Chapter 91: SIGNALING MODELS AND PRODUCT MARKET GAMES IN FINANCE: DO WE KNOW WHAT WE KNOW?*

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Abstract

Important results in a large class of financial models of signaling and product market games hinge on assumptions about the second order complementarities or substitutabilities between arguments in the maximand. Such second order relationships are determined by the technology of the firm in signaling models, and market structure in product market games. To the extent that the underlying economics (in theoretical specifications) or the data (in empirical tests) cannot distinguish between such complementarities and substitutabilities, the theoretical robustness and the empirical tests of many models are rendered questionable. Based on three well-known models from finance literature, we discuss the role that these assumptions play in theory development, and provide empirical evidence that is consistent with the arguments advanced here.

Key Words: financial signaling, second order complementarities or substitutabilities, product market games, strategic substitutes and complements, empirical evidence.

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91.1 INTRODUCTION

Central results in many models in financial economics – particularly models of interaction between product market structure and financial market behavior, as well as models of signaling – hinge on assumptions concerning second-order relationships between arguments in a function to be optimized. These second order relationships can be defined as those resulting in super- or submodularity of the maximand [Topkis (1978; 1979); Milgrom and Roberts (1990a; 1990b)], and reflect the underlying economics of the firm or the industry.

We argue below that changing the assumption from one to the other – a change that is tantamount to implicitly changing the assumptions on technology or market structure – can produce result reversals or indeterminacy, raising troublesome questions about both the theoretical robustness and the empirical testability of many well-accepted models. Data that cannot discriminate the presence of these differing technologies and market structures may produce conflicting results, or no results at all, and it possible that studying the market value announcement effects of various real and financial decisions by looking at *average* effects can be misleading.

Following Topkis (1978, 1979) and Milgrom and Roberts (1990a, 1990b), we first define super- and submodularity. We then examine its implications for signaling models and product market games. We show how the assumption of one or the other leads to result indeterminacy, or even reversals, using three examples: Miller and Rock (1985) on dividend signaling, Brander and Lewis (1986) on the effect of leverage increases, and Maksimovic (1990) on the effect of loan commitments. Based on recent empirical evidence, we argue that empirical tests that only examine *average* announcement effects may need to be reexamined by parsing data sets to reflect the presence of super- and submodularity, since the opposite effects of each may cancel the other out, or the unduly large presence of one or the other in a data set may be producing results that are not generalizable to all firms.

Many such models are making implicit assumptions about the nature of competition, or the nature of technologies in the firms and industries under study. Unless these assumptions can be validated by the data under scrutiny, we cannot generalize the results even if they appear to be consistent with the chosen model. An illustration of our point is provided by the Miller and Rock (1985) signaling model of dividends (explored in greater detail in section 3 below). We show that the choice of technology in Miller and Rock is essentially arbitrary. An equally, if not more, plausible choice of technology will give rise to a signaling equilibrium in which the strategies of firms with respect to investment policies and the announcement effects of dividend changes are the exact opposite of that obtained in the Miller-Rock equilibrium. The only rationale for the Miller-Rock choice of technology is that yields an announcement effect of dividend changes on stock prices that is consistent with the *average* (across firms) effect documented in some empirical studies. Recent studies taking a closer look at the evidence [e.g., John and Lang (1991); Lang and Litzenberger (1989), and Shih and Suk (1992)] have found both stock price increases and decreases in response to announcements of dividend increases. Moreover, such dichotomous announcement effects can be related to super- or submodularity of the firm's technology.[1]

The point of this paper is simply this: instead of making a model choice arbitrarily to yield the average effect observed empirically, it may be important to examine firm and industry characteristics closely and relate them to explanations for positive and negative effects observed in the data.

91.2 SUPERMODULARITY: DEFINITIONS

The concept of supermodularity revolves around the idea that there are second order complementarities *between* arguments in a function to be optimized, and therefore, there may be "highest" and "lowest" optimal solutions to the function. That is, the optimal solutions are such that

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[1] Such dichotomous effects have been observed – but largely left unexplained – in other related areas such as announcement effects of leverage changes [e.g., Masulis (198;3), Eckbo (1986), Mikkelson and Parch (1986)], announcement effects of investments [e.g., John and Mishra (1990)] and dividend initiations [e.g., John and Lang (1991)] or announcement effects of changes in Research and Development expenditures [e.g., Sundaram, John and John (1996)] and anti-takeover measures [e.g., John, Lang, and Shih (1991)]. For more details, see section 5 of the paper.

all the arguments are simultaneously at their highest values *or* simultaneously at their lowest values at optimum. In economic modeling, it can imply that there are Pareto-best and Pareto-worst equilibria [Milgrom and Roberts (1990b)].

Consider a function $f(x_1, x_2)$ where is $f(.)$ is twice continuously differentiable in x_1, x_2. Then supermodularity is defined as follows:

(SM) f is supermodular if and only if $\dfrac{\partial^2 f}{\partial x_1 \partial x_2} > 0$

[Topkis (1978); a weakened version of this definition can also be found in Milgrom and Roberts (1990)]. If $f(.)$ is supermodular, then $-f(.)$ is *submodular;* if $f(.)$ is both supermodular and submodular, then it is a *valuation* [Topkis (1978)].

Although we assume twice-continuously differentiable functions in the definition above (and in the rest of the paper), in general, the concept of supermodularity does not require assumptions of convexity, concavity or differentiability.[2] Further, there is no necessary connection between supermodularity and increasing or decreasing returns to scale [see Milgrom and Roberts (1990a, 1990b)].[3]

The assumption of super- or submodularity appears in a wide range of models: oligopoly games, auction games, technology adoption games, games of macroeconomic coordination, etc. [for

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[2] The generalization of similar conditions for the non-differentiable case motivates the concepts of super modularity and sub modularity. The details can be found in Milgrom and Roberts (1990b).

[3] As an example, suppose the function is of the kind:

$$f(x_1, x_2) = x_1 + x_2 + \beta x_1 x_2$$

Using (SM), we see that, if $x_1 \geq 0, x_2 \geq 0$ then $f(.)$ is supermodular if $\beta > 0$, submodular if $\beta < 0$, and a valuation if $\beta = 0$. Consider another function, the familiar Cobb-Douglas function:

$$f(x) = x_1^{\mu} x_2^{\beta}$$

Here, $f(.)$ is supermodular if, for all $x_1 \geq 0, x_2 \geq 0$, $\text{sgn}(\mu) = \text{sgn}(\beta)$. Thus even the Cobb-Douglas function can exhibit super- or submodularity, and be decreasing or increasing returns to scale under either.

a review of the various applications, see Milgrom and Roberts (1990b)]. We argue that they also apply to a wide class of product market and signaling games in finance. Using three well known examples, we analyze how the phenomena of super- and submodularity affect theoretical and empirical results.

91.3 SUPERMODULARITY IN SIGNALING MODELS

In the early signaling models, such as Leland and Pyle (1977) or Ross (1977), larger managerial holdings or larger debt levels were unambiguous signals of higher quality (i.e., higher levels of the attribute of private information). Similarly, in the dividend signaling models which followed [e.g., Bhattacharya (1979), Miller and Rock (1985), John and Williams (1985)], higher dividend payouts were assumed to connote better firm prospects. From recent work in financial signaling, it is clear that the interpretation of increases in the signal has to be made conditional on the super- or submodularity of firm technologies. In Ambarish, John and Williams (1987) and John and Mishra (1990), whether or not better firms invest larger or smaller amounts than the full information-optimal level depends on the characteristics of the firm's technology; consequently, whether announcements of investment increases will lead to increases or decreases in the firm's stock price also depends on the firm's technology.

Similarly; in John and Lang (1991) dividend increases can signal better or worse firm prospects depending on the firm's characteristics.[4] The idea can be generally understood simply as resulting from implicit assumptions concerning super- or submodularity of firm technologies. We illustrate this argument using the well-known Miller and Rock (1985) model.

91.3.1*: Dividend Signaling*

We consider an extended version of the model in Miller and Rock (1985). The investment

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[4] Empirical evidence of such dichotomous announcement effects of dividend initiations on stock prices is also presented in John and Lang (1991).

opportunity set is generalized in a straightforward way to be a function of not only investment, but also the firm quality attribute which is private information to corporate insiders.

Let $t = 0$ indicate the initial date and $t = 1$, the final date. At $t = 0$, the firm has current cash C, which includes cash from operations as well as any external financing done at the firm level selling riskless claims.[5] The firm has access to a technology (or an investment opportunity set) $f(I,\theta)$. The technology represents a set of assets such that an investment of amount I made at $t = 0$ results in a cash flow of net present value of $f(I,\theta)$ at $t = 1$ for a firm of quality θ (specified in detail below). Corporate insiders are assumed to act on behalf of current shareholders. It is common knowledge that an exogenous fraction k ($0 < k < 1$) of current shareholders will sell their shares at $t = 0$ in the market. Just prior to that, the insiders announce a dividend D, and invest I, where I is the residual amount $C - D$, in the technology of the firm.

Information about future cash flow is asymmetric. To specify this asymmetry as simply as possible, represent by θ, with $\theta \in \Theta$, the value of the single state variable affecting the firm's future profitability (for example, θ may measure its insiders' forecasted return on either its assets in place or alternatively, its opportunities to invest).[6] The firm's identity, θ, is private information; it is known to the insiders, but not observed directly by the outsiders. In particular, insiders observe the private attribute θ before picking D, whereas the outsiders must infer θ from the insiders' actions. By contrast, prior probability $\pi(\theta)$ of possessing the private attribute θ is common knowledge.[7]

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[5] The outside claims that the firm can sell are restricted to be riskless debt and equity. This assumption is made for model tractability. For convenience, we include riskless claims in C and treat separately the equity sold by current shareholders. Such an assumption is common to papers in this area, see Miller and Rock (1985), John and Williams (1985), Ambarish John and Williams (1987) and John and Mishra (1990).

[6] Although we analyze the case of one-dimensional signaling for expositional convenience, similar results obtain in more complex models of "efficient signaling" where a least-cost mix of multiple signals are used to convey one-dimensional private information. See, for example, Ambarish, John, and Williams (1987), and John and Mishra (1990).

[7] For example, the current cash C is fully revealed to outsiders through costless corporate audits; similarly, corporate audits will verify that $I = C - D$ will be the level of investment actually undertaken.

At $t = 1$, the cash flows are realized and distributed pro-rata to shareholders. By convention, we take higher θ to represent *more* favorable private information. In other words, for all feasible I, we have $f(I,\theta_1) > f(I,\theta_2)$ when $\theta_2 > \theta_1$. Also, the function $f(.,\theta)$ is positive, twice-continuously differentiable, strictly increasing and strictly concave in investment, with suitable corner conditions.

As argued in Miller and Rock (1985), corporate insiders will solve

(1) $$kP + (1-k)[D + f(C-D,\theta)]$$

which is the weighted average of the wealth of the shareholders who are selling and those who remain. The market price, P, can however only depend on common knowledge and the observable actions of insiders. In other words, the pricing function used by the market will be a function of D (and parameterized by all common knowledge). Let $P(D)$ be the cum-dividend price assigned by the market for the entire firm value. We use the signaling framework developed in Riley (1979) to analyze the model.

Following Riley (1979), let the insiders' objective be $U(\theta,D,P)$, as a function of the private information, the signal and the market pricing function. The corporate objective is now to choose the dividend payout D to maximize their objective in (1), conditional on P. This optimization is equivalent to:

(2) $$\underset{D\geq 0}{Max}\; U(\theta,D,P)$$

Let the *true* value of the firm, i.e., the value which the market would have assigned to the firm (cum-dividend) had it been informed of the value of θ be $V(\theta,D)$. Then,

(3) $$V(\theta,D) = D + f(C-D,\theta)$$

Riley (1979) defines a signaling equilibrium $\{D(\theta), P[D(\theta)]\}$ as dividend schedules $D(\theta)$ and a pricing functional $P(.)$ such that for $D(\theta)$ which solves the incentive compatibility condition (2), the following competitive rationality condition is also satisfied:

(4) $$P[D(\theta)] = V[\theta, D(\theta)]$$

The existence of the signaling equilibrium and its properties are shown in detail in Riley (1979), and we only sketch the arguments below. Let us first examine the full-information optimum investment level $I°(\theta)$ for firm type θ. It is given by the familiar condition:

$$\frac{\partial f(I°,\theta)}{\partial I} = 1 \tag{5}$$

Also, define $D°(\theta) = C - I°(\theta)$.

Now we show that the properties of the signaling equilibrium are crucially dependent on whether the firm's technology, $f(I,\theta)$, is super- or submodular as a function of I and θ; i.e., whether $\partial^2 f / \partial I \partial \theta > 0$ for all θ and all $I \geq 0$, or $\partial^2 f / \partial I \partial \theta < 0$ for all θ and all $I \geq 0$.[8]

Proposition 1: *If the firm technologies are supermodular, all but the poorest firms overinvest in the signaling equilibrium and pay lower dividends with increasing* θ; *if firm technologies are submodular, all but the poorest firms underinvest in the signaling equilibrium and pay larger dividends with larger* θ.

<u>Proof:</u>

Using Riley's assumptions (i) to (vi) in Riley (1979) we can show that when the technology is submodular, $\{D(\theta), P[D(\theta)]\}$ is an equilibrium. It is sufficient to verify Riley's crucial assumption (v). It can be verified as follows for the objective function in (1):

$$\frac{\partial(-U_D/U_P)}{\partial\theta} = \frac{\partial\{[(1-k)/k][(\partial f/\partial I) - 1]\}}{\partial\theta} < 0 \tag{6}$$

if and only if $\partial^2 f / \partial I \partial \theta < 0$, i.e., if f is submodular in I and θ. Since $I°(\theta)$ is decreasing in θ for submodular technologies, it follows that $I(\theta) < I°(\theta)$ for all types except the θ^P-type: $I(\theta^P) = I°(\theta^P)$, where θ^P represents the θ of the poorest firms. The corresponding dividend $D(\theta) = C - I(\theta)$ is *increasing* in θ. Thus all but the poorest firms *underinvest.*

Conversely, for supermodular technologies, Riley's conditions can only be satisfied for the signal $I(\theta)$ (and corresponding to a *decreasing* schedule of dividends $D(\theta) = C - I(\theta)$). The crucial Riley condition (v) can be verified only for $I(\theta) \equiv C - D(\theta)$. Now, all the but the poorest firms

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[8] It should be noted that $I°(\theta)$ is increasing in θ for supermodular technologies and decreasing in θ for submodular technologies.

overinvest in the signaling equilibrium ($I(\theta) > I°(\theta)$), and higher θ firms pay smaller dividends. *QED.*

We now examine the announcement effect of dividends on stock prices. Differentiating (1),

(7) $$k[\partial P / \partial D] - (1-k)[(\partial f / \partial I) - 1)] = 0$$

which implies

(8) $$\partial P / \partial D = [(1-k)/k][(\partial f / \partial I) - 1]$$

which, depending on $\operatorname{sgn}[(\partial f / \partial I) - 1]$, is positive for submodular firms and negative for supermodular firms. Underinvestment for submodular technologies implies $\partial f / \partial I > 1$, and overinvestment for supermodular technologies implies $\partial f / \partial I < 1$. Thus, the characteristics of the firm's technology will determine whether stock prices increase or decrease in response to announcements of dividend increases.

91.3.2: *An Example*

Let us consider a particular example of a submodular technology which is isomorphic to the one considered in the Miller-Rock paper:

(9) $$f(I,\theta) = \beta \ln(I + \theta)$$

where β is a positive constant (Miller-Rock provide an explicit solution to this technology; that this technology is submodular is easy to see: $\partial^2 f(.)/\partial I \partial \theta = -\beta/(1+\theta)^2 < 0$). This technology has the "usual" properties that one would look for in a cash flow function: it is increasing in investment level I, and the quality attribute θ; it is strictly concave, twice-continuously differentiable, and so forth. However, note that the full-information investment level $I°(\theta)$ defined by $\partial f(I°,\theta)/\partial I$ is *decreasing* in θ.[9] As we have shown in Proposition 1 [see, in particular, (6)], the firm *underinvests* in the signaling equilibrium and the announcement effect of dividend increases is *positive.*

Now consider a simple example of a supermodular technology:

(10) $$f(I,\theta) = \theta\sqrt{I}$$

(That this technology is supermodular is easy to check: $\partial^2 f(.)/\partial I \partial \theta = (0.5)I^{-(1/2)} > 0$). This

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[9] Indeed, we might argue that the optimal level of investment being lower for a higher quality firm is not an attractive characteristic.

technology also has the "usual" properties one would look for a cash flow function: it is increasing in the investment level I, and the quality attribute θ; it is strictly concave, twice-continuously differentiable, and so forth. Indeed, we MIGHT note that (10) has arguably the more attractive property that, in the full information equilibrium, the optimal investment level is increasing in firm quality, a feature that might actually make it a more plausible candidate for a cash flow technology than the one in (9). Note now that with (10) as the firm technology [and following (6)], firms will *overinvest* in the signaling equilibrium, and therefore, the announcement effects of dividend increases will be *negative.* In other words, for an equally, if not more, plausible technology to that in Miller-Rock, we get a result that is the exact opposite of theirs.

The choice of a supermodular technology such as the one in (10) is just as appropriate as the submodular one in (9). The only argument to recommend the submodular technology choice is that it yields results that are consistent with the positive *average* announcement effects that have been observed in some of the data (in particular, the earlier studies). Instead of making arbitrary *a priori* choices of technology based on average evidence, we suggest that detailed studies of technology characteristics and related announcement effects will be necessary to understand the true underlying phenomena.

91.4 SUPERMODULARITY IN PRODUCT MARKET GAMES

In games involving product market competition, supermodular games are those in which each player's strategy set is partially ordered, the marginal returns to increasing one's strategy increase in the competitor's strategy. In other words, the game is said to exhibit "strategic complementarity (SC)" [Bulow, Geanakoplos and Klemperer (1985); see also Fudenberg and Tirole (1984) and Tirole (1988)].[10] If marginal returns are decreasing in the competitor's strategy, then there is submodularity and the game is said to exhibit "strategic substitutability (SS)".

[10] It can also occur in situations in which players' strategies are multidimensional: if the marginal returns to anyone component of the player's strategy increase with increases in the other components, then the component is supermodular with respect to the other components.

Consider, for example, a duopoly. In (SM), if $f = \pi$ and if the variable of strategic choice[11] for players 1 and 2 are q_1 and q_2 then profits are (strictly) supermodular with respect to strategy q_i, i = 1,2, if and only if $\partial^2\pi / \partial q_1 \partial q_2 > 0$; if the reverse inequality holds, then profits are submodular and they are strategic substitutes.[12] In terms of reaction functions in models of oligopoly, while submodularity implies downward sloping reaction functions, supermodularity implies that reaction functions are upward sloping.

There are three variables that determine supermodularity in oligopoly games: the nature of the demand function, the nature of the cost function, and the nature of competition (for example, whether it is Cournot, Bertrand, market share, or competitive). Many of the linkages between SS/SC and these three sets of functions have been explored in some detail in Bulow, Geanakoplos, and Klemperer (1985), and we will not go into them here.

In general, Cournot and market share competition with linear demand is implicitly an assumption of SS, regardless of the shape of the cost function; on the other hand, if constant elasticity demand is assumed, then we can have SS or SC depending on the size of demand elasticity.[13] With homogeneous goods and Cournot/market share competition, demand functions that are linear or strictly concave to the origin imply SS, while demand functions that are sufficiently convex to the origin imply SC. In the case of Bertrand competition, increasing or constant marginal costs will imply *SC,* while decreasing marginal costs are likely to imply SS. Perfect competition (and Bertrand competition in homogeneous goods, which also produces the result that prices equal marginal costs) will imply SS.

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[11] If Cournot or market share competition is assumed, then the strategic variable is quantity; if Bertrand, then the strategic variable is price; under perfect competition and monopoly, the variable can be modeled either as quantity or price.

[12] The concepts of SS and SC have wide general applicability in oligopoly theory. Tirole (1988) refers to them as a "taxonomy" for business strategies, whereby strategic complements competition can be interpreted as aggressive competition, and strategic substitutes competition as passive competition.

[13] The lower the demand elasticity in a Cournot model, *ceteris paribus,* the greater the possibility that SC obtains in the product market. To the extent that lower demand elasticities characterize differentiated products, we might argue that many products in the real world are as likely to be SC as they are SS.

Thus, models cannot simply assume, say, "Cournot competition with linear demand" and attempt to draw conclusions of any reasonable empirical generality from it, since the assumption that is *really* being made is that the game exhibits SS. Whether or not this is a reasonable assumption would require prior empirical analysis of industries or firms under scrutiny.

We show how this matters in two recent models that deal with interactions between product markets and financial markets: Brander and Lewis (1986), and Maksimovic (1990).[14] In each case, we restrict ourselves to one or two key conclusions of the models, since our intention is to illustrate the broader point concerning the empirical robustness of such work.

91.4.1*: Effect of Leverage Decisions [Brander and Lewis (1986)]*

Brander and Lewis (1986) (hereafter, BL) developed a model in which financial and output decisions follow in sequence, and argued that the limited liability effect of debt may commit a leveraged firm to a more aggressive output strategy, and consequently its competitor to a more passive output strategy.

BL's setup is as follows. Firms 1 and 2 are Cournot competitors in output markets, producing competing products q_1 and q_2, respectively. The profit of firm i is $R^i(q^i, q^j, z^i)$, where z_i (independent of and identical to z_j) is a random variable that affects output, and distributed in the interval [z', z''] according to the density function $f(z_i)$. BL consider two situations with respect to the realization of z_i: $R^i_{iz} > 0$ where the marginal profits are higher in the better states of the world, and $R^i_{iz} < 0$ where the reverse obtains. For our purposes, it is sufficient to consider the former case. They also assume that $R^i < 0$ satisfies that "usual (p. 958)" properties: $R^i_j < 0$, $R^i_{ii} < 0$, and $R^i_{ij} < 0$.

From our point of view, vis-a-vis the main result of BL, the most important assumption is the third one: the assumption that the game exhibits strategic substitutability, i.e., that reaction functions are downward sloping[15] (After stating their main result and sketching the proof, we shall

[14] Our choice of these two models is for reasons of parsimony. The general issues raised below appear in many other models that consider the implications of interactions between product markets and financial markets [e.g., Rotemberg and Scharfstein (1990); Brander and Spencer (1989)].

[15] This is equivalent to the assumption that the profit function is submodular in quantities. Note, however, that, with respect to the uncertainty term z_i , BL allow for the possibility of both super- and submodularity.

show why this is a critical assumption[16]).

Firms can take on debt D^i, and the objective of firm i is to choose output levels that maximize the expected value of the firm, V^i, to its stockholders:

(11) $$V^i(q^i,q^j) = \int_{z_L}^{z''} [R^i(q^i,q^j,z^i) - D^i] f(z^i) dz^i$$

where $z_L = R^i(.) - D^i$ captures the limited liability effect. Note that the assumption that $R^i_{ij} < 0$ will imply that $V^i_{ij} < 0$ or that firm value is submodular in outputs. Similarly, the assumption that $R^i_{ii} < 0$ will imply that $V^i_{ii} < 0$; this would, for example, be the case where the demand function of the firm is concave to the origin if marginal costs are constant.

The important result of the paper is Proposition 2, which, for the case $R^i_{iz} > 0$, is as follows (BL suggest that it represents the "...key insight in the analysis. (p. 963)"): *Given* $R^i_{iz} > 0$, *a unilateral increase infirm i's debt, D^i, causes an increase in q^i and a* ***decrease*** *in q^j.*

The proof is fairly straightforward, simply requiring that the first order conditions to (11) are totally differentiated, then using Cramer's rule and assumptions concerning the signs of the variables to determine comparative statics effects. To sketch the proof, consider the case of $R^i_{iz} > 0$. If we totally differentiate the first order conditions for (11), we have

(12) $$V^i_{ii} dq^i + V^i_{ij} dq^j + V^i_{iDi} dD^i = 0$$

(13) $$V^j_{ji} dq^i + V^j_{jj} dq^j + V^j_{jDi} dD^i = 0$$

Using Cramer's rule to solve for dq^i / dD^i and dq^j / dD^i gives

(14) $$dq^i / dD^i = -V^i_{iDi} V^j_{jj} / B$$

(15) $$dq^j / dD^i = -V^i_{iDi} V^j_{ji} / B$$

where B > 0 (under the assumption of equilibrium stability) and further, V^j_{jj} is assumed to be less than zero. However, V^j_{ji} can be greater than zero or less than zero, depending on whether V is super- or submodular in quantities. Now, BL's assertion that D^i causes an increase in q^i is dependent on the

[16] BL do appear to recognize this possibility, but in passing. They note (p. 961) that the condition can be violated by feasible demand and cost structures. But they do not explore it any further.

assumption that the sign of V^j_{jj} is negative. (If V^j_{jj} is positive instead, that is, the demand function is concave to the origin and costs constant, then an increase in leverage has an output-reducing effect for firm i.) The assertion that D^i causes a *decrease* in the competitor's output is, however, dependent on the assumption that V^j_{ji} is submodular in quantities.

Now, instead, let us assume that V is supermodular in quantities, Le., that $V^j_{ji} > 0$. Then, by examining (15), we have the following proposition: *Given* $R^i_{iz} > 0$*, a unilateral increase infirm i's debt,* D^i*, causes an increase in* q^i *and an* ***increase*** *in* q^j.

The original result was that an aggressive capital structure choice by a firm will lead to a passive strategic response by its competitors. This intuition depended on the assumption of submodularity of firm value with respect to competitors' quantities. Instead, if firm value is supermodular in competitors' quantities, then aggressive capital structure choices by firms lead to an *aggressive* strategic response from their competitors. Consequently, we would observe that: (1) an increase in leverage by one firm will be followed by an increase in leverage by other firms; (2) there will be a general increase in industry output levels following leverage increases.

If such industry-wide increase in output has adverse profitability consequences for all firms in the industry and if there is a one-to-one correspondence between firm profits and stock prices, then we may well observe that leverage increases are accompanied by stock price *decreases.* In order to see this, let us examine the profits of firm i around equilibrium, when perturbed by a leverage increase (i.e., an increase in D^i):

$$(\Delta D^i)(d\pi^i / dD^i) = (\Delta D^i)[(\partial\pi^i / \partial\pi q^i)(dq^i / dD^i) + (\partial\pi^i / \partial\pi q^j)(dq^j / dD^i) + (\partial\pi^i / \partial D^i)]$$

Note that the first term on the right hand side is zero in equilibrium, and the third term is positive (from the analysis above). Now consider the second term: $\partial\pi^i / \partial q^j$ is less than zero by assumption of conventional substitutes; however, we note from (15) that dq^j / dD^i can be positive or negative depending on whether *π(.)* is supermodular or submodular. Thus, the second term (and hence the impact of leverage increases on firm profit) can be negative or ambiguous depending on whether *n(.)* is submodular (the BL case) or supermodular. Indeed, if *n(.)* is sufficiently supermodular, dq^j / dD^i can be sufficiently large and positive so as to overwhelm the effect of $\partial\pi^i / \partial D^i$; that is, leverage increases can have *negative* stock price consequences (assuming, as is normal, that stock prices covary positively with firm profits), a result that implies the reverse of the BL insight.

The crucial issue then is this: if we empirically implement the BL idea using a data set that contained products or markets where both SS and SC forms of competition are present, then we are likely to either find conflicting results in the data or unlikely to find any results at all. The only way to address this would be to empirically analyze the market prior to the analysis of valuation implications, to explore whether manifestations of SS or SC are present in the data (for example, in the choice of industries).

91.4.2 ***Loan Commitments in a Duopoly [Maksimovic (1990)]***

Maksimovic (1990) (hereafter, M) developed a model of loan commitments in a Cournot duopoly. The argument is that, when product markets are imperfect, the provision of loan commitments leads to a subsidy effect from reduced capital costs, giving the firm the incentive to increase output. Under the assumptions of the model, the increased output will lead to a reduction in the rival's output and hence, an increase in the firm's profits; consequently, the firm has the incentive to take on loan commitments. Likewise, the rival has the same set of options and will do the same.

The net result is that both firms receive loan commitments and overproduce relative to a no-commitments equilibrium, leading to a decline in industry profit. Thus, an important implication of M's model is that while" ...it is rational for a firm to obtain a loan commitment, all the firms in that industry taken together are made worse off by the existence of loan commitments (p. 1641)".

We show below that this result hinges on the implicit assumption of submodularity (i.e., strategic substitutes). As before, we present a simplified version of the model and its key result. Next, we show how results can get reversed under supermodularity.

There are two firms 1 and 2, homogeneous products Cournot competitors, facing a linear demand schedule.[17] The inverse demand function is $p = a - b(q^1 + q^2)$, where q^i, $i = 1, 2$ is the output of firm i. Marginal costs are assumed to be constant (and normalized to 1) for simplicity, and are subsumed in the intercept of the demand function, a. Firm i can obtain a loan commitment at

[17] Assuming homogeneous product Cournot competition with linear demand is implicitly an assumption of strategic substitutes; see below.

constant marginal cost r^i. The profit function for firm i is

(16) $$\pi^i(q^1,q^2) = [a - b(q^1 + q^2) - r^i]q^i$$

The first order condition for (16) gives us

(17) $$\partial\pi^i / \partial q^i = a - 2bq^1 - q^2 - r = 0$$

which, in turn, implies a downward sloping reaction function.[18] Clearly, a decrease in r (i.e., a subsidy effect resulting from the provision of a loan commitment) will lead to an increase in the firm's output and profit, and consequently, a decline in the rival's output and profit. Hence the firm will have the incentive to obtain a commitment. The rival will have the same incentive, and will also obtain a commitment. A prisoner's dilemma situation results whereby both firms produce more than the precommitment equilibrium, increasing total industry output and reducing total industry profit. This is M's main result.

Let us now generalize M's model somewhat, by specifying a general demand function (which also subsumes the normalized constant marginal cost) by writing $P = P(q^1,q^2)$, with $\partial P / \partial q^i < 0$, and retaining the assumption that the product is homogeneous (and hence, implicitly, conventional substitutes).[19] Now, the profit function can be written as

(18) $$\pi^i = P(q^1,q^2)q^i - r^i q^i \qquad i = 1,2$$

For variable X, let X_i denote the first (partial) derivative with respect to q^i, and X_{ij} denote the second (partial) derivative of X with respect to q^i and q^j, i = 1, 2 and j = 2, 1. Let X_{r^i} and X_{ir^i} similarly denote the first and second partial derivatives, respectively, with respect to r^i. The first order condition is

(19) $$\pi^i_i = P + q^i P_i - r^i = 0$$

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[18] Downward sloping reaction functions result from the assumption of strategic substitutes. Strategic substitutes should imply that $\partial^2\pi^i / \partial q^1 \partial q^2 < 0$ in (17); it is easy to verify that it is equal to –1 in M's setup. This is a consequence of assuming homogeneous products, linear demand and Cournot competition.

[19] Conventional substitutability or complementarity (that is, the first derivative of firm i's profit with respect to firm j's output) is independent of the notion of SS and SC. None of the general arguments of this paper depend the assumption of conventional substitutes or complements. It is normal to assume conventional substitutes in most economic models.

and second order conditions are assumed to be met.[20]

Strategic substitutes or complements are determined by the sign of π^i_{ij}. If $\pi^i_{ij} = P_j + q^i P_{ij} < 0$ then profit is submodular (and strategic substitutes obtain), but if $\pi^i_{ij} > 0$, then profit is supermodular (and strategic complements obtain). Note that π^i_{ij} can be of either sign since no prior restrictions are imposed on the sign of P_{ij}, the shape of the demand function.[21]

Now we are ready to explore the main result in M in some detail, by examining the movement of the key variables around equilibrium. Taking the total derivative of (19) for the two firms, we have

(20) $$\pi^1_{11} dq^1 + \pi^1_{12} dq^2 + \pi^1_{1r} dr = 0$$

(21) $$\pi^2_{21} dq^1 + \pi^2_{22} dq^2 = 0$$

(note that the partial derivative of firm 2's profit function with respect to firm 1's cost of financing, π^2_{2r} is zero, since r^i does not enter firm 2's profit function). Using Cramer's rule and simplifying,

(22) $$dq^1 / dr = \pi^2_{22} \pi^1_{1r} / (\pi^1_{12} \pi^2_{21} - \pi^1_{11} \pi^2_{22})$$

(23) $$dq^2 / dr = \pi^1_{12} \pi^1_{1r} / (\pi^1_{12} \pi^2_{21} - \pi^1_{11} \pi^2_{22})$$

We see that (23) – that is, the rival firm's reaction to an increase in the subsidy for firm 1 – depends on π_{12} and hence, on super- or submodularity of the profit function.

To examine the output effects of a change in r, we need to examine the signs of (22) and (23). dq^1 / dr will always be less than zero, since $\pi^2_{22} < 0$ by the second order condition, $\pi^2_{1r} < 0$ (output is concave in r) and equilibrium stability requires that the denominator be less than zero. However, dq^2 / dr can be greater than or less than zero, depending on whether SS or SC obtains; if SS, then dq^2 / dr will be greater than zero, and if SC then dq^2 / dr will be less than zero. The

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[20] This requires that $\pi^1_{11} < 0$ and $\pi^1_{11} \pi^2_{22} - \pi^1_{12} \pi^2_{21} > 0$.

[21] Linear demand would mean that $P_{ij} = 0$. Since $P_j < 0$ by assumption of conventional substitutes, SS always results. $P_{ij} > 0$ will imply demand functions that are strictly convex to the origin while $P_{ij} < 0$ will imply demand functions that are strictly concave to the origin. Strictly concave demand functions with homogeneous products Cournot competition will also always imply SS, while sufficiently convex demand functions could imply SC depending on the degree of convexity.

interpretation is as follows: *an increase in the subsidy to firm* 1 *(i.e., a decrease in r) will always increase firm* 1 's *output, but it could decrease or increase firm 2's output depending on whether profits are super- or submodular in quantities.* By implicitly assuming SS, M only gets the result that firm 2 will decrease output when r decreases for firm 1.

Again, as in the case of BL, we can examine how SS and SC affect industry profits. By perturbing profits around the equilibrium resulting from a small change in r_i, it can be shown that, under strategic complements, an increase in loan commitment to firm 1 could decrease the profits of firm 1. This, in turn, could not only imply that firm 1 (and similarly, firm 2, using similar arguments) will have *no* incentive to take on a loan commitment, but it could actually imply that the firms have the incentive to *make* (rather than receive) loan commitments.[22]

Our arguments in relation to both BL and M are summarized in Proposition 2 below:

***Proposition* 2:** *Under submodularity, an increase in leverage (BL) and a decrease in loan commitments (M) will result in positive firm profits; however, under supermodularity, the profit effects of leverage increases and loan commitment decreases will be ambiguous (and perhaps negative).*

91.5 EMPIRICAL EVIDENCE

In signaling models, the usual practice in the literature has been to examine the average announcement effect, and to pick a model structure that produces results consistent with this effect. However, it has been noted in a variety of contexts that even though the average effect is positive (negative), a sizeable fraction of firms in the sample have an announcement effect which is negative (positive). For example, the average positive announcement effect in the case of dividend signaling has been documented in Aharony and Swary (1980), Asquith and Mullins (1983), and Healy and Palepu

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[22] The intuition here is similar to that in the industrial organization literature on how there may be an incentive for a firm to raise its rivals' costs, since the benefits resulting from costs imposed on rivals would more than compensate for any additional costs one brings upon oneself [see, for example, Salop and Scheffman (1983)].

(1988): however, it has not been unusual for 30-40% of the firms in their samples to obtain an announcement effect that is negative. Similarly, in the case of leverage increases, Eckbo (1986) finds that leverage increases are accompanied by stock price increases for 40-48% of the firms in the sample, and decreases in the remaining cases; in an analysis of 171 straight debt public offerings, Mikkelson and Partch (1986) document 56% negative effects and the rest positive.

Results that dichotomize firms along criteria examined here produce results consistent with our arguments. For example, Lang and Litzenberger (1989) use Tobin's q to dichotomize firms, and obtain evidence consistent with our perspective; John and Lang (1991) obtain results on announcement effects of dividend initiations using a technology-based dichotomy and find similar results; using a similar classification, John, Lang, and Shih (1992) find dichotomous announcement effects from adoption of anti-takeover amendments.

In the area of product market games, Sundaram, John, and John (1996) examine the impact of R&D spending announcements on the market value of announcing firms and their competitors, using both average effects and effects when the data are parsed by SS and SC firms. They operationalize a measure of SS and SC, called "Competitive Strategy Measure (CSM)," as follows: CSM is obtained by correlating the ratio of change in the firm's own profits to the change in the firm's own revenue with the change in rest-of-industry revenues (using data for forty quarters prior to the announcement quarter), and thus they attempt to directly measure the underlying construct of the second derivative of the profit function with respect to own firm and competitor firm quantities.

For the sample taken as a whole, they find that the *average* announcement effect of R&D expenditure changes is not significantly different from zero. However, the announcement effect for the firm announcing the R&D expenditure change is negatively and significantly related to CSM, the measure of strategic interaction: firms with passive (SS) competitors have larger positive announcement effects than firms with aggressive (SC) competitors. When the firms in their sample are split by SS and SC types, they find that SS type firms have a positive and significant announcement effect, while SC type firms have a negative (although not significant) announcement effect.

These results are consistent with the arguments we advance here, which suggest that nonsignificant averages may actually hide significant effects of opposite signs if the data are parsed using underlying market structure criteria such as SS and SC. Building on the work of Sundaram, John and John(1996) there has been a number of recent papers that explore the effects of this dichotomy on a number of important firm decisions such as new product introductions [e.g., Chen, Ho, Ik and Lee (2002)], capital structure choices [Lyandres (2006) and De Jong, Nguyen and Van Dijk (2007)], and strategic IPOs (initial public offerings) [e.g., Chod and Lyanders (2008)]. Kedia (2006) examines the effect of the nature of strategic competition on the incentive features included in an optimally designed top-management compensation structure and finds empirical results in support of the dichotomy studied in this paper.

91.6 CONCLUSION

Our analysis demonstrated that the phenomena of supermodularity and submodularity have crucial implications for the generalizability of many theoretical results. Notably, we examined three cases: depending on whether *product markets* are super- or submodular, the stock price effects of leverage increases can be positive or negative; similarly, firms may have the incentive to make loan commitments rather than receive them; depending on whether firm *technologies* are super- or submodular, stock price effects of dividend increases can be positive for a broad class of firms.

The implications from all these observations are twofold. First, it is important to take a closer look at choices of technology in signaling models, and choices of market structure, demand functions, and cost functions in product market games in finance. Arbitrary modeling choices that rationalize average effects may miss crucial driving factors. Second, average effects are just that: going a step further to sort out technology types and market structure types, and systematically relating to them to the direction (positive, ambiguous, or negative) of announcement effects is likely to yield more insightful results for both researchers and practitioners.

In other words, unless our data sets are pre-analyzed and firms (or industries) classified as super- and submodular, we will obtain average results that may or may not apply to a particular firm or industry. There is also the likelihood that we obtain weak results since the effects of the two phenomena may be canceling each other out, or even the reverse of expected results if one or the other phenomenon overwhelms the data or choice of industries.

Accordingly, much research remains to be done.

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