**EXERCISES**

4.14. Delta Oil Model Revisited Revisit the refinery model of this chapter and its optimal solution.

* 1. Study the capacity of the cracker in the range from 15,000 to 30,000 barrels/day. Construct a table that traces the value of the optimal solution in this range, as well as the output quantities of premium and regular gasoline, showing the output quantities in a chart.
	2. Study the capacity of the tower in the range from 40,000 to 50,000 barrels/day. Construct a table that traces the value of the optimal solution in this range, as well as the output quantities of premium and regular gasoline, showing the output quantities in a chart.
	3. Suppose capacity expansion is contemplated at both the tower and the cracker simultaneously and in the same amount. Construct a table that traces the value of the optimal profit contribution for incremental capacities of up to 10,000 barrels/day at each facility. Extend the table to include the corresponding quantities of regular and premium gasoline.

4.15. Delta Oil Pattern Revisited Revisit the refinery model of this chapter and its optimal solution. Suppose that a capacity expansion project has raised the capacity of the cracker to 24,000 barrels/day.

* 1. Find the optimal solution to the revised profit-maximization problem.
	2. Specify the pattern in the solution of part **(a)**.

4.16. Distributing a Product The Lincoln Lock Company manufactures a commercial security lock at plants in Atlanta, Louisville, Detroit, and Phoenix. The unit cost of production at each plant is $35.50, $37.50, $37.25, and $36.25, and the annual capacities are 18,000, 15,000, 25,000, and 20,000, respectively. The locks are sold through wholesale distributors in seven locations around the country. The unit shipping cost for each plant-distributor combination is shown in the following table, along with the forecasted demand from each distributor for the coming year.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  | Tacoma | San Diego | Dallas | Denver | St Louis | Tampa | Baltimore |
| Atlanta | 2.50 | 2.75 | 1.75 | 2.00 | 2.10 | 1.80 | 1.65 |
| Louisville | 1.85 | 1.90 | 1.50 | 1.60 | 1.00 | 1.90 | 1.85 |
| Detroit | 2.30 | 2.25 | 1.85 | 1.25 | 1.50 | 2.25 | 2.00 |
| Phoenix | 1.90 | 0.90 | 1.60 | 1.75 | 2.00 | 2.50 | 2.65 |
| Demand | 5500 | 11,500 | 10,500 | 9,600 | 15,400 | 12,500 | 6600 |

1. Determine the least costly way of shipping locks from plants to distributors.
2. List the shadow prices corresponding to each plant’s capacity. Which capacity has the largest shadow price? For how large an increase does this value hold?

4.17. Distributing a Product (continued) Consider the solution to the previous exercise.

1. Describe the qualitative pattern in the solution.
2. Use the pattern in (a) to trace the effects of increasing the demands at Tacoma, San Diego and Dallas by 100 simultaneously. How will the shipping schedule change? What will be the change in the optimal total cost?
3. For how much of a change in demand in part (b) will the pattern persist?

**4**.18. Purchasing Components American Electronics Corporation (AEC) is a leading manufacturer of networked computer systems and associated peripherals. Their product line consists of two families, the Desktop (DK) family and the Workstation (WS) family. Within each family, different models are sold, as shown in the table of marketing data. The table below contains estimates of the maximum demand potential in the coming quarter for some of the individual models and for each family. In addition, information is given on minimum demand levels, which represent sales contracts already signed with major distributors.

|  |  |  |  |
| --- | --- | --- | --- |
| Model | Min. demand | Max. demand | Selling price |
| DK-1 | – | 1800 | $3000 |
| DK-2 | 600 | – | 2000 |
| DK-3 | – | 300 | 1500 |
| DK family |  | 3600 |  |
| WS-1 | 500 | – | 1500 |
| WS-2 | 400 | – | 800 |
| WS family |  | 2500 |  |

AEC is a vertically integrated firm, manufacturing many of its key components in its own factories. Recently, AEC headquarters has learned from its Semiconductor Division that the supply of their new CPU chips is quite limited. In addition, the Memory Division has capacity to produce just a finite number of disk drives, even with a two-shift production schedule, and there is industry-wide rationing of memory chips (which AEC purchases externally). This information, in the form of quarterly supply quantities, along with information on the composition of the various products, is summarized in the table below.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |   |   | Usage |   |   |  |
| Component | DK-1 | DK-2 | DK-3 | WS-1 | WS-2 | Supply limit |
| CPU chip | 1 | 1 | 1 | 1 | 1 | 6000 |
| Disk drives | 1 | 2 | 1 | 2 | 1 | 9000 |
| Memory chips | 4 | 2 | 2 | 2 | 1 | 12,000 |

In order to help understand the situation they are facing, planners at AEC have asked you to build a linear programming model. Because AEC follows a program of no layoffs, and because nearly all production costs are fixed, the model should maximize revenue for the coming quarter, subject to supply and demand constraints.

* + 1. Determine the optimal product mix. What are the maximum revenue and the corresponding mix?
		2. Describe the qualitative pattern in the solution.
		3. Use the pattern in (b) to trace the effects of increasing the Disk Drive Supply capacity by 150. How will the product quantities change? How will the optimal profit change?
		4. For how much of a change in the Disk Drive Supply capacity will the pattern persist?

4.19. Production Planning with Environmental Constraints You are the Operations Manager of Lovejoy Chemicals, Inc., which produces five products in a common production facility that will be subject to proposed Environmental Protection Agency (EPA) limits on particulate emissions. For each product, Lovejoy’s sales potentials (demand levels that Lovejoy can capture) are expected to remain relatively flat for at least the next five years. Relevant data for each product are as follows. (Note: T denotes tons.)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Product | Sales (T/year) | Variable costs ($/T) | Revenues ($/T) | Tons Emitted per T produced |
| A | 2000 | 700 | 1000 | 0.001 |
| B | 1600 | 600 | 800 | 0.0025 |
| C | 1000 | 1000 | 1500 | 0.03 |
| D | 1000 | 1600 | 2000 | 0.04 |
| E | 600 | 1300 | 1700 | 0.025 |

Your production facility rotates through the product line because it is capable of producing only one product at a time. The production rates differ for the products due to processing needs. It takes 0.3 hours to make one ton of A, 0.5 hours for B, and one hour each to make a ton of C, D, or E. The facility can be operated up to 4000 hours each year.

The EPA is proposing a “bubble policy” for your industry. In this form of regulation, imagine that a bubble encloses the manufacturing facility, and only total particulates that escape the bubble are regulated. This sort of policy replaces historical attempts by the EPA to micromanage emissions within a firm, and it allows Lovejoy to make any changes it wishes, provided the total particulate emissions from its facility are kept below certain limits. The current proposal is to phase-in strict particulate emissions limits over the next five years. These limits on total particulate emissions are shown in the table below.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Year | 1 | 2 | 3 | 4 | 5 |
| Allowable emissions (T/year)  | unlimited | 80 | 60 | 40 | 20 |

One strategy for satisfying these regulations is to adjust the product mix, cutting back on production of some products if necessary. Lovejoy wishes to explore this strategy before contemplating the addition of new equipment.

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* + - * 1. Determine the maximum profit Lovejoy can achieve from its product line in the coming year (Year 1). For each future year (Years 2–5), as emissions limits are imposed and tightened, what will Lovejoy’s profits be?
				2. Describe the qualitative pattern in the solution.
				3. Suppose the EPA were to issue transferable emissions rights to firms in the industry. Use the pattern in (b) to determine how much Lovejoy would be willing to pay in Year 4 to be allowed to emit one extra ton of particulates above the proposed limit.

4.20. Coffee Blending and Sales Hill-O-Beans Coffee Company blends four component beans into three final blends of coffee: One is sold to luxury hotels, another to restaurants, and the third to supermarkets for store-label brands. The company has four reliable bean supplies: Argentine Abundo, Peruvian Colmado, Brazilian Maximo, and Chilean Saboro. The table below summarizes the very precise recipes for the final coffee blends, the cost and availability information for the four components, and the wholesale price per pound of the final blends. The percentages indicate the fraction of each component to be used in each blend.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Component  | Hotel | Restaurant | Market | Cost per pound | Pounds available |
| Abundo | 20% | 35% | 10% | $0.60 | 40,000 |
| Colmado | 40% | 15% | 35% | $0.80 | 25,000 |
| Maximo | 15% | 20% | 40% | $0.55 | 20,000 |
| Saboro | 25% | 30% | 15% | $0.70 | 45,000 |
| Wholesale price |  |  |  |  |  |
|  per pound | $1.25 | $1.50 | $1.40 |  |  |

The processor’s plant can handle no more than 100,000 pounds per week, and Hill-O-Beans would like to operate at capacity. There is no problem in selling the final blends, although the Marketing Department requires minimum production levels of 10,000, 25,000, and 30,000 pounds, respectively, for the hotel, restaurant, and market blends.

In order to maximize weekly profit, how many pounds of each component should be purchased?

What is the shadow price (from the Sensitivity Report) on the availability of Maximo beans?

How much (per pound) should Hill-O-Beans be willing to pay for additional pounds of Maximo beans in order to raise total profit?

4.21. Make/Buy Planning The CammTex Fabric Mill is in the process of deciding on a production schedule. It wishes to know how to weave the various fabrics it will produce during the coming quarter. The sales department has confirmed orders for each of the 15 fabrics in the product line. These quarterly demands are given in the table below. Also tabulated is the variable cost for each fabric. The mill operates continuously during the quarter: 13 weeks, 7 days a week, and 24 hours a day.

CammTex uses two types of looms: general and regular. General looms can make all fabrics, and they are the only looms that can weave certain fabrics such as plaids. The production rate for each fabric on each type of loom is also given in the table. (If the production rate is zero, the fabric cannot be woven on that type of loom.) CammTex has 90 regular looms and 15 general looms.

Fabrics woven at CammTex proceed to the finishing department in the mill and then are sold. Any fabrics that are not woven in the mill because of limited capacity will be subcontracted to an outside producer and sold at the selling price. The cost of purchasing from the subcontractor is also given in the table.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|   | Demand | General | Regular | Mill | Sub. |
| Fabric | (yd) | (yd/hr) | (yd/hr) | ($/yd) | ($/yd) |
| 1 | 16,000 | 4.65 | 0 | 0.66 | 0.80 |
| 2 | 52,000 | 4.65 | 0 | 0.56 | 0.70 |
| 3 | 45,000 | 4.65 | 0 | 0.66 | 0.85 |
| 4 | 22,000 | 4.65 | 0 | 0.55 | 0.70 |
| 5 | 76,000 | 4.18 | 4.22 | 0.61 | 0.70 |
| 6 | 110,000 | 3.81 | 3.84 | 0.62 | 0.80 |
| 7 | 122,000 | 5.18 | 5.22 | 0.66 | 0.80 |
| 8 | 62,000 | 5.23 | 5.29 | 0.49 | 0.60 |
| 9 | 8000 | 5.23 | 5.29 | 0.50 | 0.70 |
| 10 | 69,000 | 5.23 | 5.29 | 0.44 | 0.60 |
| 11 | 70,000 | 3.73 | 3.29 | 0.61 | 0.80 |
| 12 | 82,000 | 4.18 | 4.22 | 0.57 | 0.75 |
| 13 | 10,000 | 4.40 | 4.44 | 0.54 | 0.70 |
| 14 | 360,000 | 5.23 | 5.29 | 0.31 | 0.45 |
| 15 | 62,000 | 4.18 | 4.22 | 0.50 | 0.70 |

1. What is minimum total cost of production and purchasing for CammTex?
2. Construct a table showing how the optimal total cost varies with the number of general looms available, in the range 10–18.
3. Describe the qualitative pattern in the solution of part (a).
4. Use the pattern in (c) to trace the effect on the optimal cost of increasing demand for fabric 6 by 100 yards.
5. Repeat (d) for fabric 5.
6. Repeat (d) for fabric 4.

4.22. Cash Planning A startup investment project needs money to cover its cash flow needs. The cash income and expenditures for the period January through April are as follows.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | Jan | Feb | Mar | Apr | Total |
| Cash flow ($000) | −150 | −450 | 500 | 250 | 150 |

At the beginning of May all excess cash will be paid out to investors. There are two ways to finance the project. One is the possibility of taking out a long-term loan at the beginning of January. The interest on this loan is 1 percent per month, payable on the first of the month for the next three months. This loan can be as large as $400,000; the principal is due April 1; and no prepayment is permitted. The alternative is a short-term loan that can be taken out at the beginning of each month. This loan must be paid back at the beginning of the following month with 1.2 percent interest. A maximum of $300,000 may be used for this short-term loan in any month. In addition, there is the possibility of investing in a money-market fund at the start of each month. This fund will pay 0.7 percent interest at the beginning of the following month.

Assume the following about the timing of cash flows.

* For months in which there is a net cash deficit, there must be sufficient funds on hand at the start of the month to cover the net outflow.
* For months in which there is a net cash surplus, the net inflow cannot be used until the end of the month (i.e., the start of the next month).
	1. What is the maximum amount that can be returned to investors?
	2. Describe the pattern in the optimal solution.
	3. Use the pattern in (b) to derive the cost of funds for each month in the planning period. That is, if there is a $1000 change in the cash flows for any month, what would be the dollar change in the amount returned to investors?
	4. Show how the shadow prices in the Sensitivity Report can be used to confirm the answers in (c).