Research Notes for Chapter 5*

Models with a Common Due Date

As mentioned in the chapter, the restricted version of the basic E/T problem is NP-hard but pseudopolynomial. A specialized dynamic programming technique that is capable of solving the problem for hundreds of jobs is due to Hall, Kubiak and Sethi (1991). The heuristic described in the chapter, which builds a V-shaped schedule based on assigning jobs in LPT order either to L or to R, is due to Sundararaghavan and Ahmed (1984). Details and references for quadratic E/T costs can be found in Vani and Raghavachari (1987), who suggest a neighborhood search approach. However, if we must resort to neighborhood searches, we might as well specify more realistic objectives. For example, Schaller (2004) proposes a more plausible model where the earliness penalty is linear and only tardiness is quadratic: that idea is applicable to both common due date and distinct due dates. Turning our attention to the unrestricted case where earliness and tardiness costs are job-dependent but symmetric, the problem is NP-hard, but Hall and Posner (1991) offer an effective dynamic programming solution capable of solving problems containing hundreds of jobs.

Scheduling with Distinct Due Dates

As noted in the chapter, timing decisions for multiple due dates lead to a block structure. Whereas finding the optimal sequence is NP-hard, for a given sequence the block scheduling problem is polynomial; that is, the optimal start time for each job (and consequently for each block of jobs) can be found in polynomial time. When the penalties are equal and symmetric, Garey et al. (1988) present a procedure with complexity $O(n \log n)$). In the general case, the complexity is less than $O(n^2)$. Szwarc and Mukhopadhyay (1995) present an $O(nm)$ implementation, where m is the number of blocks (and m cannot exceed n). As for finding the optimal sequence, Fry, Armstrong and Blackstone (1987) describe a neighborhood search procedure with neighborhoods formed by adjacent pairwise interchanges. (On the one hand, it is enough to consider interchanges within blocks: an API across a gap between blocks would make a non-early job tardier or a non-tardy job earlier. On the other hand, block membership can change during the process.) To find the optimal solution, Fry, Darby-Dowman and Armstrong (1986) have described a branch-and-bound procedure and given detailed computational results which indicate that the algorithm runs into difficulty when attempting problems larger than n = 20. Kim and Yano (1994) have dealt with symmetric and equal penalties. They solved problems of up to 28 jobs using a branch-and-bound scheme (although, again, solutions to problems larger

* The Research Notes series (copyright © 2009, 2019 by Kenneth R. Baker and Dan Trietsch) accompanies our textbook *Principles of Sequencing and Scheduling,* Wiley (2009, 2019). The main purposes of the Research Notes series are to provide historical details about the development of sequencing and scheduling theory, expand the book's coverage for advanced readers, provide links to other relevant research, and identify important challenges and emerging research areas. Our coverage may be updated on an ongoing basis. We invite comments and corrections.

Citation details: Baker, K.R. and D. Trietsch (2019) Research Notes for Chapter 5 in *Principles of Sequencing and Scheduling* (Wiley, 2019). URL: http://faculty.tuck.dartmouth.edu/principles-sequencing-scheduling/.

than 20 jobs were often difficult to obtain), and they reported that a pairwise interchange heuristic frequently finds an optimal solution.

Due Dates as Decisions

In the common due date case, models that treat the due date as a decision are essentially identical to models with a given due date. For distinct due dates treated as decisions by making them part of the objective function, Seidmann, Panwalkar and Smith (1981) consider the objective function

$$f(S) = \sum_{j=1}^{n} [\alpha E_j + \beta T_j + \gamma (d_j - d_0)^+].$$

The optimal schedule is SPT starting at time zero. Constructing this schedule establishes the completion time for each job. Then, if $\gamma \le \beta$, the optimal choice of due dates is $d_j = C_j$; otherwise, the optimal choice is $d_j = \min\{d_0, C_j\}$.

Extensions

As mentioned in the chapter, Hassin and Shani (2005) suggest a new model formulation, where—for a price—jobs may be rejected and only accepted jobs are subject to early/tardy penalties. Their paper also summarizes the complexity status of several variants of the problem (as well as ones with common due date). Hoogeveen (2005) cites earlier models with this type of rejection cost (but not specifically with E/T costs). Indeed, one can view the U-problem as one involving a constant rejection cost for each tardy job where the objective is to minimize the total rejection cost. Given an E/T context, the problem becomes the minimization of the total earliness, tardiness, and rejection penalties. Furthermore, whereas modeling earliness cost as linear makes practical sense, we may consider more general penalty functions for tardiness. If we limit ourselves to relatively simple tardiness penalty function forms for this purpose, we might consider including a fixed element, a linear element and a quadratic element (but not for earliness, per our argument in Section 5.5). Thus we obtain a quite general objective to minimize,

$$f(S) = \sum_{j=1}^{n} [I(j)(w_j F_j + u_j \delta(T_j) + \alpha_j E_j + \beta_j T_j + \kappa_j T_j^2) + (1 - I(j))v_j] \qquad \text{(RN5.1)}$$

where $I(j) = 1$ if job j is processed and 0 if it is rejected; $w_j \ge 0$ measures the weighted flow time penalty (so $w_j F_j$ is a part of F_w); $\delta(T_j) = 1$ if $T_j > 0$ and 0 otherwise, and $u_j \ge 0$ is the fixed part of the delay penalty (so $u_j \delta(T_j)$ is a part of U_w); $v_j \ge 0$ is the economic benefit of processing job j; and $\kappa_j \ge 0$ penalizes quadratic tardiness. As before, $\beta_j \ge 0$ is a linear tardiness penalty rate, and when $\alpha_j > 0$ it penalizes earliness. But $\alpha_j \le 0$ is allowed; e.g., $\alpha_j = -\beta_j$ implies that the lateness of job j is weighted by β_j. If we set $I(j) = 1$, (RN5.1) essentially generalizes most of our previous models. Furthermore, several new models can be generated by various combinations of elements. But when we treat $I(j)$ as a decision, we can think about the economic profit by performing the jobs as $\sum v_j$ minus (RN5.1). Under this interpretation, if we reject job j, thus setting $I(j) = 0$, the contribution of job j to (RN5.1) is v_j so the net economic contribution of the job is $v_j - v_j = 0$. But if we accept the job, $I(j)$

= 1 and the job contributes $v_j - (w_jF_j + u_j\delta(T_j) + \alpha_j E_j + \beta_j T_j + \kappa_j T_j^2)$. If this contribution is negative, the schedule cannot be optimal. We can also say that v_j is the rejection penalty, which becomes justified if the penalty function is reduced by more than v_j as a result. In such models, due dates may be given or decisions.

The various models that can be generated by activating subsets of parameters in Equation (RN5.1) all have single objectives, although these objectives can be quite complex. Another approach to modeling with complex objectives is to use multiple criteria. We encountered one example of a bicriteria model in Section 2.4.1, where we presented Smith's rule for minimizing F as a secondary measure among solutions that minimize the maximal tardiness (the primary measure). This approach is *hierarchical*: the primary measure takes precedence, and the secondary measure is optimized by selecting among solutions that are optimal for the first measure. In this chapter, Algorithms 5.1* and 5.1** exemplify hierarchical bicriteria models. But it is also possible to address bicriteria problems by minimizing some function of both measures, and this can be done by selecting appropriate parameters in Equation (RN5.1). Indeed, the basic E/T model can be interpreted as an example, where the two criteria are total earliness and total tardiness (with or without weights) and the bicriteria function is obtained by adding them. More generally, if all we know is that the function is monotone nondecreasing in both measures, then it can be shown that the optimal solution must be Pareto-optimal. A solution is *Pareto-optimal* if it is impossible to decrease one of the measures without increasing the other. Hoogeveen (1992, 2005) discusses such models.

In the hierarchical approach, we apply two (or more) objectives to the same set of jobs. A somewhat related approach partitions the jobs into two (or more) subsets, and applies a different measurement to each subset. In such a model, the first subset might represent jobs required by a customer concerned about on-time performance. The other subset might adopt the objective of minimizing total flowtime. In the literature, such models are referred to as *multi-agent* models, where each set of jobs is represented by an independent agent. As such, they are related to game theory, because at issue is how to support negotiation between the agents. The hierarchical approach can be slightly generalized by using constraints on the objective function of higher-level agents. Thus, each agent optimizes his or her schedule subject to the constraints imposed by higher-level agents' schedules (Agnetis et al. 2004). As in the multicriteria case, the optimal solution for a multi-agent problem must be Pareto-optimal: an optimal sequence cannot be improved for one agent without harming at least one other agent. Indeed, any weighted combination of the measurements of the agents yields a special case of the single-agent multicriteria model. (See, for example, Yuan et al., 2005). Agnetis et al. (2004) and Hoogeveen (2005) discuss further similarities between the multi-agent and multicriteria models and provide earlier references.

References

Agnetis, A., P.B. Mirchandani, D. Pacciarelli and A. Pacifici (2004), "Scheduling Problems with Two Competing Agents," *Operations Research* 52(2), 229-242.

Bagchi, U., F. Julien and M. Magazine (1994) "Due-Date Assignment to Multi-job Customers," *Management Science* 40, 1389-1392.

Cheng, T.C.E. (1987) "An Algorithm for the CON Due Date Determination and Sequencing Problem," *Computers and Operations Research* 14, 537-542.

De, P., J. Ghosh and C. Wells (1990) "CON Due-Date Determination and Sequencing," *Computers and Operations Research* 17, 333-342.

De, P., J. Ghosh and C. Wells (1994) "Due-Date Assignment and Early/Tardy Scheduling on Identical Parallel Machines," *Naval Research Logistics* 41, 17-32.

Emmons, H. (1987) "Scheduling to a Common Due Date on Parallel Common Processors," *Naval Research Logistics Quarterly* 34, 803-810.

Fry, T., K. Darby-Dowman and R. Armstrong (1988) "Single Machine Scheduling to Minimize Mean Absolute Lateness," Working Paper, College of Business Administration, University of South Carolina, Columbia, S.C.

Garey, M., R. Tarjan and G. Wilfong (1988) "One-Processor Scheduling with Symmetric Earliness and Tardiness Penalties," *Mathematics of Operations Research* 13, 330-348.

Hoogeveen, J.A. (1992), *Single-Machine Bicriteria Scheduling,* PhD Thesis, Technical University of Eindehoven, Netherlands.

Hoogeveen, J.A. (2005) "Multicriteria Scheduling," *European Journal of Operational Research* 167(3), 592-623.

Hoogeveen, J.A., H. Oosterhout and S.L. van de Velde (1994) "New Lower and Upper Bounds for Scheduling around a Small Common Due Date," *Operations Research* 42, 102-110.

Kim, Y. and C. Yano (1994) "Minimizing Mean Tardiness and Earliness in Single-Machine Scheduling Problems with Unequal Due Dates," *Naval Research Logistics* 41, 913-933.

Panwalkar, S., M. Smith and A. Seidmann (1982) "Common Due Date Assignment to Minimize Total Penalty for the One Machine Scheduling Problem," *Operations Research* 30, 391-399.

Schaller, J. (2004) “Single Machine Scheduling with Early and Quadratic Tardy Penalties,” *Computers & Industrial Engineering* 46, 511-532.

Seidmann, A., S. Panwalkar and M. Smith (1981) "Optimal Assignment of Due-Dates for a Single Processor Scheduling Problem," *International Journal of Production Research* 19, 393-399.

Vani, V. and M. Raghavachari (1987) "Deterministic and Random Single Machine Scheduling with Variance Minimization," *Operations Research* 35, 111-120.

Yuan, J.J., W.P. Shang, and Q. Feng (2005) "A Noted on the Scheduling with Two Families of Jobs," *Journal of Scheduling* 8, 537-542.