Online Appendix for Carry-Along Trade

Andrew B. Bernard*
Emily J. Blanchard†
Ilke Van Beveren‡
Hylke Vandenbussche§

September 2017

1 Theory Appendix

This appendix offers more detailed treatment of the theoretical exercise in the main body of the paper, including two extensions. To minimize the need to cross reference with the main text, the basic set-up is repeated here, together with the formalized results. The extensions consider the possibilty of diseconomies of scope in sourcing and quantity cannibalistic preferences. See also the working paper version of this paper, Bernard et al. (2012), for further discussion of these and additional extensions, including the possibility of a fixed cost of sourcing.

1.1 The Basic Model

Firms. As in the main text, the model features a continuum of atomistic firms, indexed by $j \in [0, 1]$, each of which may provide multiple unique products to the market. Each firm has a 'core' product indexed by $i = 0$; remaining products are indexed by their distance from the core according to $i \in (0, k(j)]$, where $k(j)$ also denotes the (endogenous) equilibrium scope of products provided to the market by firm j. Each good in the market is uniquely identified by the product-firm pair, ij.

Firms can serve the market by producing goods in-house, by sourcing from arms-length suppliers, or some combination of the two: producing some goods and sourcing others. Firm j's marginal cost of producing product i in-house is given by $c(j, i)$ where: $\frac{\partial c(j,i)}{\partial i} > 0$, $\frac{\partial c(j,i)}{\partial j} > 0$ and $c(\cdot) \in C^1$. The sourcing technology has constant returns to both scale and scope and is given by $\hat{c}(j, i) = \hat{c}(j)\ \forall i, j$. Notice that each product will be either produced in-house or sourced from arms-length suppliers, but not both.

All firms face a product-specific per-unit distribution cost, $\delta(i, j)$, which is independent of whether a good is made in-house or sourced from an arms-length supplier, constant with respect to quantity within a product, and additive with the direct (constant) marginal cost of producing or sourcing the product. The distribution cost increases with a product's distance from the firm's core product so that $\delta'(i) > 0$. From a modeling perspective, $\delta(\cdot)$ ensures that firms do not expand scope infinitely.

*100 Tuck Hall, Hanover, NH 03755, USA, *tel:* +1 603 646 0302, *email:* andrew.b.bernard@tuck.dartmouth.edu
†100 Tuck Hall, Hanover, NH 03755, USA, *tel:* +1 603 646 8962, *email:* emily.j.blanchard@tuck.dartmouth.edu
‡Korte Nieuwstraat 33, 2000 Antwerp, Belgium, *tel:* +32 3 201 18 33, *email:* ilke.vanbeveren@kuleuven.be
§Naamsestraat 69, 2000 Leuven, Belgium, *tel*: +32 16 32 69 20, *email*: hylke.vandenbussche@kuleuven.be

Customers. A mass of identical customers has non-degenerate preferences over differentiated goods, where goods are both firm (j) and product (i) specific. Customers care only about the firm-product pair and do not differentiate between goods produced in-house versus sourced from a supplier. Income effects are absorbed in total market quantity, Q, which is taken as given by atomistic firms.

Each firm j faces inverse market demand for product ij of the form:

$$p_{ij} \equiv p_j(q_{ij}, Q_j, Q),$$

where $\frac{dp_{ij}}{dq_{ij}} < 0$, $\frac{\partial p_{ij}}{\partial Q_j} \leq 0$, and $p_j(\cdot) \in C^1$.

1.2 Firm Behavior and Selection

In equilibrium, each firm makes three decisions (1) *entry:* whether to enter the market or not, (2) *product scope:* how many products to sell, and at what prices and quantities, and (3) *make-or-source:* which products to make in-house and which to source from upstream suppliers. We consider each decision in turn, beginning with the last.

Make-or-Source Decision. For each product i it sells, a given firm j decides whether to produce in-house at constant marginal cost $c(j,i)$, or to source from a supplier at constant marginal cost $\hat{c}(j)$. Customers make no distinction between in-house produced goods and CAT goods, distribution costs are independent of the make-or-source decision and the production and sourcing cost structures are independent of total product scope. Thus, the make-or-source decision for any given product i is simply a choice of the lowest marginal cost means of procurement. Because in-house production exhibits decreasing returns to scope whereas the sourcing technology has constant returns to scope, it is immediate that every firm will have a unique make-or-source threshold that delineates produced goods from sourced goods. This threshold, which we denote by $\hat{k}(j)$, is independent of total firm scope and is defined implicitly by:

$$c(j, \hat{k}(j)) = \hat{c}(j). \tag{1}$$

Lemma 1. *In equilibrium, each firm j will produce in-house all products $i \leq \hat{k}(j)$ and will source the remaining products $(i > \hat{k}(j))$ from arms-length suppliers.*

For products closest to a firm j's core production competency ($i \leq \hat{k}(j)$), the firm has an in-house cost advantage relative to the pool of homogeneous arms-length suppliers; for products farther from firm j's core competency, the marginal cost of in-house production rises, eventually reaching a point at which the marginal cost of buying from an arms-length supplier is less than the cost of producing in-house.

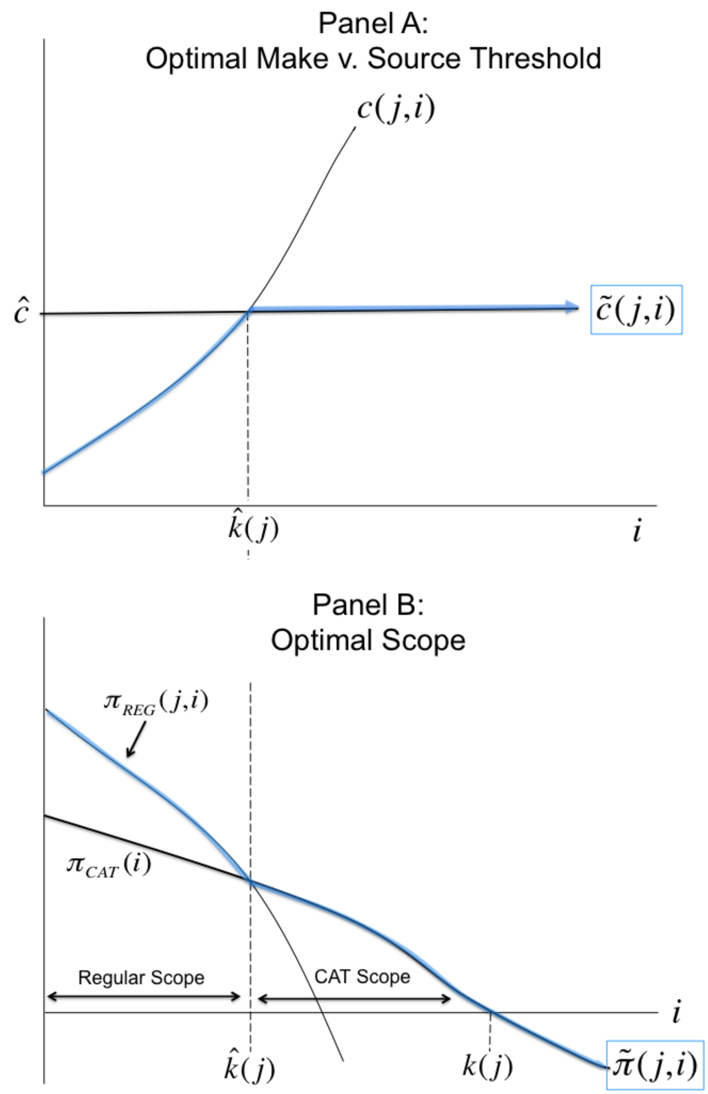
The optimal cost function for each firm-product pair may then be written $\tilde{c}(j,i) \equiv \min\{c(j,i), \hat{c}(j)\}$ and is simply the lower envelope of the in-house and CAT-sourced cost curves over the support of products. Notice that this minimized cost function is strictly increasing in i until $i \geq \hat{k}(j)$ and constant thereafter, as shown in Panel A of Figure 1.

Optimal Scope. From here we can define the profit function for any given firm-product pair:

$$\tilde{\pi}(j,i) = \max_{q_{ij}} [p(q_{ij}, Q_j, Q) - \tilde{c}(j,i) - \delta(i,j)] q_{ij}. \tag{2}$$

Notice that, one, the firm-product profit function embodies the optimal make-or-source decision, $\hat{k}(j)$, through $\tilde{c}(\cdot)$; and two, more remote varieties are less profitable $\frac{\partial \tilde{\pi}(j,i)}{\partial i} < 0$ due to the symmetric demand structure and decreasing returns to scope. We can summarize the *set* of products sold by a firm as the total product *scope*, k, where the firm will sell all products $i \leq k$, and no others.

Figure 1: Firm j's Optimal Make-or-Source Decision for Each Product i



Aggregating a firm's profit function for each product over all offered products $i \in [0, k]$, firm j's total return as a function of scope and productivity is defined as:

$$\Pi(j, k) \equiv \int_0^k \tilde{\pi}(j, i)di. \tag{3}$$

Taking the derivative with respect to k yields the first order condition that defines implicitly the firm's optimal scope, $k(j)$. The FOC implicitly defines firm j's optimal scope, $k(j)$:

$$\frac{\partial \Pi(j, k)}{\partial k} = \underbrace{\int_0^{k(j)} \frac{\partial \tilde{\pi}(j, i)}{\partial k} di}_{\text{infra-marginal spillovers}} + \tilde{\pi}(j, k(j)) = 0. \tag{4}$$

That is, if the profitability of an infra-marginal product is independent of the firm's total product scope, the firm will optimally continue adding products until the last product added yields zero profit. More generally, the firm will continue to add products until the marginal cost (benefit) if infra-marginal spillovers are just equal to the marginal profit (losses) earned by the most remote (k^{th}) variety.

The second order condition is then just that the profitability of the marginal (k^{th}) product is decreasing as scope rises $\frac{\partial^2 \Pi(j,k)}{\partial k^2} \leq 0$. We assume that diseconomies of scope in distribution are sufficiently strong to ensure this second order condition for all firms. (The importance of assuming $\delta'(i) > 0$ is evident. Without it, in the absence of (negative) intra-marginal spillovers, there is no reason that an interior equilibrium necessarily should exist for firms engaged in CAT at the margin.)

Panels A and B in Figure 1 summarize. For any given product i, firm j can choose between producing in-house at constant marginal cost $c(i,j)$ or sourcing a carry-along product from an upstream supplier at cost $\hat{c(j)}$. At $\hat{k}(j)$ the two costs are equal, as shown in Panel A. Below this threshold, the cost is lower (and profit is higher) via regular production; above it, marginal cost is lower and profit is greater via Carry-Along Trade. In Panel B, we label the (hypothetical) firm-product profit function from regular production by $\pi_{REG}(i,j)$, while the CAT profit function is denoted by $\pi_{CAT}(i,j)$.[1] The firm's profit for each variety given the optimal make-or-source decision, $\tilde{\pi}(j,i)$, is then simply the upper envelope of the two potential profit functions under either provisioning strategy, producing in-house (π_{REG}) or sourcing from an upstream supplier (π_{CAT}). In the absence of infra-marginal spillovers, the firm will optimally continue to add products until the profit of the marginal product is zero, i.e. $\tilde{\pi}(j,i) = 0$, which is labeled $k(j)$ in Panel B. More generally with intra-firm spillovers, the profit of the marginal variety is weighed against the infra-marginal spillovers.

Entry. Firms will enter a market if their realized profit is sufficient to cover an exogenous, homogeneous fixed cost of entry, F. There is no firm-level idiosyncratic component to profit apart from firm productivity, so only sufficiently productive firms will enter the market. Defining each firm's total profit to be the sum of returns to each product given the optimal scope, $\tilde{\Pi}(j) \equiv \int_0^{k(j)} \tilde{\pi}(j,i)di$, the least productive firm to enter the market, firm $\bar{j}$, is given implicitly by $\tilde{\Pi}(\bar{j}) = F$.

In the model, equilibrium product scope is equivalent to the *mass* of products supplied to the market by a firm. In equilibrium, firm j sells a mass of produced products, $\hat{k}(j)$ (the set $i \in [0, \hat{k}(j)]$), to the market, with a total product mass of $k(j)$ (the set $i \in [0, k(j)]$). The mass of products sourced via Carry-Along Trade is then simply $k(j) - \hat{k}(j)$ (the set $i \in (\hat{k}(j), k(j))$). In what follows, we refer to product scope recognizing that it is shorthand for both the mass and the set of products offered to the market.

1.3 Equilbrium Existence and Uniqueness

Industry equilibrium is characterized by (i) the set of active firms, (ii) the set of products produced by each firm and (iii) the equilibrium price and quantity for each (firm specific) product sold to the market; together, $(i) - (iii)$ determine the (fixed point) equilibrium aggregate industry output, Q.

Beginning with (iii), the equilibrium price-quantity pair $(q^o_{ij}(Q), p^o_{ij}(Q))$ for any (firm specific) product ij is simply that which maximizes firm $j's$ profit. If there are no supply or demand side spillovers across products within the firm, and if demand, sourcing costs, and distribution costs are symmetric as assumed, the profit maximizing price and quantity choices are also independent across products within (and across) firms. More generally, the optimal quantity of product i sold by firm j

[1] Where $\pi_{REG}(i,j) \equiv \max_{q_{ij}}[p(q_{ij}) - c(j,i) - \delta(j,i)]q_{ij}$ and $\pi_{CAT}(i,j) \equiv \max_{q_{ij}}[p(q_{ij}) - \hat{c} - \delta(j,i)]q_{ij}$. Note that $\pi_{REG}(i,j)$ is necessarily steeper than $\pi_{CAT}(i,j)$ for all $i \leq \hat{k}(j)$ as drawn: regular production exhibits diseconomies of scope in production and distribution, whereas CAT has diseconomies only in distribution. $\frac{\partial \pi_{REG}(j,i)}{\partial i} = -q^{REG}_{ij}\left(\frac{\partial c(j,i)}{\partial i} + \delta'(j,i)\right) < \frac{\partial \pi_{CAT}(j,i)}{\partial i} = -q^{CAT}_{ij}\delta'(j,i) < 0$, given that $\forall i \leq \hat{k}(j)$, $q^{REG}_{ji} \geq q^{CAT}_{ji} > 0$.

depends on the net profit from sales of product i and potential positive or negative spillover effects on the set of the firm's other products (below,$/i$) ; formally:

$$q_{ij}^o(Q) \equiv \arg\max_{q_{ij}} [p(q_{ij}, Q_j, k(j), Q) - \tilde{c}(j,i) - \delta(j,i)]q_{ij} + \int_{/i} \pi_{hj}(j,h;Q,q_{ij})dh. \quad (5)$$

This optimal quantity will be unique under our assumptions of (strictly) downward sloping own-demand $\frac{dp(\cdot)}{dq_{ji}} < 0$ and constant returns to scale within product $(\frac{d\tilde{c}(j,i)}{dq_{ij}} = 0)$, as long as intra-firm spillovers, captured in the last term, are not both positive and too large to swamp the typical downward slope of demand. Existence of q_{ij}^o requires that there exists some positive, finite $\hat{q}_{ij}$ s.t. $p'(q_{ij})\hat{q}_{ij} + p(q_{ij})|_{q=\hat{q}} + \frac{d[\int_{/i} \pi_{jh}(j,h;Q,q_{ij})dh]}{dq_{ij}} \leq \tilde{c}(j,i) + \delta(i,j)$ (i.e. marginal revenue, net of any spillovers, eventually falls below marginal cost), which can be assured by sufficiently large production, sourcing, or distribution costs. The equilibrium price for each variety is then given by inverse demand, s.t. $p_{ji}^o(Q) \equiv p(q_{ji}^o, Q)$.

Optimal scope (ii): Given the existence of a unique profit maximizing price-quantity pair for every firm-product product ij given Q, the equilibrium scope of products $k(j;Q)$ produced by any given firm j is defined implicitly by the first order condition:

$$\frac{\partial \Pi(j,k)}{\partial k} = \underbrace{\int_0^k \frac{\partial \tilde{\pi}(j,i)}{\partial k} di}_{\equiv S(j,k)} + \tilde{\pi}(j,k) = 0. \quad (6)$$

Where we use $S(j,k)$ to represent the inframarginal spillovers on firm j's total profit from adding a k-th product. Uniqueness of the optimal scope decision for each firm j is ensured by the second order condition:

$$\frac{\partial^2 \Pi(j,k)}{\partial k^2} = \frac{\partial S(j,k)}{\partial k} + \frac{\partial \tilde{\pi}(j,k)}{\partial k} = \frac{\partial S(j,k)}{\partial k} - q_{ji}^o \underbrace{\left(\frac{\partial \tilde{c}(j,i)}{\partial i} + \delta'(i) \right)}_{+} \Bigg|_{i=k} \leq 0,$$

which obtains as long as the diseconomies of scope are sufficiently strong relative to any potential positive infra-marginal spillovers. (In the absence of positive spillovers, any $\delta'(i) > 0$ is sufficient to ensure uniqueness). An interior optimal scope decision $k(j,Q)$ exists as long as there exists some finite scope $\hat{i}$ such that $\tilde{\pi}(j,\hat{i};Q) + S(j,\hat{i};Q) \leq 0 \forall j$. Sufficiently large and increasing distribution costs, decreasing marginal returns to demand-scope complementarity, or quantity cannibalization would all suffice.[2]

Entry: The set of firms in the market is the set $\Omega(Q) \equiv [0, \bar{j}(Q)]$ where $\bar{j}(Q)$ denotes the least productive firm to enter the market and is given implicitly by $\tilde{\Pi}(\bar{j}(Q)) = F$ where $\tilde{\Pi}(j,Q) \equiv \int_0^{k(j)} \tilde{\pi}(j,i;Q)di$. Uniqueness and existence of $\bar{j}(Q)$ are ensured by the assumptions that $\frac{\partial c(j,i)}{\partial j} < 0$ and sufficiently large F to exclude wholesalers.

Aggregate equilibrium quantity Q is then simply the fixed point solution to $(i)-(iii)$: $q_{ij}^o(Q), p_{ij}^o(Q)$ $\forall j \in \Omega(Q), i \leq k(j;Q)$, where $Q \equiv \int_0^{\bar{j}(Q)} \int_0^{k(j;Q)} q_{ij}^o(Q)didj$.

1.4 Firm Behaviour and Selection in a Benchmark Case

The model's predictions for the relationship between firm productivity, regular product scope, CAT scope and (total) product scope follow directly. In characterizing the relationship between firm

[2] Note that with super-convex preferences (Mrazova and Neary (2011)), one needs to introduce a small positive fixed cost for adding each variety to ensure that firms do not expand scope infinitely. Such a fixed cost would complicate exposition but otherwise would not qualitatively change the results.

productivity and the extent of produced and Carry-Along Trade across firms, we begin with a benchmark case in which demand, sourcing, and distribution costs are symmetric across firms. In the main text of the paper, we relax this assumption to allow for ex-ante heterogeneity in firm demand, distribution, or sourcing costs.

Productivity and Regular Product Scope. Regular product scope is increasing in firm productivity. Given that the most productive (lowest j) firms have the lowest marginal cost of in-house production for any given product i, while the marginal cost of carry-along products is identical and constant across firms and products, it is immediate that the least productive firms (highest j) will be the first to switch from producing in-house to sourcing via CAT:

Lemma 2. *$\hat{k}(j)$ is strictly decreasing in j.*

Proof. From the implicit definition of $\hat{k}(j)$ in equation 1 and the implicit function theorem: $\hat{k}'(j) = -\frac{\frac{\partial c(j,i)}{\partial j}}{\frac{\partial c(j,i)}{\partial i}} < 0$. ⋄

Productivity and (total) Product Scope. Optimal (total) product scope is also increasing with firm productivity, but weakly. Among firms that produce everything in-house (i.e. those for which $\hat{k}(j) \geq k(j)$), the profit of the each additional (produced) product is strictly increasing with firm productivity via a marginal cost advantage. In contrast, among firms engaged in Carry-Along Trade total product scope is independent of productivity *in the benchmark case*, since demand, sourcing, and distribution costs are (here) assumed to be symmetric across firms. Thus, for any firm i, $\tilde{\pi}_{CAT}(i) = \max_{q_{ji}}(p(q_{ji}) - \hat{c} - \delta(i))q_{ji}$ and the equilibrium (total) product scope will be the *same* for all firms engaged in Carry-Along Trade, regardless of their initial productivity. Formally:

Lemma 3. *$k(j)$ is weakly decreasing in j.*

(i) For firms that produce only in-house ($\forall j$ s.t. $\hat{k}(j) \geq k(j)$): total product scope is strictly increasing with firm productivity: $k'(j) < 0$.

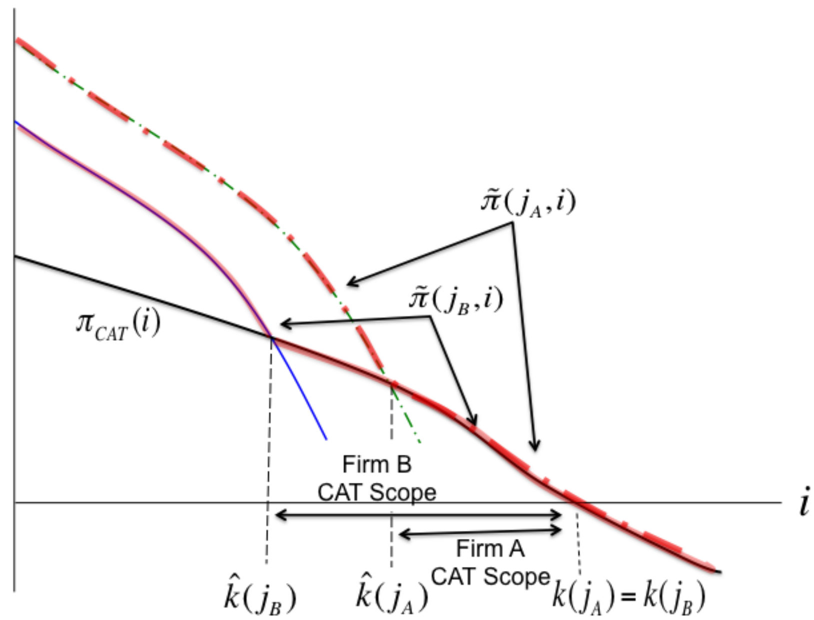
(ii) For firms that engage in CAT ($\forall j$ s.t $\hat{k}(j) < k(j)$), total product scope is independent of firm productivity: $k(j) \equiv k_{CAT}$ and thus $k'(j) = 0$.

Proof. From the implicit definition of $k(j)$ in equation 4 and the envelope condition for equation 2: $k'(j) = -\left.\frac{\frac{\partial \tilde{\pi}(i,j)}{\partial j}}{\frac{\partial \tilde{\pi}(i,j)}{\partial i}}\right|_{i=k} = -\left.\frac{\frac{\partial \tilde{c}(i,j)}{\partial j}}{\frac{\partial \tilde{c}(i,j)}{\partial i} + \delta'(i)}\right|_{i=k}$. For firms that produce only in-house, the marginal product k is produced in-house, so that $\tilde{c}(j, i = k) = c(j, k)$ and thus $k'(j) = -\left.\frac{\frac{\partial c(i,j)}{\partial j}}{\frac{\partial c(i,j)}{\partial i} + \delta'(i)}\right|_{i=k} < 0$. For CAT firms, the k^{th} product is sourced from arm-length upstream suppliers at constant marginal cost $\hat{c}$ and so $k'(j) = -\left.\frac{0}{\delta'(i)}\right|_{i=k} = 0$. ⋄

Productivity and CAT Scope. Comparing two firms with different core productivities, the more productive (lower j) firm will produce a greater range of products in-house and thus will highest in-house production costs that will source – and thus engage in CAT – the most. Conversely, the most productive firms have the highest opportunity cost of Carry-Along Trade relative to in-house production and will use it the least.

Figure 2 illustrates the relationship between firm productivity and product scope for regular products, CAT products and all products in equilibrium. Firm A has greater core efficiency ($j_A < j_B$) and thus greater profits than Firm B for any given product i produced in-house. This means that Firm A will find it profitable to produce a greater range of products in-house than Firm B, so that Firm A's regular product scope is greater than Firm B's: $\hat{k}(j_A) > \hat{k}(j_B)$. However, since both firms have identical sourcing technologies, their marginal profit from any given CAT product $i > \hat{k}(j_A) > \hat{k}(j_B)$ will be the same. If both firms engage in any CAT activity, they will have the

Figure 2: High (A) versus Low (B) Productivity Firms



same optimal total product scope: $k(j_A) = k(j_B)$. CAT product scope is then necessarily smaller for the more productive firm, $k(j_A) - \hat{k}(j_A) < k(j_B) - \hat{k}(j_B)$.

Formalizing the central results of in this baseline scenario yields the following proposition:[3]

Proposition 1. *If firms have equal access to a constant sourcing technology and demand is symmetric and independent across firms and products:*

i) Regular product scope, $\hat{k}(j)$, is increasing in productivity; firm productivity and produced scope are super-modular in firm payoffs;

ii) Total product scope, $k(j)$, is identical for all firms engaged in carry-along-trade; firm productivity and total product scope are modular in firm payoffs for firms with at least one CAT product (for whom $\hat{k}(j) \leq k(j)$).

iii) Sourced (CAT) product scope, $k(j) - \hat{k}(j)$, is decreasing in firm productivity; firm productivity and Carry-Along Trade are sub-modular in firm payoffs.

Proof. Parts (i) and (ii) follow directly from lemmas 2 and 3 respectively. Part (iii) follows from (i) and (ii).

Furthermore:

Corollary 1. *Sales volumes are uniformly higher for regular products than for carry-along products. Within regular products, sales volumes will be highest for the most productive firms and for products closest to a firm's core product. Within CAT products, sales volumes are highest for products closest to a firms' core product, but otherwise independent of firm productivity.*

Proof. Given symmetric demand, the within-product scale, q^o_{ji} is inversely related to product-specific marginal cost, but otherwise symmetric across firms. The marginal cost of bringing a

[3]We confirm in Lemma 4 that diseconomies of scope in sourcing generate the same basic prediction as Proposition 1, with the caveat that (ii) becomes a strict inequality (i.e. $k'(j) > 0$) and (iii) becomes weak (i.e. $k'(j) - \hat{k}'(j) \leq 0$).

product to the market, $\tilde{c}(j,i)+\delta(i)$, is by definition lower for all products made in-house than for CAT products within a given firm j. Among products made in-house, costs are strictly higher for less productive firms for any given product distance i, and are decreasing in i for any given firm j. Within products made in-house, diseconomies of scope arise in both direct production cost and distribution, whereas only the distribution cost component obtains for CAT products. $\diamond$

The predictions encompassed in Proposition 1 and Corollary 1 are at odds with what we observe in the data. The baseline symmetric no-spillovers model is, however, consistent with the data in predicting that sales volumes for in-house production and the scope of products made in-house are higher for more productive firm.

1.5 Diseconomies of Scope in Sourcing

This section demonstrates the robustness of the key results in Proposition 1 to an environment with diseconomies of scope in sourcing. We assume that the cost of sourcing each marginal product increases with the number (mass) of products that the firm has already sourced, but is otherwise independent of i (the distance from the core product).[4] This framework is consistent with an environment in which a firm is able to reach lower marginal cost suppliers initially, but must turn to increasingly higher cost suppliers as it continues to expand the set of sourced products. With diseconomies of scope in sourcing, we no longer need to assume diseconomies of scope in distribution to ensure existence; thus we set $\delta(i)=0 \quad \forall i,j$ to reduce notation. All other modeling assumptions are exactly as in the baseline (symmetric no-spillovers) case.

This revised framework warrants a subtle expositional shift. As before, we can define precisely the marginal cost of providing the k^{th} product to the market (which remains independent of total product scope), but whether that product is produced in-house or sourced depends on the total product scope.[5] For this reason and because distance from the core is irrelevant to sourcing costs, we now write the minimum value cost function, $\tilde{c}(\cdot)$ and the maximum value profit function, $\tilde{\pi}(\cdot)$ in terms of k rather than i.

As in the baseline case, sourcing technology is assumed to be common across all firms. Denote the marginal cost to any firm j of sourcing its k^{th} sourced product, k^s, by $\hat{c}(k^s)$. Diseconomies of scope in sourcing imply $\hat{c}'(k^s)>0$. For any given product scope, a cost minimizing firm will: (i) make in-house those products closest to the core and source those more distant;[6] (ii) set the make-or-source threshold so that the marginal cost of sourcing an additional product will equal the marginal cost of producing the next product in-house. Together, (i) and (ii) yield the implicit definition of firm j's optimal make-or-source threshold as a function of total product scope; $\hat{k}(j,k)$ is the solution to:[7]

$$c(j,\hat{k}) \leq \hat{c}(k-\hat{k}) \text{s.t.} \hat{k} \leq k, \tag{7}$$

which holds with equality if any products are sourced via CAT in equilibrium. The marginal cost

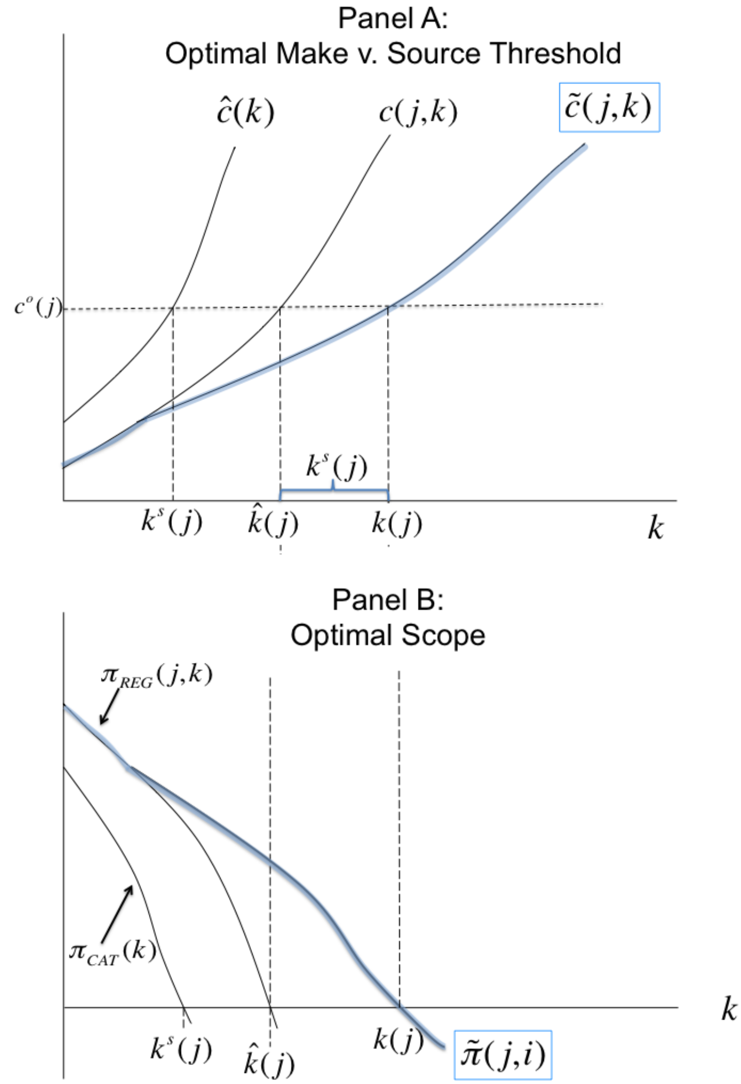
[4] This assumption implies that the notion of a firm's "core competency" extends to production but not sourcing. Adopting an environment in which sourcing cost increases with i, distance from the core product, is a trivial extension of the baseline case. The make or source decision remains independent of total firm scope and is unique as long as the sourcing cost schedule $\hat{c}(i)$ crosses the in-house production cost schedule $c(j,i)$ only once from above in $\{c,i\}$ space, so that $\tilde{c}(j,i)$ remains the lower envelope of the two cost schedules as in Panel A of Figure 1.

[5] In equilibrium, we can then define the *set* of products produced in house as $i \in [0,\hat{k}(j)]$, while products $i \in (\hat{k}(j),k(j)$ are sourced. Out of equilibrium, the mapping from scope to set varies according to cost minimization as described below.

[6] This follows directly from the assumption that in-house production costs rise as products are more distant from the core while the sourcing technology is independent of distance.

[7] As in the baseline case, we continue to assume that the cost of producing in house is lower than the cost of sourcing for product $i=0$, which rules out pure wholesalers.

Figure 3: Firm j's Optimization Problem with Diseconomies of Scope in Sourcing



at which firm j can provide the k^{th} product to the market is then:

$$\tilde{c}(j,k) = c(j, \hat{k}(j,k)) \leq \hat{c}(k - \hat{k}(j,k)). \tag{8}$$

Intuitively, as total scope (k) increases, the firm will increase both regular and CAT product scope subject to the cost minimization solution in (7)-(8). Graphically, $\tilde{c}(j,k)$ is the horizontal sum of the in-house and sourced marginal cost curves in $\{k, c\}$ space as shown in Panel A of Figure 3. From existing assumptions over production and sourcing cost structures, $\frac{\partial \tilde{c}(j,k)}{\partial k} > 0$ and $\frac{\partial \tilde{c}(j,k)}{\partial j} > 0$.

The profit function for firm j's k^{th} product is:

$$\tilde{\pi}(j,k) = \max_{q_{jk}} [p(q_{jk}) - \tilde{c}(j,k)] q_{jk}. \tag{9}$$

Graphically, the profit function for product k is the horizontal sum (rather than the upper envelope, as in the baseline case) of the in-house produced and CAT-sourced profit functions, as shown in

Panel B of Figure 3.[8] Aggregating over the set of offered products, firm j's total return as a function of scope and productivity is then:

$$\Pi(j,k^o) \equiv \int_0^{k^o} \tilde{\pi}(j,k)dk. \tag{10}$$

Taking the derivative with respect to k yields the first order condition that defines implicitly the firm's optimal scope, $k(j)$. As before, in the absence of infra-marginal spillovers across products, the firm will optimally add products until the last product added yields zero profit. Firm j's optimal total scope is given implicitly by:

$$\frac{\partial \Pi(j,k^o)}{\partial k^o} = \tilde{\pi}(j,k(j)) = 0. \tag{11}$$

Note that $\tilde{\pi}(j,k(j)) = \pi_{REG}(j,\hat{k}(j)) = \pi_{CAT}(k(j)-\hat{k}(j)) = 0$; at the optimal scope, the marginal profit of adding one more product via in-house production or sourcing is also (by definition) zero.

We can now demonstrate the robustness of the results in Proposition 1 to this environment with diseconomies of scope in sourcing.

Lemma 4. *For the augmented model in which firms have equal access to a common 'sourcing' technology with diseconomies of scope, $\hat{c}(k^s)$ where $\hat{c}'(k^s) > 0$:*
i Total product scope, $k(j)$, is increasing in firm productivity;
ii CAT product scope, $k(j) - \hat{k}(j)$, is (weakly) decreasing in firm productivity; and
iii Regular product scope, $\hat{k}(j)$, is increasing in firm productivity.

Proof. [*i*] Total product scope is increasing in firm productivity if $dk(j)/dj < 0$. Taking the total derivative of the implicit definition of $k(j)$ in (11) yields:

$$\frac{\partial \tilde{\pi}(j,k)}{\partial k} dk^o + \frac{\partial \tilde{\pi}(j,k)}{\partial j} dj \quad = \quad 0 \tag{12}$$

$$\Rightarrow \quad \frac{dk(j)}{dj} = - \left. \frac{\partial \tilde{\pi}(j,k)/\partial j}{\partial \tilde{\pi}(j,k)/\partial k} \right|_{k=k^o} = - \left. \frac{\partial \tilde{c}(j,k)/\partial j}{\partial \tilde{c}(j,k)/\partial k} \right|_{k=k^o} < 0 \qquad \diamond(13)$$

[*ii*] CAT product scope is weakly decreasing in firm productivity if $dk^s(j)/dj \geq 0$, where $k^s(j) \equiv k(j) - \hat{k}(j)$. (8), the marginal cost of the last product provided to the market pins down the optimal CAT product scope according to: $c^o(j) \equiv \tilde{c}(j,k(j)) = c(j,\hat{k}(j)) = \hat{c}(k^s(j))$. (Panel A of Figure 3 illustrates.) Taking the total derivative of $c^o(j) = \hat{c}(k^s)$ with respect to j and k yields:

$$\frac{dc^o}{dj} dj \quad = \quad \hat{c}'(k^s) dk^s \tag{14}$$

$$\Rightarrow \quad \frac{dk^s(j)}{dj} = \underbrace{\frac{1}{\hat{c}'(k^s)}}_{+} \frac{dc^o(j)}{dj}. \tag{15}$$

From the definition of $c^o(j) \equiv \tilde{c}(j,k(j))$,

$$\frac{dc^o}{dj} \quad = \quad \underbrace{\frac{\partial \tilde{c}(j,k)}{\partial k}}_{+} \underbrace{\frac{dk^o(j)}{dj}}_{-} + \underbrace{\frac{\partial \tilde{c}(j,k)}{\partial j}}_{+} \tag{16}$$

[8] Given the symmetric demand structure and decreasing returns to scope, (i.e. $\frac{\partial \tilde{c}(j,k)}{\partial k} > 0$), we have that $\frac{\partial \tilde{\pi}(j,k)}{\partial k} < 0$ as shown.

Substituting for $\frac{dk^o(j)}{dj}$ from (13):

$$\frac{dc^o}{dj} = \frac{\partial\tilde{c}(j,k)}{\partial k}\Big(-\frac{\partial\tilde{c}(j,k)/\partial j}{\partial\tilde{c}(j,k)/\partial k}\Big) + \frac{\partial\tilde{c}(j,k)}{\partial j} = 0. \tag{17}$$

Substituting back into (15) yields the result, $\frac{dk^s(j)}{dj} = 0$.[9] $\diamond$

[*iii*] Regular product scope is increasing in firm productivity if $d\hat{k}(j)/dj < 0$. The result follows immediately from (i), (ii) and the adding up condition s.t. $\hat{k}'(j) = k'(j) - k^{s\prime}(j) < 0.\diamond$

1.6 Quantity Cannibalistic Preferences

In this section, we demonstrate that the addition of negative demand side spillovers embodied in quantity cannibalistic preferences cannot generate predictions consistent with the sourcing patterns observed in the data.

Formally, we refer to preferences as *quantity cannibalistic* if they generate demand that depends on a firm's total quantity of output supplied to the market, so that inverse demand may be written $p_{ji} = p(q_{ji}, Q_j)$, where $Q_j \equiv \int_0^{k(j)} q_{ji} di$. In the literature including Feenstra and Ma (2008), Eckel and Neary (2010), Dhingra (2013) and Arkolakis et al. (2014), preferences generate a *negative* cannibalization spillover, so that inverse demand satisfies $\frac{\partial p(q_{ji},Q_j)}{\partial Q_j} < 0\ \forall ji$.

In deciding how many products to sell to the market, each firm now weighs the direct benefit of producing the k^{th} product ($\tilde{\pi}(j,k)$) against the indirect cost of expanding firm scope (the demand-side spillover cost via within-firm quantity cannibalization). The make-or-source decision remains unchanged as long as demand-side spillovers remain independent of whether products are made in-house or supplied by upstream firms. The revised first order condition for optimal firm scope is:

$$\begin{aligned}\frac{\partial\Pi(j,k)}{\partial k} &= \int_0^k \frac{d\tilde{\pi}(j,i;k)}{dk} di + \tilde{\pi}(j,k) = 0 \\ &= \underbrace{\int_0^k \frac{\partial p(q_{ji},Q_j)}{\partial Q_j}\frac{\partial Q_j}{\partial k} q^o_{ji} di}_{\text{demand spillover}(-)} + \tilde{\pi}(j,k) = 0. \end{aligned} \tag{18}$$

The demand-side spillover imposes a larger cost on more productive firms: more productive firms have a lower marginal cost of production for any given product and thus produce a greater quantity of each product ($\frac{\partial q_{ji}}{\partial j} < 0$) . For any given scope, k, the same quantity cannibalization demand shift embodied in $\frac{\partial p(q_{ji},Q_j)}{\partial Q_j}\frac{\partial Q_j}{\partial k}$ is applied to more units of production for more productive firms, resulting in a larger negative demand-side spillover. It is immediate that the optimal total scope is strictly decreasing in firm productivity among the set of firms engaged in Carry-Along Trade. Regular scope and firm productivity remain super-modular; the model predicts that CAT scope must be decreasing with firm productivity, counter to the data.

1.7 Response to Trade Shocks

Here we derive the effect of an exogenous shock, x, on total exports by a given firm j. Total firm exports, Q_j, can be written as the sum of regular and CAT sales (if any): $Q_j(x) \equiv$

[9] If we add diseconomies of scope in distribution; i.e. $\delta'(k) > 0$ as in the baseline model, the inequality holds with strict inequality; i.e. $\frac{dc^o}{dj} = \frac{\partial\tilde{c}(j,k)}{\partial k}\Big(-\frac{\partial\tilde{c}(j,k)/\partial j}{\partial\tilde{c}(j,k)/\partial k + \delta'(k)}\Big) + \frac{\partial\tilde{c}(j,k)}{\partial j} > 0$.

$\int_0^{\hat{k}_j(x)} q_j^{REG}(i;x)di + \int_{\hat{k}_j(x)}^{k_j(x)} q_j^{CAT}(i;x)di$. (To economize notation, we have moved the firm identifier, j, to a subscript.) Using Leibnitz' rule, the response of firm j's export quantity to the shock x can be written:

$$\begin{aligned} \tfrac{d}{dx}Q_j(x) = \int_0^{\hat{k}_j(x)} \tfrac{dq_j^{REG}(i;x)}{dx}di \quad & + \int_{\hat{k}_j(x)}^{k_j(x)} \tfrac{dq_j^{CAT}(i;x)}{dx}di + q_j^{CAT}(\cdot)k_j'(x)... \\ & + [q_j^{REG}(\hat{k_j};x) - q_j^{CAT}(\hat{k_j};x)]\hat{k}_j'(x), \end{aligned} \tag{19}$$

Where the last term will be zero if (and only if) the trade shock does not affect the make or source margin: i.e. $\hat{k}_j'(x) = 0$. In general, the make or source margin will change in response to supply side-shocks that differentially affect the marginal cost of producting in house versus sourcing. In contrast, demand or distribution shocks generally will not affect the optimal (cost minimizing) make or source decision.

Finally, for completeness, notice that for a firm that is not engaged in Carry-Along Trade, the derivative of sales with respect to a shock x is simply:

$$\frac{d}{dx}Q_j(x) = \int_0^{k_j(x)} \frac{dq_j^{REG}(i;x)}{dx}di + q_j^{REG}(k_j;x)k_j'(x). \tag{20}$$

References

Arkolakis, Costas, Sharat Ganapati, and Marc Muendler, "The Extensive Margin of Exporting Goods: A Firm-level Analysis," *Working Paper*, 2014. 1.6

Bernard, Andrew B., Emily J. Blanchard, Ilke Van Beveren, and Hylke Y. Vandenbussche, "Carry-Along Trade," Technical Report, NBER Working Paper No. 18246 2012. 1

Dhingra, Swati, "Trading Away Wide Brands for Cheap Brands," *American Economic Review*, February 2013, *103(6)*, 2554–2584. 1.6

Eckel, Carsten and Peter Neary, "Multi-product firms and flexible manufacturing in the global economy," *Review of Economic Studies*, 2010, *77* (1), 188–217. 1.6

Feenstra, Robert C. and Hong Ma, "Optimal Choice of Product Scope for Multiproduct Firms under Monopolistic Competition," in Elhanan Helpman, Dalia Marin, and Thierry Verdier, eds., *The Organization of Firms in a Global Economy*, Harvard University, 2008, chapter Optimal Choice of Product Scope for Multiproduct Firms under Monopolistic Competition. 1.6

Mrazova, M. and J. P. Neary, "Selection effects with heterogeneous firms," Technical Report, Discussion paper 2011. 2