}{\theta}}}{(Q^d_{HF})^{-\frac{1}{\sigma}} P^d_{FF} \left(Q^d_{FF}\right)^{\frac{1}{\sigma}}}\right] \\
&+\mu_{LC} \left[\frac{F^d_{L^d_H} \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)}{F^d_{Q^u_{HH}} \left(L^d_H, Q^u_{HH}, Q^u_{FH}\right)} - \hat{A}^u \left(L^u_H\right)\right]
\end{aligned}$$

The first order conditions associated with the choices of Q^d_{HH}, Q^d_{FH}, Q^d_{HF}, Q^u_{HH}, Q^u_{FH}, Q^u_{HF}, L^d_H, and L^u_H are as follows:

$$
\begin{aligned}
U_{Q_{HH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_d && \text{(D.9)}\\
U_{Q_{FH}^d}\left(Q_{HH}^d, Q_{FH}^d\right) &= \mu_{TB} P_{FH}^d && \text{(D.10)}\\
\mu_d &= \mu_{TB} \frac{\sigma-1}{\sigma} P_{HF}^d - \mu_{SB} \frac{1}{\sigma} \frac{1}{Q_{HF}^d} \frac{P_{HF}^u}{P_{HF}^d} && \text{(D.11)}\\
\mu_u &= \mu_d \left(1+\gamma^d\right) \bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{HH}^u}^d(\cdot) \\
&\quad +\mu_{SB} \bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{HH}^u}^d(\cdot) \\
&\quad \times \left[\gamma^d \frac{F_{Q_{HH}^u}^d(\cdot)}{F^d(\cdot)} + \frac{F_{Q_{HH}^u, Q_{HH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)}\right] \\
&\quad +\mu_{LC} \left[\frac{F_{L_H^d, Q_{HH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)} - \frac{F_{L_H^d}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)} \frac{F_{Q_{HH}^u, Q_{HH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)}\right] && \text{(D.12)}\\
\mu_{TB} P_{FH}^u &= \mu_d \left(1+\gamma^d\right) \bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{FH}^u}^d(\cdot) \\
&\quad +\mu_{SB} \bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{HH}^u}^d(\cdot) \times \left[\gamma^d \frac{F_{Q_{FH}^u}^d(\cdot)}{F^d(\cdot)} + \frac{F_{Q_{HH}^u, Q_{FH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)}\right] \\
&\quad +\mu_{LC} \left[\frac{F_{L_H^d, Q_{FH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)} - \frac{F_{L_H^d}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)} \frac{F_{Q_{HH}^u, Q_{FH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)}\right] && \text{(D.13)}\\
\mu_u &= \mu_{TB} \frac{\theta-1}{\theta} P_{HF}^u + \mu_{SB} \frac{1}{\theta} \frac{1}{Q_{HF}^u} \frac{P_{HF}^u}{P_{HF}^d} && \text{(D.14)}\\
\mu_L &= \mu_u \left(1+\gamma^u\right) \bar{A}_H^u \left(L_H^u\right)^{\gamma^u} - \mu_{LC} \gamma^u \bar{A}_H^u \left(L_H^u\right)^{\gamma^u - 1} && \text{(D.15)}\\
\mu_L &= \mu_d \left(1+\gamma^d\right) \bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{L_H^d}^d(\cdot) \\
&\quad +\mu_{SB} \hat{A}_H^d \left[\gamma^d \frac{F_{L_H^d}^d(\cdot)}{F^d(\cdot)} F_{Q_{HH}^u}^d(\cdot) + F_{Q_{HH}^u, L_H^d}^d(\cdot)\right] \\
&\quad +\mu_{LC} \left[\frac{F_{L_H^d, L_H^d}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)} - \frac{F_{L_H^d}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)} \frac{F_{Q_{HH}^u, L_H^d}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)}\right] && \text{(D.16)}
\end{aligned}
$$

In these derivations, note that we have used

$$
\begin{aligned}
\frac{\partial\left(\bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{HH}^u}^d(\cdot)\right)}{\partial Q_{HH}^u} &= \bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{HH}^u}^d(\cdot) \\
&\quad \times \left[\gamma^d \frac{F_{Q_{HH}^u}^d(\cdot)}{F^d(\cdot)} + \frac{F_{Q_{HH}^u, Q_{HH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)}\right]
\end{aligned}
$$

and

$$
\begin{aligned}
\frac{\partial\left(\bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{HH}^u}^d(\cdot)\right)}{\partial Q_{FH}^u} &= \bar{A}_H^d \left(F^d(\cdot)\right)^{\gamma^d} F_{Q_{HH}^u}^d(\cdot) \\
&\quad \times \left[\gamma^d \frac{F_{Q_{FH}^u}^d(\cdot)}{F(\cdot)} + \frac{F_{Q_{HH}^u, Q_{FH}^u}^d(\cdot)}{F_{Q_{HH}^u}^d(\cdot)}\right].
\end{aligned}
$$

Useful Expressions with Our Functional Forms Remember that technology is given by

$$F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) = \left(L_H^d\right)^\alpha \left(\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^u\right)^{\frac{\theta-1}{\theta}}\right)^{\frac{\theta(1-\alpha)}{\theta-1}},$$

and define

$$\pi_{HH}^u \equiv \frac{\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}}}{\left(Q_{HH}^u\right)^{\frac{\theta-1}{\theta}} + \left(Q_{FH}^u\right)^{\frac{\theta-1}{\theta}}}$$

and

$$X_H^d \equiv \bar{A}_H^d \left(F^d\left(Q_{HH}^u, Q_{FH}^u\right)\right)^{1+\gamma^d}.$$

We next note that:

$$\begin{aligned}
F_{L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha \frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d} \\
F_{Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{Q_{HH}^u}\pi_{HH}^u \\
F_{Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{Q_{FH}^u}\left(1-\pi_{HH}^u\right) \\
F_{L_H^d, L_H^d}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha(\alpha-1)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d L_H^d} \\
F_{Q_{HH}^u, Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{Q_{HH}^u}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[-\alpha\pi_{HH}^u - \frac{1}{\theta}\left(1-\pi_{HH}^u\right)\right] \\
F_{Q_{HH}^u, Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= (1-\alpha)F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)\frac{1-\pi_{HH}^u}{Q_{FH}^u}\frac{\pi_{HH}^u}{Q_{HH}^u}\left(\frac{1}{\theta}-\alpha\right) \\
F_{L_H^d, Q_{HH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha(1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d}\frac{\pi_{HH}^u}{Q_{HH}^u} \\
F_{L_H^d, Q_{FH}^u}^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right) &= \alpha(1-\alpha)\frac{F^d\left(L_H^d, Q_{HH}^u, Q_{FH}^u\right)}{L_H^d}\frac{1-\pi_{HH}^u}{Q_{FH}^u}
\end{aligned}$$

First-Order Conditions with Functional Forms We can now plug some of the above expressions into our first-order conditions

$$U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_d \tag{D.17}$$
$$U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right) = \mu_{TB}P^d_{FH} \tag{D.18}$$
$$\mu_d = \mu_{TB}\frac{\sigma-1}{\sigma}P^d_{HF}-\mu_{SB}\frac{1}{\sigma}\frac{1}{Q^d_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \tag{D.19}$$
$$\begin{aligned}\mu_u = {} & \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}+\mu_{SB}(1-\alpha)\frac{X^d_H}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\gamma^d(1-\alpha)\pi^u_{HH}-\alpha\pi^u_{HH}-\frac{1}{\theta}\pi^u_{FH}\right]\\ & +\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{(1-\alpha)}\left[1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right]\end{aligned} \tag{D.20}$$
$$\begin{aligned}\mu_{TB}P^u_{FH} = {} & \mu_d\left(1+\gamma^d\right)(1-\alpha)X^d_H\frac{1-\pi^u_{HH}}{Q^u_{FH}}+\mu_{SB}(1-\alpha)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]\\ & +\mu_{LC}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{1-\pi^u_{HH}}{Q^u_{FH}}\frac{Q^u_{HH}}{\pi^u_{HH}}\end{aligned} \tag{D.21}$$
$$\mu_u = \mu_{TB}\frac{\theta-1}{\theta}P^u_{HF}+\mu_{SB}\frac{1}{\theta}\frac{1}{Q^u_{HF}}\frac{P^u_{HF}}{P^d_{HF}} \tag{D.22}$$
$$\mu_L = \mu_u(1+\gamma^u)\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}-\mu_{LC}\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1} \tag{D.23}$$
$$\mu_L = \mu_d\left(1+\gamma^d\right)\frac{\alpha X^d}{L^d_H}+\mu_{SB}\left(1+\gamma^d\right)\frac{\alpha(1-\alpha)X^d_H}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}}-\mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H L^d_H}\frac{Q^u_{HH}}{\pi^u_{HH}} \tag{D.24}$$

Manipulating the First-Order Conditions From equations (D.17), (D.18), and (D.19), we obtain:

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)} = \frac{P^d_{FH}}{P^d_{HF}}\left(\frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{FH}}\frac{P^u_{HF}}{P^d_{HF}}\right).$$

Because in a competitive equilibrium with tariffs we have

$$\frac{U_{Q^d_{FH}}\left(Q^d_{HH},Q^d_{FH}\right)}{U_{Q^d_{HH}}\left(Q^d_{HH},Q^d_{FH}\right)} = \left(1+\tau^d_H\right)\frac{P^d_{FH}}{P^d_{HF}}, \tag{D.25}$$

we can establish that

$$1+\tau^d_H = \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q^d_{FH}}\frac{P^u_{HF}}{P^d_{HF}}. \tag{D.26}$$

Also, note from equations (D.17), (D.18), and (D.25) that we have

$$1+\tau^d_H = \frac{\mu_{TB}P^d_{HF}}{\mu_d}, \tag{D.27}$$

and in a competitive equilibrium with import tariffs (but no export taxes)

$$\frac{F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right)}{F^d_{Q^u_{FH}}\left(Q^u_{HH},Q^u_{FH}\right)} = \frac{P^u_{HF}}{\left(1+t^u_H\right)P^u_{FH}}. \tag{D.28}$$

Furthermore, the penultimate constraint in the initial optimization can be written as:

$$\frac{X^d_H}{F^d\left(Q^u_{HH},Q^u_{FH}\right)}F^d_{Q^u_{HH}}\left(Q^u_{HH},Q^u_{FH}\right) = \frac{P^u_{HF}}{P^d_{HF}}, \tag{D.29}$$

and the last one as

$$\frac{\alpha}{1-\alpha}\frac{Q^u_{HH}}{\pi^u_{HH}L^d_H}=\frac{X^u_H}{L^u_H}. \tag{D.30}$$

Now combine equations (D.21), (D.27), (D.28), (D.29), and (D.30)

$$\frac{1+t^d_H}{1+t^u_H}=1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\gamma^d\left(1-\alpha\right)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\mu_{LC}}{\mu_c}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{P^d_{HF}}{P^u_{HF}}. \tag{D.31}$$

We next plug equation (D.21) into equation (D.20) to obtain

$$\mu_{TB}P^u_{FH}\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{Q^u_{FH}}{1-\pi^u_{HH}}=\mu_u+\mu_{SB}\left(1-\alpha\right)\frac{X^d_H}{Q^u_{HH}}\frac{\pi^u_{HH}}{Q^u_{HH}}\frac{1}{\theta}-\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{1-\alpha}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}$$

Next, plugging μ_u from equation (D.22), and invoking equation (D.28) and (D.29), we obtain

$$\mu_{TB}P^u_{HF}\left[\frac{1}{1+t^u_H}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}\left(1-\alpha\right)X^d_H\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\mu_{LC}\frac{1}{L^d_H}\frac{\alpha}{(1-\alpha)}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}$$

Next, invoking equation (D.28) and (D.29), we can simplify this to

$$\mu_{TB}P^u_{HF}\left[\frac{1}{1+t^u_H}-\frac{\theta-1}{\theta}\right]=\mu_{SB}\frac{1}{\theta}X^d_H\left(1-\alpha\right)\frac{\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\mu_{LC}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}.$$

And, plugging in (D.27). this delivers

$$\frac{1+\tau^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+\tau^d_H\right)=\frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\right]-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{1}{L^d_H}\frac{1}{\theta}\frac{1}{\pi^u_{HH}}\frac{P^d_{HF}}{P^u_{HF}}. \tag{D.32}$$

We finally seek to solve for μ_{LC} as a function of μ_{SB}. We begin with equation (D.15) and (D.24)

$$\frac{\mu_u}{\mu_d}\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}=\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\frac{\alpha X^d}{L^d_H}+\frac{\mu_{SB}}{\mu_d}\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\frac{\alpha\left(1-\alpha\right)X^d_H}{L^d_H}\frac{\pi^u_{HH}}{Q^u_{HH}}+\frac{\mu_{LC}}{\mu_d}\frac{1}{1+\gamma^u}\left[\gamma^u\bar{A}^u_H\left(L^u_H\right)^{\gamma^u-1}-\frac{\alpha}{1-\alpha}\frac{1}{L^d_HL^d_H}\frac{Q^u_{HH}}{\pi^u_{HH}}\right].$$

Next, plug equation (D.12) and using (D.30), we obtain

$$\begin{aligned}&\frac{\left(1+\gamma^d\right)\gamma^u}{\left(1+\gamma^u\right)}X^d_H+\frac{\mu_{SB}}{\mu_d}\left(1-\alpha\right)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi^u_{HH}\right)}{\pi^u_{HH}}\right)\right]\\ =\;&\frac{\mu_{LC}}{\mu_d}\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^uL^d_H}{L^u_H}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right],\end{aligned}$$

which we can express as

$$\frac{\mu_{LC}}{\mu_d}=\frac{\frac{\left(1+\gamma^d\right)\gamma^u}{\left(1+\gamma^u\right)}X^d_H+\frac{\mu_{SB}}{\mu_d}\left(1-\alpha\right)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{\left(1+\gamma^d\right)}{\left(1+\gamma^u\right)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{\left(1-\pi^u_{HH}\right)}{\pi^u_{HH}}\right)\right]}{\bar{A}^u_H\left(L^u_H\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^uL^d_H}{L^u_H}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}. \tag{D.33}$$

Recap of Key Equations

$$
\begin{aligned}
1+\tau_H^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}\frac{1}{\sigma-1}\frac{1}{Q_{FH}^d}\frac{P_{HF}^u}{P_{HF}^d} \\
\frac{1+t_H^d}{1+t_H^u} &= 1+\gamma^d+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\mu_{LC}}{\mu_c}\frac{\alpha}{1-\alpha}\frac{\theta-1}{\theta}\frac{1}{L_H^d}\frac{P_{HF}^d}{P_{HF}^u} \\
\frac{1+\tau_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_H^d\right) &= \frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\right]-\frac{\mu_{LC}}{\mu_d}\frac{\alpha}{1-\alpha}\frac{1}{L_H^d}\frac{1}{\theta}\frac{1}{\pi_{HH}^u}\frac{P_{HF}^d}{P_{HF}^u} \\
\frac{\mu_{LC}}{\mu_d} &= \frac{\frac{(1+\gamma^d)\gamma^u}{(1+\gamma^u)}X_H^d+\frac{\mu_{SB}}{\mu_d}(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{(1+\gamma^d)}{(1+\gamma^u)}\gamma^u-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}\frac{1}{1+\gamma^u}\left(\frac{\gamma^u L_H^d}{L_H^u}-1\right)-\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]}
\end{aligned}
$$

We have not been successful in proving any general results, so let us study some special cases.

B. Second-Best Import Tariffs with No Scale Economies

Given the above derivations, it is straightforward to prove that Proposition 7 applies even when $\alpha > 0$. Simply set $\gamma^d = \gamma^u = 0$ in equations (D.31), (D.32)and (D.33). First note, equation (D.33) becomes

$$\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X_H^d\pi_{HH}^u}{Q_{HH}^u}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}\right)\right]}{\bar{A}_H^u\left(L_H^u\right)^{\gamma^u}\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}\right)\right]},$$

and plugging (D.30),

$$\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}(1-\alpha)^2X_H^dL_H^d\left(\frac{\pi_{HH}^u}{Q_{HH}^u}\right)^2\frac{1+\frac{1}{\theta}\frac{(1-\pi_{HH}^u)}{\pi_{HH}^u}}{1+\frac{\alpha}{\theta}\frac{1-\pi_{HH}^u}{\pi_{HH}^u}}.$$

Plugging this expression for $\frac{\mu_{LC}}{\mu_d}$ into (D.31), delivers

$$\frac{1+t_H^d}{1+t_H^u}=1+\frac{\mu_{SB}}{\mu_d}\frac{\pi_{HH}^u}{Q_{HH}^u}(1-\alpha)\left[\frac{\alpha+\pi_{HH}^u(1-\alpha)}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right]$$

And finally, plugging $\frac{\mu_{LC}}{\mu_d}$ into equation (D.32) delivers

$$\frac{1+\tau_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_H^d\right)=\frac{\mu_{SB}}{\mu_d}\frac{1}{\theta}\left[\frac{1}{Q_{HF}^u}+\frac{1}{Q_{HH}^u}\frac{(1-\alpha)\theta\pi_{HH}^u}{\alpha+\pi_{HH}^u(\theta-\alpha)}\right].$$

In sum, we can write the system as

$$
\begin{aligned}
1+\tau_{FH}^d &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}A \\
\frac{1+\tau_{FH}^d}{1+t_H^u} &= 1+\frac{\mu_{SB}}{\mu_d}B \\
\frac{1+\tau_H^d}{1+t_H^u}-\frac{\theta-1}{\theta}\left(1+\tau_H^d\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}
$$

where

$$
\begin{aligned}
A &= \frac{1}{\sigma-1}\frac{1}{C_{HF}}\frac{P^u_{HF}}{P^d_{HF}} > 0 \\
B &= \frac{\pi^u_{HH}}{Q^u_{HH}}(1-\alpha)\left[\frac{\alpha+\pi^u_{HH}(1-\alpha)}{\alpha+\pi^u_{HH}(\theta-\alpha)}\right] > 0 \\
C &= \frac{1}{\theta}\left[\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\frac{(1-\alpha)\,\theta\pi^u_{HH}}{\alpha+\pi^u_{HH}(\theta-\alpha)}\right] > 0
\end{aligned}
$$

So we have

$$
\begin{aligned}
1+\tau^d_{FH} &= \frac{\sigma}{\sigma-1}+\left[\frac{1+\tau^d_{FH}}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+\tau^d_{FH}\right)\right]\frac{A}{C} \\
\frac{1+\tau^d_{FH}}{1+t^u_H} &= 1+\left(\frac{1+\tau^d_{FH}}{1+t^u_H}-\left(1+\tau^d_{FH}\right)\frac{\theta-1}{\theta}\right)\frac{B}{C}
\end{aligned}
$$

When solving this system, we obtain

$$
\frac{1+\tau^d_{FH}}{1+t^u_H}=\frac{1-\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta}\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}}{1-\frac{B}{C}+\frac{\theta-1}{\theta}\frac{A}{C}},
$$

which is higher or lower than 1 depending on the relative size of σ and θ. More specifically, when $\sigma>\theta$, $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta}<1$, and we have tariff escalation. But when $\sigma<\theta$, then $\frac{\sigma}{\sigma-1}\frac{\theta-1}{\theta}>1$, and we have tariff de-escalation.

C. Second-Best Import Tariffs with No Scale Economies Upstream ($\gamma^u=0$)

We next study the case in which $\gamma^d>0$ but $\gamma^u=0$. In that case, equation (D.33) reduces to

$$
\frac{\mu_{LC}}{\mu_d}=\frac{\mu_{SB}}{\mu_d}\frac{(1-\alpha)\frac{X^d_H\pi^u_{HH}}{Q^u_{HH}}\left[\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)\right]}{\bar{A}^u_H\left[\frac{1}{\alpha}+\frac{1}{1-\alpha}\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]},
$$

and we can write

$$
\begin{aligned}
1+\tau^d_H &= \frac{\sigma}{\sigma-1}+\frac{\mu_{SB}}{\mu_d}A \\
\frac{1+t^d_H}{1+t^u_H} &= 1+\gamma^d+\frac{\mu_{SB}}{\mu_d}B \\
\frac{1+\tau^d_H}{1+t^u_H}-\frac{\theta-1}{\theta}\left(1+\tau^d_H\right) &= \frac{\mu_{SB}}{\mu_d}C
\end{aligned}
$$

with

$$\begin{aligned}
B &= \frac{\pi^u_{HH}}{Q^u_{HH}}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{X^d_H\left[\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)\right]}{\bar{A}^u_H\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}\alpha\frac{\theta-1}{\theta}\frac{1}{L^d_H}\frac{P^d_{HF}}{P^u_{HF}}\right] \\
&= \frac{\pi^u_{HH}}{Q^u_{HH}}\left[\left[\gamma^d(1-\alpha)+\left(\frac{1}{\theta}-\alpha\right)\right]+\frac{\left[\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)\right]}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}\frac{\theta-1}{\theta}\right] \\
&= \frac{\pi^u_{HH}}{Q^u_{HH}}(1-\alpha)\frac{\pi^u_{HH}(\theta-1)+\alpha\sigma+\pi^u_{HH}\sigma(1-\alpha)}{(\sigma-1)\left(\alpha(1-\pi^u_{HH})+\pi^u_{HH}\theta\right)}>0.
\end{aligned}$$

and

$$C=\frac{1}{\theta}\frac{1}{Q^u_{HF}}+\frac{1}{Q^u_{HH}}\frac{1}{\theta}\left[1-\frac{\left(1+\frac{1}{\theta}\frac{(1-\pi^u_{HH})}{\pi^u_{HH}}\right)}{\left[\frac{1-\alpha}{\alpha}+\left(1+\frac{1}{\theta}\frac{1-\pi^u_{HH}}{\pi^u_{HH}}\right)\right]}\right]>0.$$

Given $B>0$ and $C>0$, it is then straightforward to use the same steps as in the proof of the $\alpha=0$ case in Online Appendix C.4 to show that

$$1+\tau^d_H>\frac{\sigma}{\sigma-1}$$

and

$$\frac{1+t^d_H}{1+t^u_H}>1+\gamma^d.$$

E Data Appendix

E.1 Data Construction for Figure 2

US Tariff Data.

- We use US import tariff data at the 8-digit level from the US Harmonized Tariff Schedule (HTS) available at https://dataweb.usitc.gov/tariff/annual. We use the most-favored-nation (MFN) ad valorem tariff rate whenever possible. In approximately 25% of the cases, the MFN ad valorem rate is not available and instead a "specific" tariff rate is applied such as "68 cents/head", "1 cents/kg", "0.9 cents each" etc. In these cases we perform an imputation by calculating an ad valorem equivalent tariff rate using unit values obtained from the US Census Bureau.

- In a next step we use the imputed ad valorem tariff rate to calculate applied MFN ad valorem tariff rates for all goods, taking trade agreements between the US and other countries into account. This is, we calculate the applied MFN ad valorem tariff rate as an import weighted average of the MFN ad-valorem rate and the tariff rate that is paid by countries that are members of a trade agreement.[5] US import data for the year 2015 come from the US Census Bureau.

- Data on tariffs imposed in February and March 2018 on almost all countries (washers; solar panels; iron and steel; aluminum) come from Fajgelbaum et al. (2020) and all subsequent tariffs imposed on imports from China throughout 2018 and 2019 from Chad Bown (available here).

ROW Tariff Data.

- We use tariff data for 115 countries plus the European Union at the 6-digit HS code level from the WTO Tariff Download Facility available at http://tariffdata.wto.org/default.aspx. We use the most-favored-nation (MFN) ad valorem tariff rate which constitutes the simple average duty of all products within a 6-digit HS code classification.

- We use data on retaliatory tariffs imposed by China throughout 2018 and 2019 from Chad Bown (available here). Data on retaliatory tariffs imposed by the European Union, Canada, Mexico, India and Turkey stem from Li (2018). Using data on these tariff waves we adjust the MFN applied tariff rates taking 2015 US export value weighted averages with US export data coming from the US Census Bureau.

Intermediate and Final Goods Classification

- We classify goods into intermediate and final goods using the UN Broad Economic Categories (BEC). The cross-walk between HTS10 codes and end-use categories is available here). We classify goods as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods (including capital goods) start with BEC code 41, 521, 112, 321, 522, 61, 62, 63. All other goods have no classification.

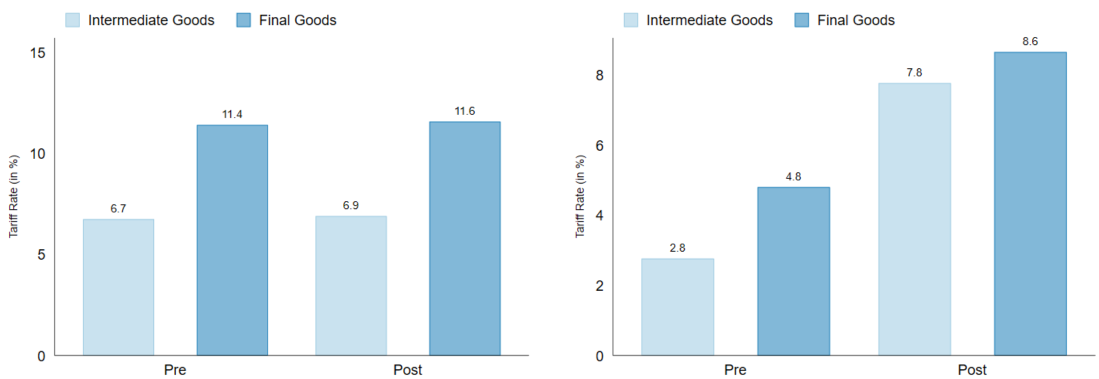
[5]We currently account for the following trade agreements: Generalized System of Preferences (GSP, 41 countries), The Agreement on Trade in Civil Aircraft (32 countries), NAFTA (3 countries), Caribbean Basin Initiative (CBI, 17 countries), African Growth and Opportunity Act (AGOA, 40 countries), Caribbean Basin Trade Partnership Act (CBTPA, 8 countries), Dominican Republic-Central America FTA (6 countries) and the Agreement on Trade in Pharmaceutical Products (7 countries).

Tariff Escalation Unweighted

- As alternative to Figure 2 which shows trade-weighted tariff rates, Figure E.1 displays an unweighted version of the tariff increase on intermediate and final goods by the ROW on imports from the US throughout the trade war and vice versa.

Figure E.1: Comparison ROW and US (Unweighted)

(a) ROW average tariffs on US **(b)** US average tariffs on ROW



Notes: Pre: Tariffs in January 2018, Post: Tariffs in December 2019. Tariff data from WTO and USITC. Goods are classified as intermediate goods when their BEC code starts with 111, 121, 21, 22, 31, 322, 42 and 53. Final goods start with BEC code 41, 521, 112, 321, 522, 61, 62, 63 (including capital goods). All other goods have no classification.

E.2 Elasticity Estimation

We now explain the estimation of elasticities of substitution in the upstream and downstream sectors using three different approaches: trade elasticity approach, sectoral markup approach and scale elasticity approach. We present results for all three approaches and demonstrate how they differ.

Sectoral Markup Approach Our first elasticity estimation approach relies on sectoral markups. Information on firm-level markups allow us to derive elasticities in a straightforward manner since equations (A.1) and (A.2) illustrate that $markup = \frac{elasticity}{elasticity-1}$. We thus compile data for this exercise as follows:

We obtain upstream/downstream sector classifications using WIOD. We use 2014 sales of the US to the US and RoW to calculate the share of total sales per sector that goes to final consumers. We then classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median. This yields a dataset which shows upstream and downstream classifications for 87 sectors at the 2-digit NACE level (European industry classification). This 2-digit NACE data we combine with a NACE-NAICS concordance file that maps 4-digit NACE (we only use the first 2 digits) to 6-digit NAICS. If there exists multiple NACE 2-digit codes for a NAICS 6-digit code we choose the NACE 2-digit code that has larger total US sales. This yields a final dataset that shows upstream and downstream classifications for 1,175 different NAICS 6-digit codes. This dataset we combine with data kindly provided by Baqaee and Farhi (2020) (BF) based on 6-digit NAICS codes. The BF data lists markups and sales for 31,683 different firms from 1978 âĂŞ 2018. They provide three different types of markups calculated based on a user cost / production function / or accounting profits method. We select their data between 2012 and 2017 and focus on the markups calculated using the production function estimation approach. We further exclude firms that have markups smaller than 1 (14% of all firm-year observations).

Table E.1: Elasticities

	mean	sd	min	p5	p25	p50	p75	p95	max	count
Upstream	4.43	4.26	1.10	1.15	1.60	2.75	5.04	16.50	16.50	11045
Downstream	6.44	6.05	1.29	1.46	2.44	4.03	7.49	22.24	22.24	14773

Notes: The table shows weighted mean elasticities for upstream and downstream sectors between 2012 and 2017 across all firms in the WIOD that have markups greater than 1. Elasticities stem from the production function estimation approach. Weights represent the share of firm sales in total sales. We winsorize elasticities and sales at the 5-95th percentile by sector.

We then calculate firm-level elasticities as $elasticity = \frac{markup}{markup-1}$ and winsorize elasticities and sales at the 5-95th percentile by sector. Finally, we calculate weighted mean elasticities for upstream and downstream sectors across all firms where weights represent the share of firm sales in total sales. Table E.1 presents elasticities for upstream and downstream sectors pooling all years from 2012 to 2017.

Trade Elasticity Approach In our second elasticisty estimation approach, we estimate elasticities in the upstream and downstream sectors by measuring the response of imports in the up-and downstream sectors to changes in import tariffs. More specifically, we calculate the changes in US import values in both sectors during the US-China trade war (January 2018 to December 2019) that raised US import tariffs on upstream goods by 4.1 percentage points and downstream goods by 4.4 percentage points. We obtain data on import values at the country-HTS10-month level from the US Census Bureau's Application Programming Interface (API). Data on US import tariffs is constructed as described in Section E.1.

We regress 12-month log changes in import values on 12-month log changes in tariff rates via the following regression specification:

$$\Delta ln(v_{ijt}) = \alpha_j + \tau_{it} + \beta \Delta ln(1 + Tariff_{ijt}) + \omega_{ijt} \tag{E.1}$$

where i indicates foreign countries, j denotes products and t corresponds to time; α_j is a product fixed effect; τ_{it} is a country-time fixed effect; and ω_{ijt} is a stochastic error. We denote import values by v_{ijt}. We estimate equation E.1 separately for intermediate and final goods using both log differences and the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$.[6] The results are presented in Table E.2.

Column 1 (3) suggests that a one percent increase in tariffs on intermediate (final) goods is associated with a 1.05 (1.81) percent decrease in import value. However, since tariffs can lead to zero imports, which will be dropped from the regression, columns 2 and 4 perform the same regression this time using the inverse hyperbolic sine instead of the log change. This adjustments leads to greater trade elasticities for both types of goods. A one percent increase in tariffs on intermediate (final) goods is associated with a 2.35 (3.08) percent decrease in import value. Note that the estimates from this specification correspond to an elasticity of substitution between intermediate (final) goods of 2.35 (2.08).

[6] Note that regression coefficients based on the hyperbolic sine transformation are sensitive to the scale of the import values. This is, results vary depending on whether import values are measured in thousands, millions, etc. Following Amiti et al. (2019), we measure import values in single US dollars.

Table E.2: Impact of US Tariffs on Import Values

	Intermediate Goods		Final Goods	
	(1) Log Change Import Value $\Delta ln(v_{ijt})$	(2) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$	(3) Log Change Import Value $\Delta ln(v_{ijt})$	(4) Inv. Hyperb. Import Value $\Delta ln(v_{ijt})$
log change tariff				
$\Delta ln(1 + Tariff_{ijt})$	-1.05***	-2.35***	-1.81***	-3.08***
	(0.07)	(0.44)	(0.08)	(0.35)
N	1302744	2220920	1253577	2251844
R2	0.027	0.048	0.022	0.045

Notes: Observations are at the country-HTS10-month level for the period January 2018 to December 2019. Since the specification is in 12-month changes, the data includes obesrvations from January 2017 onwards. Robust standard errors in parentheses. Variables are in twelve-month log change. All columns include product-level and country-time fixed effects. The dependent variables are the log change and the change in the inverse hyperbolic sine of US import values of intermediate and final goods, respectively. We use the inverse of the hyperbolic sine transformation, $log[x + (x^2 + 1)^{0.5}]$, to be able to estimate changes when import values are zero in t or $t - 12$. *$p < 0.05$, **$p < 0.01$, ***$p < 0.001$.

Scale Elasticity Approach Our final elasticity estimation approach exploits the isomorphism of our model to a model with external economies of scale. As discussed in Section 2, these models are isomorphic provided that the following restrictions between the external economies of scale parameters and the elasticities of substitution across varieties hold: $\gamma^u = 1/(\theta - 1)$ and $\gamma^d = 1/(\sigma - 1)$. Data on γ^u and γ^d thus allow us to easily derive information on elasticities.

Data on scale elasticities comes from Bartelme et al. (2019). The authors provide 2SLS estimates on scale elasticities for 15 manufacturing industries presented in Table E.3. We classify these industries into upstream and downstream industries following the same procedure as in the *Sectoral Markup Approach* and then calculate the average scale elasticity in those sectors.

Table E.3: Scale Elasticities

Industry	NACE Rev. 2	WIOD class.	Scale elast.
Food products, beverages and tobacco	10, 11, 12	downstream	0.16
Textiles	13, 14, 15	downstream	0.12
Wood and products of wood and cork	16	upstream	0.11
Paper products and printing	17, 18	upstream	0.11
Coke and refined petroleum products	19	upstream	0.07
Chemicals and pharmaceutical products	20, 21	upstream	0.2
Rubber and plastic products	22	upstream	0.25
Other non-metallic mineral products	23	upstream	0.13
Basic metals	24	upstream	0.11
Fabricated metal products	25	upstream	0.13
Computer, electronic and optical products	26	downstream	0.13
Electrical equipment	27	upstream	0.09
Machinery and equipment, nec	28	downstream	0.09
Motor vehicles, trailers and semi-trailers	29	downstream	0.15
Other transport equipment	30	downstream	0.16

Notes: Industries and 2SLS scale elasticities stem from Bartelme et al. (2019). Upstream and downstream classifications stem from WIOD where we classify a sector as upstream when the share of total sales to final consumers is below the median across all sectors and as downstream when the share is above the median.

For the upstream sector we obtain an average scale elasticity of 0.133 and for the downstream sector an average scale elasticity of 0.135. Exploiting the isomorphism between this setup and our framework with monopolistic competition and free entry, we convert these to $\theta = 8.52$ and $\sigma = 8.41$.

E.3 Share of Inputs in the Downstream Sector

As in the "Sectoral Markup Approach," we classify sectors into upstream and downstream depending on whether he share of total sales to final consumers is below or above the median across all sectors. From the WIOD in 2014 we calculate the share of inputs in the downstream sector as the ratio of intermediate inputs to sales in the downstream sectors leading to an estimate of $1 - \alpha = 0.45$.

F Additional Quantitative Results

F.1 Robustness

Table F.1: Calibrated Parameters - Robustness

	τ^d	τ^u	A^d_{RoW}	A^u_{RoW}
$\theta = 4.43$ and $\sigma = 6.44$	1.787	2.2986	0.289	0.114
$\theta = 2.35$ and $\sigma = 3.08$	5.021	8.536	0.248	0.102
$\theta = 8.52$ and $\sigma = 8.41$	1.508	1.446	0.301	0.121
$\theta = 2.5$ and $\sigma = 4$	3.007	6.878	0.260	0.103
$\theta = 5.5$ and $\sigma = 4$	3.007	1.877	0.281	0.116
$\alpha = 0.75$	2.249	2.040	0.196	0.114
$\alpha = 0.25$	2.239	2.042	0.119	0.124
$\alpha = 0$	2.375	2.073	0.598	0.128

Notes: This table reports the re-calibrated parameters in our robustness exercise.