**EXERCISES**

* 1. **Finding a Lost Plane** A private plane went down off the coast and sank during a bad storm although rescuers were able to save its crew. Aboard the plane was a transmitter that was able to send out a signal for 72 hours after the plane went down. When the weather cleared, searchers went out in three different boats carrying equipment that could detect the signal and estimate its distance from the transmitter. The locations of the three boats on an *x*-*y* grid and their distance estimates (in miles) are shown in the table below.

|  |  |  |  |
| --- | --- | --- | --- |
| *Searcher* | 1 | 2 | 3 |
| *x-location* | 25 | 35 | 70 |
| *y-location* | 60 | 40 | 10 |
| *Estimate* | 29.3 | 34.7 | 15.5 |

The estimates are known to be imperfect, but the information is sufficient to locate the sunken plane, at least approximately. What is its *x*-*y* location?

* 1. Curve Fitting for Costs Newton Manufacturing Company has reached a stable volume in the last couple of years and is interested in developing a planning model for its production levels, based on aggregate units of output. One element will be a cost model that describes the relationship of unit cost to production volume. Newton’s capacity is thought to be around 2500 aggregate units at current equipment and labor force levels, and it is well known that unit costs tend to rise when output volumes are significantly above or below this figure. For volumes above this figure, costs rise due to overtime premiums and to high congestion levels in the plant. For volumes below the nominal figure, costs rise due to inefficiencies in production. Analysts at Newton have therefore decided that some type of quadratic function would be a suitable model for unit costs, and they have proposed the form *ax*2 + bx + *c*, where *x* represents the aggregate number of output units, and the parameters a, b,and c remain to be determined. For this purpose, the model will be fit as closely as possible to the last 12 months of observed data, as reproduced in the table below.

|  |  |  |
| --- | --- | --- |
| Month | Aggregate output | Unit cost |
| 1 | 2350 | $53.35 |
| 2 | 2200 | 54.60 |
| 3 | 2450 | 49.62 |
| 4 | 2600 | 53.62 |
| 5 | 2550 | 49.69 |
| 6 | 2400 | 51.18 |
| 7 | 2300 | 53.25 |
| 8 | 2650 | 51.91 |
| 9 | 2700 | 54.23 |
| 10 | 2750 | 50.06 |
| 11 | 2500 | 49.08 |
| 12 | 2250 | 54.46 |

* 1. What values of the three parameters provide the best fit to the data, as measured by the minimum sum of squared differences?
  2. What does the model in (a) predict as the unit cost for an output of 2500?
  3. Supply Chain Design Muslin Office Furniture manufactures a popular line of filing cabinets and has a very strong competitive position in its market. The company sells its product to a number of wholesale distributors who, in turn, sell to retail customers. In this environment, the company faces a demand curve of the following form



where P1 denotes its selling price and *Q*1 denotes the volume (in thousands) sold at that price. Muslin also experiences increasing marginal costs of the form 0.8*Q*1. (This means that its total cost is 0.8(*Q*1)2/2.) Increasing marginal costs occur because of quality losses and congestion on the shop floor as volume rises.

One of Muslin’s distributors is a subsidiary known as New England Supply. They represent Muslin’s exclusive distributor in the northeast, and the parent company allows them to operate as an independent entity, focused on distribution. They buy filing cabinets from Muslin and sell them to retail customers in the northeast. In that market, New England Supply faces its own demand curve as follows



where P2 denotes the retail selling price and Q2 denotes the volume (in thousands) sold in the northeast at that price. New England Supply incurs its own operating costs, in addition to the cost of purchasing the product from Muslin, so that its marginal cost function takes the form P1 + 0.4*Q*2. This means that its total cost is P1Q2 + 0.4(*Q*2)2/2.

1. Suppose that Muslin Office Furniture and New England Supply each analyze their own pricing strategies separately. That is, Muslin finds its profit-maximizing price. Then New England Supply, whose cost is influenced by Muslin’s price, maximizes its own profits. What is each firm’s optimal price and how much profit is earned between the two companies?
2. Suppose instead that the two firms make coordinated decisions. In other words, they choose a pair of prices, one wholesale and one retail, aimed at maximizing the total profit between the two firms. What is each firm’s optimal price in this coordinated environment? How much profit is earned between the two companies?

8.14. Estimating Beta In finance it is important to be able to predict the return on a stock from the return on the market, that is, on a market index such as the S&P 500 index. It is often hypothesized that a particular prediction equation exists



where y is the return on a stock during a time period, x is the return on the market index during the same time period, and α and β are constants that must be estimated. The value of β is of particular interest, because it indicates how closely the returns on a particular stock tend to follow the returns on the market as a whole.

If our knowledge of the parameters α and β were perfect, then we could predict individual stock returns accurately from the behavior of the market. Typically, such knowledge does not exist, and our values of α and β are imperfect. In other words, when we use them, we encounter errors in our predictions. The best we can do is to choose the estimates α and β in order to make prediction errors close to zero.

Find data on returns for Coca-Cola stock on a monthly basis for the period January 2, 2001 to December 1, 2006, and returns for the S&P 500 index for the same 72 months. Fit the linear prediction equation to this set of data. Use as a criterion the minimum sum of squared differences between the actual stock returns and their predicted values. For the historical data, estimate the parameters of the prediction equation for Coca-Cola stock.

1. What is the estimated value of *β*, for Coca-Cola stock?
2. Repeat the estimation process for Microsoft stock. What do you expect to find in terms of the relationship between the *β* for Microsoft and the *β* for Coke?

8.15. Production Smoothing A supplier of raw material has made plans to provide monthlydeliveries to a customer. The customer’s requirements are shown in the following table.

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Month | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| Units | 100 | 200 | 300 | 400 | 100 | 100 | 500 | 300 |

The raw material can be processed and prepared for delivery in any volume because part-time labor can be used, and the labor pool is quite large. However, changes in month-to-month production volumes can be costly. When production levels increase, costs must be incurred in acquiring and training new workers. When production levels decrease, costs are incurred due to layoff policies.

Based on historical data, the cost estimate for increasing production from one month to the next is $15 per unit increase in capacity. In the other direction, reducing production from one month to the next incurs a cost of $10 per unit reduction in capacity. The other relevant cost is the cost of inventory: each unit held in stock incurs a cost of $20 per month held.

Entering month 1, the starting inventory is 80 units, and the production level has been steady at 100 units. To make sure the plans can be extended into the future, inventory is required to be at least 50 units at the end of the eighth month, and the planned production level for month 9 is 200.

What is a minimum-cost production plan for the supplier?

8.16. Political Redistricting Based on the new census information, it is time to redraw the boundaries of the political districts in the state of Idazona. Each district will have one representative in the next Congress, and Idazona has been allocated four representatives based on its share of the national population. The state is made up of nine counties, with populations (in thousands) shown in the table. (See the state map below.)

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**Figure 8.20.** State map of ldazona.

The main requirement in the formation of districts is that they produce equal populations, or as close to equal as possible. Furthermore, the districts must be composed of adjacent counties without splitting any county between two or more districts. Officials in Idazona interpret the requirement to mean that if a district is created from two counties, then those two counties must share a border. Furthermore, if a district is created from three counties, then at least one of the counties must be adjacent to the other two. No district is permitted to have exactly one county or more than three counties.

Mathematically, officials are seeking a districting plan for which the maximum deviation between a district population and the average district population will be as small as possible.

What assignment of counties to districts will satisfy the desired conditions?

**8.17. Scheduling the Factory** The major production operations at Madison Engine Works are performed at a machining cell. This is a large piece of multi-purpose equipment that can perform several operations on a particular metal component. Madison Engine actually owns three of these machines with slightly different characteristics, but all of the waiting jobs can be processed, in whole or in part, on any of the three machines.

Each waiting job consists of a number of different parts. Before running a job—or part of one—on a particular machining cell, one of the plant’s setup teams must prepare the cell. If a job is split and processed by two cells, then each cell requires a setup. Because the machines have different ages, the setup time for a particular job is different at one cell than at another cell.

To describe this week’s scheduling problem at the three machining centers, Madison Engine has downloaded data from its information system, as shown below.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  | *Job Number* | |  |  |  |  |  |
|  | *1* | *2* | *3* | *4* | *5* | *6* | *7* | *8* | *9* | *10* | *Setup time* |
| Cell 1 | 17 | 10 | 10 | 25 | 28 | 30 | 25 | 11 | 22 | 16 | 12 |
| Cell 2 | 32 | 48 | 35 | 47 | 40 | 50 | 41 | 47 | 48 | 34 | 8 |
| Cell 3 | 70 | 70 | 55 | 70 | 71 | 58 | 52 | 78 | 64 | 58 | 6 |

Each job has a processing time, which represents the number of hours it would take to run the job, in its entirety, on a particular cell. However, it is also possible to split the job, in virtually any fractional amount, between two or even three cells, as long as the setup is carried out at every machine that works on part of the job. What is the shortest time in which the entire set of jobs can be completed?

8.18. Miles Manufacturing (Revisited) Revisit the scenario of Example 7.4, and associate a weighting factor with each job. Now, we can take our objective to be the maximum weighted tardiness in the schedule, where a job’s weighted tardiness is the product of its tardiness and its weight. The weights of the jobs are listed in the full table of data given below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Job number | 1 | 2 | 3 | 4 | 5 | 6 |
| Processing time (hours) | 5 | 7 | 9 | 11 | 13 | 15 |
| Weight | 8 | 8 | 6 | 6 | 4 | 2 |
| Due date (hours from now) | 28 | 35 | 24 | 32 | 30 | 40 |

What is the maximum weighted tardiness for the sequence (5-3-1-2-4-6) found in Example 7.4?

What is the optimal value of total weighted tardiness?

**8.19. Designing a HVAC system** Gaillard College is renovating its Engineering building. A key component of the design is the deployment of its air conditioning (AC) units. The building contains 20 distinct spaces to be air conditioned. Each space imposes a particular average daily load (ADL), which is a function of volume of the space and the equipment used in it. The following table gives the daily load estimates for each of the spaces.

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Space | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| ADL | 0.7 | 1.4 | 0.8 | 0.7 | 1.0 | 2.4 | 0.1 | 0.3 | 0.2 | 0.4 |
| Space | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| ADL | 0.2 | 0.4 | 0.4 | 0.1 | 2.0 | 1.5 | 0.3 | 0.4 | 0.3 | 1.6 |

The AC will be provided by six separate units, each one serving a particular zone in the building. Four of the AC units currently exist, and two larger units will be purchased as part of the project.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| Unit | 1 | 2 | 3 | 4 | 5 | 6 |
| Capacity | 4 | 4 | 5 | 5 | 3 | 3 |

The design problem is to assign each space to a zone and thereby to a specific AC unit. However, it is not desirable to allow all possible unit-space assignments because the resulting ductwork might be unrealistic. Instead, a “layout array” shows which spaces can feasibly be assigned to which zones. In the array, an entry of 1 signifies a feasible assignment.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Space | |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| Unit | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 4 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 6 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 |

Given an assignment, the ADL on a unit is the sum of the ADL’s from the spaces assigned to its zone. To keep the system balanced, the design objective is to minimize the maximum excess capacity (beyond total assigned ADL) for any of the AC units. What is the optimal value of the maximum excess capacity for any of the AC units and which unit carries the largest excess?