**EXERCISES**

* 1. **DEA Efficiency with Side Constraints** Review Exercise 5.5. After looking at the analysis, your corporate client proposes a refinement. Since the corporation has emphasized market share in other divisions, the following constraint is suggested: the virtual output for market share should be at least 150 percent of the virtual output from either profit or growth.

1. With the side constraints added, calculate the efficiencies of the five restaurants.
2. List one advantage and one disadvantage of using the side constraint.
   1. **DEA with Cross-Efficiencies** Review Exercise 5.5. After looking at the analysis, your corporate client proposes yet another refinement to break ties in the efficiency ratings. The evaluation of the *k*th DMU produces a set of “optimal prices” that are as favorable as possible to unit *k.* Suppose we call these “set *k* prices.” Now, to evaluate DMU *k*, compute the value of its efficiency (output value divided by input value) under each of the price sets (set 1 prices through set 5 prices, in this case). Then, average the five efficiency values. This average value is called the cross-efficiency.
3. Rank the DMUs based on cross-efficiency.
4. What are the advantages and disadvantages of using the cross-efficiency measure?
   1. **Evaluating Universities** A study of Finnish universities is aimed at determining an efficiency ranking. Two inputs are used: the university’s annual budget and the selectivity rating (for which higher is better). Four outputs are used: the number of graduates (receiving their primary degree), the number of advanced graduates (receiving a post-graduate degree), a progress index (measured against a standard rate of progress toward the corresponding degree), and a completion index (measuring the propensity of students to finish their degree requirements). The data are provided in Table 5.5.
5. Perform a DEA evaluation for the universities, listing the efficiency for each.
6. How many of the 20 universities are efficient?

**Table 5.5.** Data on 20 Finnish Universities

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  | *Inputs* |  | *Outputs* |  |  |  |
| ***University*** | Budget | Selectivity | Graduates | Advanced | Progress | Completion |
| **1** | 1.000 | 0.090 | 1.000 | 1.000 | 0.715 | 0.577 |
| **2** | 0.290 | 0.099 | 0.455 | 0.324 | 0.835 | 0.705 |
| **3** | 0.419 | 0.059 | 0.504 | 0.348 | 0.770 | 0.650 |
| **4** | 0.149 | 0.069 | 0.288 | 0.158 | 0.835 | 0.704 |
| **5** | 0.163 | 0.073 | 0.155 | 0.152 | 0.835 | 0.583 |
| **6** | 0.380 | 0.094 | 0.516 | 0.318 | 0.770 | 0.668 |
| **7** | 0.281 | 0.105 | 0.423 | 0.276 | 0.770 | 0.528 |
| **8** | 0.172 | 0.036 | 0.221 | 0.155 | 0.715 | 0.689 |
| **9** | 0.060 | 0.063 | 0.133 | 0.033 | 1.000 | 0.726 |
| **10** | 0.069 | 0.118 | 0.132 | 0.030 | 1.000 | 1.000 |
| **11** | 0.047 | 0.146 | 0.019 | 0.006 | 0.715 | 0.753 |
| **12** | 0.388 | 0.051 | 0.424 | 0.473 | 0.665 | 0.669 |
| **13** | 0.174 | 0.054 | 0.213 | 0.212 | 0.835 | 0.692 |
| **14** | 0.088 | 0.034 | 0.111 | 0.033 | 0.770 | 0.618 |
| **15** | 0.107 | 0.091 | 0.172 | 0.052 | 0.835 | 0.707 |
| **16** | 0.042 | 0.051 | 0.082 | 0.018 | 0.910 | 0.533 |
| **17** | 0.044 | 0.065 | 0.124 | 0.027 | 0.910 | 0.921 |
| **18** | 0.075 | 0.327 | 0.070 | 0.006 | 0.715 | 0.684 |
| **19** | 0.092 | 0.113 | 0.052 | 0.006 | 0.665 | 0.515 |
| **20** | 0.037 | 1.000 | 0.011 | 0.000 | 0.910 | 0.359 |

* 1. **Evaluating Municipal Departments** An effort is underway to evaluate several municipal departments in the UK that collect taxes related to property. One input is used: the annual cost of operating the municipal office. Four outputs are measured, relating to different activities carried out in each of the departments. The data are shown in Table 5.6.

1. Perform a DEA evaluation for the departments, listing the efficiency for each unit.
2. How many of the 62 DMUs are efficient?

**Table 5.6.** Data on 62 Municipal Departments

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Inputs | Outputs | | | |
| ID | Costs | Inheritances | Rebates | Warrants | Collections |
| **1** | 9.13 | 7.525 | 34.114 | 21.958 | 3.840 |
| **2** | 13.60 | 8.301 | 23.270 | 35.966 | 8.632 |
| **3** | 5.76 | 10.909 | 13.392 | 11.527 | 4.931 |
| **4** | 11.24 | 16.621 | 36.817 | 27.552 | 9.522 |
| **5** | 15.57 | 22.809 | 95.776 | 23.611 | 12.266 |
| **6** | 5.65 | 1.777 | 0.156 | 1.314 | 39.011 |
| **7** | 21.60 | 15.107 | 70.958 | 54.216 | 10.809 |
| **8** | 8.57 | 7.919 | 48.688 | 14.032 | 5.923 |
| **9** | 6.01 | 7.066 | 36.304 | 5.445 | 2.936 |
| **10** | 8.02 | 8.858 | 43.610 | 13.774 | 4.274 |
| **11** | 9.93 | 8.999 | 36.852 | 20.661 | 8.151 |
| **12** | 7.90 | 8.278 | 45.222 | 6.191 | 5.327 |
| **13** | 5.15 | 6.763 | 18.704 | 10.620 | 3.540 |
| **14** | 6.42 | 8.984 | 13.600 | 12.319 | 3.752 |
| **15** | 5.94 | 7.686 | 25.906 | 8.242 | 2.483 |
| **16** | 8.68 | 7.227 | 16.965 | 17.581 | 6.274 |
| **17** | 4.86 | 3.356 | 23.672 | 4.298 | 2.482 |
| **18** | 10.33 | 8.558 | 30.540 | 17.770 | 8.005 |
| **19** | 21.97 | 12.234 | 92.020 | 29.530 | 14.763 |
| **20** | 9.70 | 7.674 | 41.162 | 13.272 | 4.503 |
| **21** | 6.34 | 8.168 | 16.613 | 8.264 | 5.047 |
| **22** | 7.70 | 7.884 | 15.749 | 14.502 | 3.034 |
| **23** | 5.99 | 5.666 | 27.546 | 5.243 | 3.410 |
| **24** | 5.20 | 6.923 | 12.613 | 4.298 | 3.040 |
| **25** | 6.36 | 7.352 | 23.510 | 5.744 | 4.207 |
| **26** | 8.87 | 6.456 | 38.100 | 9.645 | 3.093 |
| **27** | 10.71 | 13.642 | 23.862 | 14.631 | 4.631 |
| **28** | 6.49 | 7.675 | 17.972 | 8.269 | 2.756 |
| **29** | 15.32 | 15.341 | 55.415 | 16.361 | 12.530 |
| **30** | 7.00 | 8.369 | 14.918 | 9.883 | 4.328 |
| **31** | 10.50 | 9.608 | 37.910 | 13.493 | 5.035 |
| **32** | 10.88 | 10.648 | 36.962 | 14.248 | 4.844 |
| **33** | 8.52 | 8.967 | 24.672 | 11.841 | 3.753 |
| **34** | 7.61 | 6.111 | 31.734 | 7.657 | 2.872 |
| **35** | 10.91 | 9.778 | 42.725 | 12.169 | 4.657 |
| **36** | 9.72 | 7.713 | 5.897 | 14.600 | 9.251 |
| **37** | 12.63 | 11.082 | 41.586 | 16.420 | 5.647 |
| **38** | 11.51 | 9.066 | 28.491 | 16.284 | 5.962 |
| **39** | 6.22 | 6.627 | 14.667 | 7.703 | 3.083 |
| **40** | 5.29 | 3.958 | 20.416 | 1.961 | 1.835 |
| **41** | 8.78 | 6.558 | 31.720 | 8.596 | 4.831 |
| **42** | 13.50 | 4.769 | 26.469 | 20.877 | 4.170 |
| **43** | 12.60 | 6.680 | 30.280 | 9.085 | 19.449 |
| **44** | 8.10 | 8.103 | 9.708 | 8.534 | 7.502 |
| **45** | 9.67 | 6.004 | 19.460 | 10.708 | 8.033 |
| **46** | 12.37 | 11.253 | 28.500 | 12.528 | 6.741 |
| **47** | 9.50 | 8.674 | 23.542 | 8.992 | 3.664 |
| **48** | 11.47 | 10.300 | 15.576 | 13.740 | 6.458 |
| **49** | 11.78 | 12.221 | 14.325 | 10.100 | 5.021 |
| **50** | 12.57 | 10.432 | 18.306 | 16.387 | 3.924 |
| **51** | 50.26 | 32.331 | 150.000 | 45.099 | 19.579 |
| **52** | 12.70 | 9.500 | 22.391 | 14.900 | 5.803 |
| **53** | 13.30 | 7.530 | 21.990 | 14.655 | 8.324 |
| **54** | 5.60 | 3.727 | 12.208 | 5.388 | 2.837 |
| **55** | 11.75 | 5.198 | 13.280 | 13.618 | 7.104 |
| **56** | 8.47 | 6.149 | 19.453 | 6.505 | 3.300 |
| **57** | 8.36 | 5.959 | 17.110 | 4.655 | 3.077 |
| **58** | 11.07 | 7.247 | 16.338 | 8.686 | 6.620 |
| **59** | 10.38 | 7.761 | 16.440 | 6.014 | 3.313 |
| **60** | 11.83 | 5.347 | 12.410 | 12.238 | 4.567 |
| **61** | 12.71 | 6.320 | 13.632 | 8.530 | 5.161 |
| **62** | 11.19 | 6.578 | 10.900 | 3.523 | 3.456 |