**EXERCISES**

7.11. **Stock Cutting** Lumber is supplied to a medium-sized furniture workshop in standard lengths of 100 inches. Different designs call for pieces of specified lengths that do not exceed 100 inches. Nevertheless, the shop wants to use a minimum number of standard-length master pieces to accommodate a given list of required pieces. Today's list is shown in the table below.

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Design | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
| Length | 33 | 14 | 63 | 84 | 39 | 94 | 54 | 41 | 50 | 71 | 56 |

1. Develop a rule of thumb to solve this problem. How many standard length pieces are needed, using your rule of thumb?
2. What is the optimal solution? How close did your rule of thumb in (a) come to the optimum?

7.12. **Assigning Groups** A project course in the Engineering Department has 22 students enrolled. The students must be assigned to teams that will meet each week for half a day in their laboratory to work on their projects. The laboratory is reserved for this purpose at seven times (T1 through T7); these are: morning and afternoon on Mondays, Tuesdays, and Wednesdays, as well as morning on Fridays.

The students have been asked for their time preferences on a scale of 10 (with a larger score meaning higher preference). These preferences are shown in the table below, where zero is entered if the student has an existing conflict on the calendar. Students who are Engineering majors are identified in the table with the letter M. At least one Engineering major is required for each team.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | Student | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 |  |
| *Times* |  | M | M | M |  |  | M |  |  | M |  |  | M |  |  | M |  |  |  |  | M |  | M | *limit* |
| Mam | T1 | 10 | 0 | 0 | 10 | 0 | 0 | 8 | 8 | 0 | 0 | 10 | 0 | 0 | 8 | 10 | 0 | 0 | 10 | 0 | 0 | 8 | 0 | 5 |
| Mpm | T2 | 8 | 10 | 0 | 0 | 0 | 7 | 0 | 8 | 10 | 0 | 0 | 0 | 7 | 0 | 8 | 10 | 0 | 0 | 0 | 7 | 0 | 7 | 4 |
| Tam | T3 | 6 | 5 | 5 | 0 | 10 | 0 | 10 | 6 | 5 | 5 | 0 | 10 | 0 | 10 | 6 | 5 | 5 | 0 | 10 | 0 | 10 | 0 | 3 |
| Tpm | T4 | 6 | 0 | 0 | 8 | 0 | 0 | 0 | 10 | 0 | 0 | 8 | 0 | 0 | 0 | 6 | 0 | 0 | 8 | 0 | 0 | 0 | 0 | 4 |
| Wam | T5 | 0 | 5 | 0 | 6 | 6 | 10 | 5 | 0 | 5 | 0 | 6 | 6 | 10 | 5 | 0 | 5 | 0 | 6 | 6 | 10 | 5 | 10 | 5 |
| Wbm | T6 | 0 | 0 | 0 | 9 | 0 | 0 | 9 | 0 | 0 | 0 | 8 | 0 | 0 | 9 | 0 | 0 | 0 | 5 | 0 | 0 | 9 | 0 | 4 |
| Fam | T7 | 0 | 0 | 10 | 0 | 6 | 0 | 0 | 0 | 0 | 10 | 0 | 6 | 0 | 0 | 0 | 0 | 10 | 0 | 6 | 0 | 0 | 0 | 5 |

Some projects are designed for five members, others for just three or four. However, each team must have at least three members. As a consequence, there will be fewer than seven groups when the schedule is worked out. The goal, however, is to maximize the sum of preferences in the assignments.

1. What is the maximum sum of preferences that can be achieved?
2. In the optimal solution to (a), how many students are assigned to their highest preference?

7.13. **Assigning Groups (continued)** What is the maximum number of students who could be assigned to their first choice, if that were the only criterion?

7.14. Taking Discounts Universal Technologies, Inc. has identified two qualified vendors with the capability to supply some of its electronic components. For the coming year, Universal has estimated its volume requirements for these components and obtained price-break schedules from each vendor. (These are summarized as “all-units” discounts in the table below.) Universal’s engineers have also estimated each vendor’s maximum capacity for producing these components, based on available information about equipment in use and labor policies in effect. Finally, because of its limited history with Vendor A, Universal has adopted a policy that permits no more than 60% of its total unit purchases on these components to come from Vendor A. What is the minimum-cost purchase plan for Universal?

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | Vendor A | | | | Vendor B | | | |
| Product | Requirement | Unit price | | Volume required | | Unit price | Volume required | | |
| 1 | 500 | $225 | 0–250 | | $224 | | 0–300 |
|  |  | $220 | 250–500 | | $214 | | 300–500 |
| 2 | 1000 | $124 | 0–600 | | $120 | | 0–1000 |
|  |  | $115 | 600–1000 | |  | | (no discount) | |
| 3 | 2500 | $60 | 0–1000 | | $54 | | 0–1500 |
|  |  | $56\* | 1000–2000 | | $52 | | 1500–2500 |
|  |  | $51 | 2000–2500 | |  | |  |
| Total capacity (units) | |  | | 2500 | |  | 2000 | | |

\*For example, if 1400 units are purchased from Vendor A, they cost $56 each, for a total of $78,400.

* 1. Taking Discounts (continued) In the previous exercise, suppose that Vendor A provides a new price-discount schedule for component 3. This one is an “incremental” discount, as opposed to an “all-units” discount, as follows.

Unit price = $60 on all units up to 1000

Unit price = $56 on the next 1000 units

Unit price = $51 on the next 500 units

1. With the change in pricing at Vendor A, what is the minimum purchasing cost for Universal?
2. Which purchase quantities involving components 1 and 2 change as a result of the new price schedule?

7.16. Routing Flights Coastal Air offers discount fares to travelers in the East, including service to its main base in New York City (*LGA*). The airline serves six cities and wants its aircraft to land at each city once each day, returning to its starting point. The task is to find a route with the minimal total flying time. The flying time between city pairs in minutes is given in the table below. (Times are asymmetric due to different take-off and landing patterns, as well as different traffic conditions at the various airports.)

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  | *To* | *City* |  |  |
|  |  | *GSO* | *CLT* | *JAX* | *RDU* | *BWI* | *LGA* |
|  | *GSO* | 0 | 45 | 114 | 42 | 81 | 114 |
|  | *CLT* | 38 | 0 | 88 | 53 | 93 | 145 |
| *From* | *JAX* | 109 | 102 | 0 | 107 | 176 | 187 |
| *City* | *RDU* | 44 | 51 | 111 | 0 | 82 | 109 |
|  | *BWI* | 86 | 100 | 160 | 80 | 0 | 57 |
|  | *LGA* | 130 | 128 | 204 | 118 | 67 | 0 |

1. What is the minimum flying time of a route that starts and ends in New York (LGA) and visits all of the other cities once?
2. What is the optimal route corresponding to the flying time in (a)?

**7.17. Touring Florida** A long-time professor, planning life in retirement, has decided to relocate to Florida. Following up on information found at a website, she has decided to visit a dozen Florida locations to look for a place to live. The website provides information on driving times between the various cities and towns, as shown in the table below. (Driving times are given in hours.) She would like to visit each of the places once and return to Miami, her starting point.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | BR | CL | CB | DB | FL | FM | FP | GV | JV | KW | KS | MI |
| Bradenton | 0.0 | 0.9 | 3.3 | 3.4 | 4.2 | 1.7 | 3.0 | 3.3 | 4.7 | 7.0 | 2.1 | 4.4 |
| Clearwater | 0.9 | 0.0 | 3.3 | 3.3 | 5.3 | 2.5 | 3.0 | 2.5 | 4.0 | 8.8 | 2.1 | 5.5 |
| Cocoa Beach | 3.3 | 3.3 | 0.0 | 1.5 | 3.8 | 4.0 | 1.3 | 3.8 | 3.5 | 7.8 | 0.3 | 4.0 |
| Daytona Beach | 3.4 | 3.3 | 1.5 | 0.0 | 5.0 | 4.3 | 2.6 | 2.0 | 2.0 | 9.0 | 0.7 | 5.5 |
| Ft. Lauderdale | 4.2 | 5.3 | 3.8 | 5.0 | 0.0 | 2.8 | 1.9 | 6.5 | 6.8 | 4.0 | 4.2 | 0.5 |
| Ft. Myers | 1.7 | 2.5 | 4.0 | 4.3 | 2.8 | 0.0 | 2.5 | 4.8 | 6.0 | 6.0 | 2.7 | 3.0 |
| Ft. Pierce | 3.0 | 3.4 | 1.3 | 2.6 | 2.0 | 2.5 | 0.0 | 4.3 | 4.4 | 5.4 | 2.0 | 2.3 |
| Gainesville | 3.2 | 2.5 | 3.8 | 2.0 | 6.5 | 4.8 | 4.3 | 0.0 | 1.5 | 10.0 | 3.3 | 7.0 |
| Jacksonville | 4.7 | 4.0 | 3.5 | 2.0 | 6.8 | 6.0 | 4.4 | 1.5 | 0.0 | 11.3 | 3.3 | 7.3 |
| Key West | 7.0 | 8.8 | 7.8 | 9.0 | 4.0 | 6.0 | 5.5 | 10.0 | 11.3 | 0.0 | 8.5 | 3.5 |
| Kissimmee | 2.1 | 2.0 | 1.0 | 1.4 | 3.9 | 2.9 | 2.5 | 2.5 | 3.0 | 7.2 | 0.0 | 4.3 |
| Miami | 4.4 | 5.5 | 4.0 | 5.5 | 0.5 | 3.0 | 2.3 | 7.0 | 7.3 | 3.5 | 4.5 | 0.0 |

1. What is the minimum amount of driving time required to visit all of the locations once and return to Miami?
2. What is the tour that achieves the minimum time?

**7.18. Sequencing Tasks** Eight insurance policies are in the queue waiting to be evaluated by the underwriting department. Each policy has a known processing time (in hours), a due date (derived from customer expectations) and a penalty factor (which is the Marketing Department’s importance weighting for the customer.)

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ***Job*** | ***1*** | ***2*** | ***3*** | ***4*** | ***5*** | ***6*** | ***7*** | ***8*** |
| ***Process time*** | 13 | 9 | 8 | 12 | 7 | 10 | 14 | 11 |
| ***Due date*** | 55 | 13 | 27 | 51 | 43 | 24 | 32 | 62 |
| ***Penalty factor*** | 5 | 7 | 3 | 4 | 4 | 8 | 6 | 6 |

In this situation, the objective is to minimize the penalty-weighted *number* of late tasks. In other words, if a policy evaluation is completed by its due date, then no penalty is incurred. If the evaluation completes after its due date, the penalty is the value listed in the table. The measure of scheduling effectiveness is the sum of the penalties for the late policies—that is, the total penalty.

1. What is the minimum possible value of the total penalty?
2. What sequence achieves the minimum penalty in (a)?

7.19. Locating Emergency Services The Southeast Emergency Management Agency is planning to establish a number of helicopter bases in a hurricane-prone part of the country. There are 25 sites under consideration, but the agency has funds to install only three bases. Using a specialized map, the agency has identified counties that can be served from each site in less than 15 minutes of response time. (This response time is considered the maximum desired for the purposes of servicing the critical emergencies that might follow a hurricane.)

The data describing the potential sites consist of a census list and an accessibility matrix. The census list gives the population of each county. The accessibility matrix contains a 1 as its (*k*, j)th element if county k can be serviced from a base at site j within 15 minutes.

1. At present, the agency has funds to install just three of the bases, and it wishes to maximize the population served by the bases. At which sites should the bases be established and what is the corresponding population served?
2. The agency is close to receiving some generous grant support that will enable it to install more than three bases. Construct a chart showing how the maximum population served varies with the number of bases.

