Diversification and Performance:

Linking Relatedness, Market Structure and the Decision to Diversify

Ron Adner and Peter Zemsky*

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Abstract

An extensive empirical literature in strategy and finance studies the performance implications of corporate diversification. Two core debates in the literature concern the existence of a diversification discount and the relative importance of industry relatedness and market structure for the performance of diversifiers. We address these debates by building a formal model in which the extent of diversification is endogenous and depends on the degree of industry relatedness. Firms' diversification choices affect both their own competitiveness and market structure. We find a non-monotonic effect of relatedness on performance: while greater relatedness increases the competitiveness of diversified firms, it can also spur additional diversification, thereby eroding market structure and performance. In addition, our model elucidates the emergence of heterogeneity in firm scope strategies. We use the model to generate data and show how the negative effect of relatedness on market structure can give rise to spurious inference of a diversification discount in cross-sectional regressions.

Key words: diversification discount, horizontal scope of the firm, formal foundations of strategy

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1. Introduction

A fundamental question in corporate strategy is the choice of horizontal scope – the set of industries and market segments in which a firm competes. Governing this choice is a trade-off between the threat of losing focus and the opportunity to grow and exploit synergies. This trade-off raises the question of whether and when diversification is profitable. Understanding the drivers of successful diversification has been a pillar of the strategy research agenda from the founding of the field (e.g. Penrose, 1959; Ansoff, 1957).

In this paper, we seek to elucidate the relationship between diversification and performance by developing a formal model in which diversification decisions are endogenous. In so doing, we contribute to the growing literature on the formal foundations of strategy, which has so far largely ignored issues of corporate strategy. Prior work on the formal foundations of strategy has focused on the general issue of value creation and value capture (Brandenberger and Stuart, 1996; Lippman and Rumelt, 2003; MacDonald and Ryall, 2004), the workings of strategic factor markets in which firms acquire valuable resources (Makadok, 2001; Makadok and Barney, 2001) and on the sustainability of competitive advantage at the business unit level (Adner and Zemsky, 2006). Given the extensive attention paid to corporate level issues in the strategy field, we think that extending formal work into the realm of corporate strategy is a natural and important next step.

Empirical work on the relationship between performance and diversification has a long history in both the strategy literature (e.g., see Ramanujam and Varadarajan, 1989; and Montgomery, 1994 for reviews) and in the finance literature (e.g., see Martin and Sayrak, 2003 for a review). Work in finance, in particular, has centered on a debate between two competing views of diversified firms. One view is that diversified firms are able to exploit superior information to make better resource allocation choices through their internal capital markets than could financial markets (Caves, 1971; Myers and Majluf, 1984). A competing view is that diversified firms are plagued by inefficiencies due to agency problems and that resources would be better allocated between businesses by financial markets (Amihud and Lev, 1981; Schliefer and Vishney 1989). The observation that diversified firms trade at a discount to their more focused peers is taken as evidence of unresolved agency problems and poor corporate governance. Numerous studies have supported the existence of such a diversification discount (e.g., Montgomery and Wernerfelt, 1988; Lang and Stulz, 1994; Berger and Ofek, 1995).

The strategy literature, with its fundamental concern with firm heterogeneity, has had a different focus. It has sought to explain differences in performance among diversified firms. Early contributions focused on the extent to which relatedness among corporate businesses was associated with higher returns (Rumelt, 1974; Bettis, 1981). Later contributions sought to link the nature of firm resources with the type of diversification in which firms engaged (Montgomery and Wernerfelt, 1988; Chatterjee and Wernerfelt, 1991). The basic notion, going back to Penrose (1959), is that the greater the relatedness among the markets within which the firm competes, the greater the scope for sharing resources across business units and hence the greater the performance of diversified firms. Competing with this internal, resource-based perspective on diversification performance, is a research stream that emphasizes the importance of external competitive pressures, specifically the importance of industry attractiveness and market structure (Christensen and Montgomery, 1981; Montgomery, 1985) and the potential to manage competition through multi-market contact (Karnani and Wernerfelt, 1985; Gimeno and Woo, 1999).

In both strategy and finance, early empirical work took the decision to diversify as exogenous. More recent empirical contributions have explicitly incorporated the endogeneity of the diversification decision. Several papers use empirical methods (e.g. Heckman, 1979; Deheja and Wahba, 2001) that control for endogeneity by explicitly allowing for the possibility that underlying differences among firms affect both firm performance *and* the decision to diversify (Campa and Kedia, 2002; Villelonga, 2004a, 2004b; Graham et al., 2002). These papers suggest that diversifiers are different from non diversifiers. When researchers control for these differences, they fail to find a diversification discount, and in some cases, they find a diversification premium. In other words, they argue that weaker firms are inherently more likely to diversify, and that it is this underlying weakness that is responsible for their low performance, rather than their diversification strategy *per se*. The debate, however, is not yet settled (see for example the round-table discussion in Villelonga, 2003).

In parallel to these empirical studies, several theoretical papers in finance have modeled the diversification decisions of firms. Borrowing from the strategy literature, these papers usually start with a set of firms that vary in their capabilities. They then examine the diversification decisions of individual firms in isolation from their peers. In Maksimovic and Phillips (2002) and in Gomes and Livdan (2004) firms with high productivity specialize in a single industry while those with lower productivity chose to diversify. In Matsusaka (2001) and in Bernardo and Chowdhry (2002) firms choose to engage in costly diversification as a way to search for new opportunities to leverage their capabilities. Consistent with the recent empirical findings, all of these theories predict a spurious diversification discount. That they obtain these results with rational, profit maximizing firms calls into question prior claims about the pervasiveness of agency problems in corporate strategy (Jensen, 1986).

In contrast with the recent finance literature, which has focused almost exclusively on the impact of firm heterogeneity, the strategy literature on diversification has emphasized three distinct drivers of diversification performance: firm heterogeneity, industry relatedness and the extent of competitive pressures. Clearly, each of these drivers of profitability should impact the decision to diversify. The received theory, with its exclusive focus on the decision of isolated firms, has overlooked the effects of competitive interactions. Recent empirical studies in strategy (Stern and Henderson, 2004, examining the US personal computer industry; Bowen and Wiersema, 2005, examining the entry of foreign-based rivals) have begun to explicitly link competition and endogenous diversification decisions. We hope that further theory development can help to guide future empirical work.[1]

We contribute to the received literature by developing a formal model of diversification decisions by multiple competing firms that simultaneously considers industry relatedness and market structure. The main elements of our model are as follows. There are two industries and firms have a choice between a specialist strategy, which involves competing in only one industry, and a diversification strategy, which involves competing in both industries and incurring additional fixed costs. We allow firms to vary in their competitiveness, either due to differences in marginal costs or due to differences in consumer willingness to pay for their offer. The degree of industry relatedness determines whether diversification has a positive effect

[1] In a recent working paper, Levinthal and Wu (2005) formalize ideas from Penrose (1959) by considering the effect of capacity constrained capabilities on diversificaiton decisions and performance. Their focus on capabilities and relatedness in a single firm model complements our focus on competitive interactions and relatedness in a multi firm model.

on firm competitiveness ("synergies") or a negative effect ("loss of focus"). Market structure depends on the number of rivals in each industry and on their competitiveness.

We decompose the decision to diversify into three elements: the fixed cost associated with diversification (F), the revenue growth from entry into the second industry (G), and the effect on revenues in the home industry due to shifts in competitiveness (H). A firm chooses to diversify when the additional fixed cost burden is less than the net increase in revenue ($F < G + H$).

A key building block in the analysis is the effect of relatedness on the decision to diversify. Increased relatedness between the industries has two effects. First, increased relatedness creates more opportunities to share fixed costs across businesses, which lowers the fixed cost burden associated with diversification. Second, increased relatedness enhances the competitiveness of diversified firms relative to specialized firms by creating more opportunities at the corporate level to reduce marginal costs or increase consumer willingness to pay. Increased competitiveness increases revenues in both the home market and the target market. Hence, increased relatedness makes it more likely that diversification is profitable.

Now consider the impact of increases in relatedness on performance. Holding fixed the number of diversified and specialized firms, the profits of diversified firms increase with relatedness. At the same time, the profits of specialized firms decrease due to the increased competitiveness of their diversified rivals. However, this does not account for the endogeniety of diversification decisions. With sufficient increases in relatedness, firms that would have previously chosen to specialize now choose to diversify and enter a second industry. This deterioration in market structure lowers the profits of both diversifiers and specialists. Thus, while the effect of increased relatedness on the profits of specialized firms is unambiguously negative, the effect on diversified firms is non-monotonic. Overall, however, as relatedness increases from a low level up to a high level we find that the profits of all firms fall.

In many industries one observes differences in firm scope strategies. For example, automakers vary in the range of vehicles they produce and the geographies that they serve; some information technology firms like IBM and HP pursue broad strategies while other like SAP and Sun Microsystems pursue more focused strategies. The resource-based view of strategy explains such differences in product market positions with differences in the underlying resource-base of the firms (Wernerfelt, 1984).

Our theory explains firm heterogeneity in scope strategy even though all firms are initially the same. While one might expect that, absent resource heteroegneity, either all firms would chose to diversify or none would, we show that this need not be the case. Rather, we show that the number of diversified firms increases incrementally with increases in the degree of market relatedness. The level of diversification increases incrementally in our model because each new diversifier increases competition and this lowers the returns from additional diversification. Hence, not all firms will find it profitable to bear the fixed costs of diversifying. The more related are the two industries, the greater the returns to diversification and the greater the number of firms that choose to diversify. Because diversification strategy affects competitiveness, the differences in firm scope strategies give rise to heterogeneity in market shares and profits.

In our model, diversification can either enhance or reduce the competitiveness of diversifiers relative to specialists. We show that firms may rationally diversify even in the case where diversification decreases their competitiveness, thus lowering their revenues in the home market, because of offsetting revenue growth in the new market. We consider how outcomes depend on whether diversification increases or decreases competitiveness. When competitiveness decreases, we find that either specialists or diversifiers can have higher profits, depending on the level of relatedness. We also find that firms face a coordination problem in that there may be multiple possible levels of diversifications. This is because diversification by a given firm weakens it in its home market, which can, in turn, induce another firm to diversify to exploit this weakness. In contrast, when competitiveness increases with diversification, we find that the profits of diversified firms are always higher than those of specialists and that there is a unique level of diversification for a given level of relatedness.

To make explicit the implications of our theory for empirical work on the diversification discount we use our model to generate cross-sectional data. We analyze the data with different OLS regression models. The data are composed of fifty industry pairs that vary in their degree of relatedness. Within each pair, there are four firms, which behave according to our theoretical model. We focus on parameters such that diversifiers have higher profits than specialists within any given industry pair. However, industry pairs with more diversified firms (due to higher levels of relatedness) tend to have lower profits due to increased competitive pressures. We show that a spurious inference of a diversification discount can arise. This occurs with empirical specifications that do not include effective controls for industry relatedness. We show that controlling for relatedness will correctly identify the underlying diversification premium, but that, absent controls for market structure, the regression analysis will lead to incorrect inference regarding the effect of industry relatedness.

As in any formal modeling exercise, we have had to make important simplifying assumptions. These assumption contribute to the model's tractability and make the mechanisms underlying the results more transparent. First, we assume that the two industries are symmetric in terms of demand and cost conditions. Second, we assume that firms are initially homogeneous (but we do show how heterogeneity emerges from firms' diversification choices). Third, we take the number of firms as exogenous, which means that we can not address issues of entry and exit or of mergers and acquisitions. However, our model is highly tractable, and as we discuss in the conclusion, all of these simplifications are potential avenues for further developing a formal theory of corporate strategy.

The paper proceeds as follows. Section 2 describes the model. Section 3 considers the benchmark where diversification decisions are exogenous. Section 4 characterizes the drivers of a firm's diversification decision and then Section 5 characterizes how the equilibrium level of diversification varies with market relatedness. Section 6 considers the relative profitability of specialized and diversified firms. Section 7 presents the analysis of the simulated data and Section 8 contains a concluding discussion.

2. The Model

We develop a simple, formal model. There are two markets which we label A and B. These could be entirely different industries or they could be two market segments within a single industry. We assume that the two markets have the same underlying attractiveness including the same number of potential buyers, S. The actual attractiveness and realized size of each market will, however, vary with the number of firms that enter. There are $n > 1$ firms that are initially identical. Firm heterogeneity arises from differences in their choice of scope strategy.

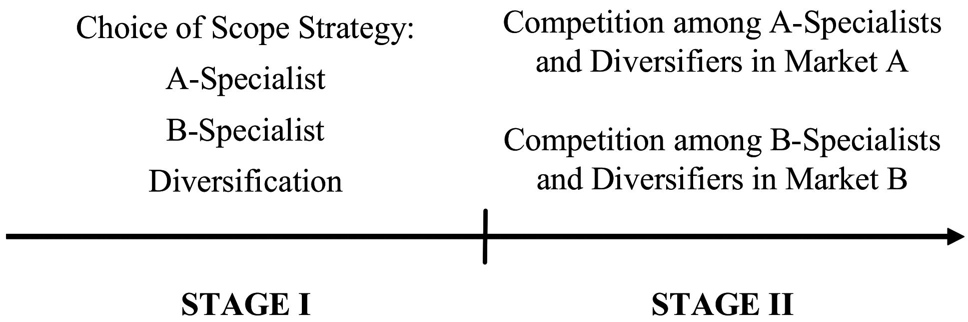

Figure 2.1: The timing of the model

2.1. Model Timing

Our model is a two-stage game. In two-stage games, the first stage incorporates the decisions of interest and the second stage incorporates competitive interactions. The timing of the model is illustrated in Figure 2.1. In the first stage, the n firms simultaneously decide on one of three scope strategies: specialize in market A, specialize in market B or diversify by entering both A and B. Let n_A denote the number of firms specializing in market A, let n_B denote the number of firms specializing in market B, and let $n_D = n - n_A - n_B$ denote the number of firms that diversify. In the second stage profits are determined by competition among those firms active in each market. Thus, the n_D diversified firms and the n_A specialized firms compete in market A; the n_D diversified firms and the n_B specialized firms compete in market B.

For a two-stage model to be appropriate, it is important that the first stage decisions be harder to change than second stage decisions because firms are assumed to be committed to their first stage decisions when they compete in the second stage. In our model, diversification decisions in the first stage are stickier and longer term decisions than are price and output choices in the second stage. Hence, a two-stage model seems appropriate.

We follow the standard approach in studying two-stage games of focusing on subgame perfect equilibria, which rules out equilibria that can only be supported by non-credible threats. In addition, we focus on pure strategy equilibria so that firms are not randomly choosing their scope strategies.

2.2. Stage I: The Effects of Diversification

A firm's choice of scope in stage I affects both its fixed costs and its competitiveness. The degree of relatedness between the two markets determines exactly how fixed costs and competitiveness are impacted by the decision to diversify.

Firms incur a fixed cost of $\hat{F} \geq 0$ when they diversify and enter both markets. $\hat{F}$ represents the additional fixed costs required for entry into the second industry, such as new product design, advertising and qualifying new suppliers. The more related are the two industries – the more they share technologies and customers – the lower is $\hat{F}$.

A firm's competitiveness in a market is the level of value that it creates with its product and service offering. Value creation is the gap between consumers' willingness to pay for the firm's offer and the firm's marginal cost of production (Brandenberger and Stuart, 1996). We let v_{ij} be an index of firm i's value creation for a consumer in market j. A firm's horizontal scope strategy determines its value creation as follows. Firms specialized in market A have $v_{iA} = v_S > 0$ and $v_{iB} = 0$ while firms specialized in market B have $v_{iA} = 0$ and $v_{iB} = v_S$, where v_S is the value creation of specialized firms in their home market. Diversified firms have $v_{iA} = v_{iB} = v_D$, where v_D is the value creation of diversified firms.

We make no restriction on the whether v_S or v_D is larger. Let $k = v_D/v_S$ be the competitiveness of diversified firms relative to specialized firms. The case of $k < 1$ corresponds to diseconomies of scope where diversified firms are less competitive than specialized firms, which could result, for example, from a loss of focus. The case of $k > 1$ corresponds to synergies where diversified firms are more competitive than specialized firms, which could result, for example, from the ability to better utilize production capacity or to offer customers the convenience of a "one-stop-shop". The more related are the two markets in terms of shared technologies and customers the greater is k.

While we make no restriction on whether k is greater or less than one, we simplify the analysis by restricting the range of its possible values to assure that no firm has such low competitiveness that it is forced out of a market. Specifically we assume that

$$\frac{n}{n-1} > k > \frac{n+1}{n+3}. \tag{2.1}$$

For example, with four firms we have that $1.33 > k > 0.625$.[2]

In summary, our model follows the strategy literature in viewing industry relatedness as a key mediating construct in determining the effect of diversification. Greater relatedness results in both a decrease in the fixed costs of diversifying ($\hat{F}$) and an increase in the relative competitiveness of diversified firms (k).

2.3. Stage II: The Outcome of Competition

One challenge with two-stage games is the need to specify the nature of competition in the second stage. There are a wide range of possible models of market competition including Hotelling models of horizontal differentiation, Cournot models of quantity competition with homogeneous products, and models of price competition with vertically differentiated products. In this paper we use Cournot competition with linear demand and heterogenous firms to model competitive interactions among the firms in stage II. For our study, the advantage of a Cournot specification is that competitive pressures are moderate. Thus, a firm can earn some profits even if it is not the most efficient firm in the market. This is important because we want to allow for the possibility that firms diversify even when there are diseconomies of scope (i.e. $k < 1$). Moreover, we want to allow for profits to decline gradually as more firms pursue a given scope strategy. A Cournot model with linear demand is the simplest model that delivers these properties.

As detailed in Appendix I, Cournot profits can be expressed as a function of the value creation of the firms competing in a market. Specifically, the profits of firm i in market j depend on the number of rivals in the market (N), the firm's value creation for a consumer in the market (v_{ij}), its value creation relative to each of its rivals ($\sum_{m \neq i}(v_{ij} - v_{mj})$), and the size of the market (S):

$$\pi_{ij} = S \left(\frac{v_{ij} + \sum_{m \neq i}(v_{ij} - v_{mj})}{N+1} \right)^2 . \tag{2.2}$$

Note two key properties of the Cournot profit function (2.2). First, profits are falling in the number of competitors N. Second, profits depend on both absolute value creation (v_{ij}) and relative value creation ($v_{ij} - v_{mj}$), which is what makes it possible for moderately inefficient

[2] See Appendix I for a derivation of condition (2.1).

firms to still produce and earn some profits.

Substituting v_S and v_D into (2.2), the profit of a specialized firm is given by

$$\pi_S(n_S, n_D) = S\left(\frac{v_S + n_D(v_S - v_D)}{n_S + n_D + 1}\right)^2, \tag{2.3}$$

where n_S is the number of other firms specializing in the same market and n_D is the number of diversifiers. The profit of a diversified firm, which is active in two markets and must incur the fixed cost $\hat{F}$, is

$$\pi_D(n_D, n_A, n_B) = S\left(\frac{v_D + n_A(v_D - v_S)}{n_A + n_D}\right)^2 + S\left(\frac{v_D + n_A(v_D - v_S)}{n_A + n_D}\right)^2 - \hat{F}. \tag{2.4}$$

3. Exogenous Diversification

We start the analysis by considering a baseline case where firm scope strategies are exogenous. We take as given the number of firms n_D, n_A and n_B pursuing each strategy and characterize the effect of changes in relatedness on the profits of diversified and specialized firms. The analysis is straight forward.

Recall that industry relatedness affects both the fixed costs $\hat{F}$ and competitiveness $k = v_D/v_S$. The profits of specialized firms π_S are given by (2.3). This is independent of the fixed costs associated with diversification $\hat{F}$. To see the effect of k we can rewrite (2.3) as

$$\pi_S(n_S, n_D) = Sv_S^2\left(\frac{1 + n_D(1 - k)}{n_S + n_D + 1}\right)^2, \tag{3.1}$$

which is falling in the competitiveness of diversified firms. As one would expect, the more competitive are diversified firms, the lower the profits of specialists.

Now consider the profits of diversified firms π_D as given by (2.4), which we can rewrite as

$$\pi_D(n_D, n_A, n_B) = Sv_S^2\left(\frac{k + n_A(k - 1)}{n_A + n_D + 1}\right)^2 + Sv_S^2\left(\frac{k + n_B(k - 1)}{n_B + n_D + 1}\right)^2 - \hat{F}, \tag{3.2}$$

which is decreasing in $\hat{F}$ and increasing in k. The greater the additional fixed cost of diversification, the lower the profits of diversified firms; the greater the competitiveness of diversifiers, the higher their profits. Thus we have the following:

Proposition 3.1. *Holding fixed the number of firms pursuing each strategy: (i) The profits of diversified firms are increasing in market relatedness. (ii) The profits of specialized firms are decreasing in market relatedness.*

As for the relative profits of specialists and diversifiers, there are too many degrees of freedom to make any prediction. For $\hat{F}$ sufficiently high, $\pi_S > \pi_D$, while for $k \geq 1$ and $\hat{F}$ sufficiently low $\pi_D > \pi_S$.

A central objective of this paper is to understand how the effect of relatedness on the absolute and relative performance of specialists and diversifiers changes when one accounts for the endogeneity of firm scope strategies.

4. The Decision to Diversify

A useful first step towards endogenizing firm scope is to examine the incentives for a single firm to choose to diversify, holding fixed the scope strategies of the other firms. Three key elements that shape a firm's decision to diversify, which we label F, G and H.

Consider a situation where there are $n_A \geq 1$ firms specialized in market A, one of which is considering whether or not to diversify. This focal firm increases its profits by diversifying if and only if

$$\pi_S(n_A - 1, n_D) < \pi_D(n_D + 1, n_A - 1, n_B). \tag{4.1}$$

We can use (3.1) and (3.2) to rewrite (4.1) as

$$F < G + H$$

where F is the increase in fixed costs required to diversify into market B relative to the market size, G is the increase in profits coming from growth in market B, and H is the change in profits in the home market A. Specifically, we have that

$$F = \frac{\hat{F}}{S} \frac{1}{v_S^2} \geq 0$$

and what matters is the extent of fixed costs $\hat{F}$ relative to market size, where the relevant measure of market size depends on the number of consumers (S) and the value created for consumers (v_S).[3]

The growth in profits from entry into the target market B is

$$G = \left(\frac{k(n_B + 1) - n_B}{n_D + n_B + 2} \right)^2 > 0$$

which is positive and increasing in k the competitiveness of diversifiers.[4] Finally, the effect of diversification on profits in the home market A is

$$H = \frac{(kn_A - (n_A - 1))^2 - (n_D + 1 - kn_D)^2}{(n_D + n_A + 1)^2}.$$

H is increasing in k and for $k = 1$ we have $H = 0$. Thus for $k < 1$ we have that $H < 0$ and the effect on the home market serves to discourage diversification. For $k > 1$ we have that $H > 0$ and the effect in the home market encourages diversification.

A widely cited managerial prescription for making diversification decisions is to consider Porter's three tests (Porter, 1987): the cost-of-entry test, the better-off test and the attractiveness test. A benefit of our formal treatment is that it clarifies some of the ways in which the factors highlighted in the three tests interact with each other in determining the desirability of diversification. Our $F < G + H$ formula overlaps with Porter's tests as follows. The quantity F captures the cost of entry, as well as including the impact of market size, which is a key component of industry attractiveness. The better-off test concerns the effect of diversification on competitiveness in the target and the existing business, which we capture with k. A key component of the industry attractiveness test is market structure, which in our model is given by the number of specialized (n_A, n_B) and diversified (n_D) rivals. Our terms G and H are determined by the interaction of competitiveness (k) with market structure (n_A, n_B and n_D).

While F and k are exogenous to the model, G and H depend on n_D, n_A and n_B and hence on the choices of the other firms. The next section disentangles this strategic interdependence and characterizes the equilibrium level of diversification.

[3] Note that F falls with industry relatedness just as $\hat{F}$ does. Henceforth, we refer to F as the fixed costs of diversifying, leaving implicit that what matters is fixed costs relative to market size.

[4] The lower bound on k in (2.1) assures that $G > 0$.

5. The Level of Diversification

We now characterize the equilibrium level of diversification and how it varies with the extent of market relatedness. We also consider whether there is a unique equilibrium level of diversification or whether strategic interdependencies create multiple possible levels of diversification. Why is multiplicity more than a technical matter? Consider how the sentiment towards diversification has varied over time, from the 1960s, when diversification was regarded with favor, to the 1980s, when it was regarded with suspicion (Schliefer and Vishny, 1991). The existence of multiple equilibria matters because it means that coordination among firms matters. It means external influences, such as popular sentiment towards diversification, can shift behavior, even with profit maximizing firms.

The decision of a single firm to diversify as characterized in Section 4 can be expressed in terms of just two exogenous parameters, namely k and F. We focus the analysis and exposition on these two parameters, which both vary with industry relatedness.

We begin the analysis with some technical preliminaries, which established two key properties. First, specialized firms spread out as evenly as possible between the two markets in order to minimize competitive pressures. The implication is that identifying the equilibrium number of diversifiers is sufficient for characterizing the equilibrium outcome. Second, an equilibrium must satisfy a pair of conditions similar to (4.1), which jointly define a range of F values.

5.1. Technical Preliminaries

Denote by n_D^*, n_A^* and n_B^* the equilibrium number of firms pursuing each diversification strategy. In principle, the number of possible equilibria increases exponentially in the total number of firms, n. The number of diversifiers n_D^* can take values from 0 to n and then for each value of n_D^* the remaining firms can be divided among A specialists and B specialists in $n - n_D^* + 1$ different ways. Fortunately, the following lemma greatly simplifies the analysis.

Lemma 5.1. *The specialist firms spread themselves as evenly as possible across the two markets so that the difference is at most one (i.e., $|n_A^* - n_B^*| \leq 1$).*

Profits in each market are falling in the number of competitors. There cannot be an equilibrium where the difference in the number of specialists firm is greater than one, because a firm from the more crowded market would increase its profits by specializing in the less crowded market. Hence, given a value of n_D^* there is essentially only one possibility for the configuration of the specialist firms.[5] The number of possible equilibria is then linear in n.

Thus we have simplified the analysis of the model so that we only need to characterize how the equilibrium level of diversification n_D^* depends on the level of market relatedness as reflected in k and F. We now identify the two conditions that an equilibrium level of diversification must satisfy.

Lemma 5.2. *Necessary and sufficient conditions for $0 < n_D^* < n$ to be an equilibrium level of diversification for $n_A^* \geq n_B^*$ are*

$$\pi_D(n_D^*, n_A^*, n_B^*) \geq \pi_S(n_B^* + 1, n_D^* - 1), \tag{5.1}$$

$$\pi_S(n_A^*, n_D^*) \geq \pi_D(n_D^* + 1, n_A^* - 1, n_B^*). \tag{5.2}$$

The necessary and sufficient condition for $n_D^* = n$ to be an equilibrium is

$$\pi_D(n_D^*, 0, 0) \geq \pi_S(1, n_D^* - 1). \tag{5.3}$$

The necessary and sufficient condition for $n_D^* = 0$ is

$$\pi_S(n_A^*, 0) \geq \pi_D(1, n_A^* - 1, n_B^*). \tag{5.4}$$

Condition (5.1) assures that the firms choosing to diversify would not increase their profits by switching to a specialist strategy. Condition (5.2) assures that firms choosing to specialize in market A would not increase their profits by diversifying. This condition assures that firms in market B, who are in the more attractive market with less competition, do not want to diversify either.[6] Condition (5.3) assures when all firms are diversified it is not profitable for

[5] If $n - n_D^*$ is even, then by Lemma 5.1 $n_A^* = n_B^* = (n - n_D^*)/2$. If $n - n_D^*$ is odd, then either $n_A^* = n_B^* + 1$ or $n_A^* = n_B^* - 1$. For our purposes, it does not matter whether the extra firm is in market A or B.

[6] Given the symmetry of demand in the two markets, Lemma 5.2 focuses without loss of generality on the case where there are at least as many firms specialized in market A as in market B.

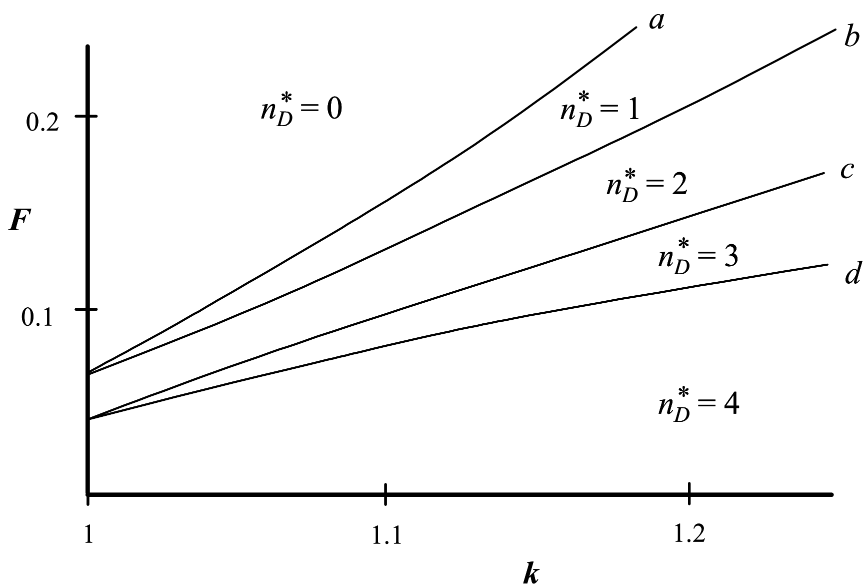

Figure 5.1: Parameter regions that support different equilibirum levels of diversification (n_D^*) for the case of $n = 4$ and $k \geq 1$. Relatedness increases from the upper left to the lower right.

any firm to deviate to a specialist strategy. Condition (5.4) assures that when all firms are specialists, it is not profitable for a firm specialized in market A to diversify.

Conditions (5.1) and (5.3) is of the same nature as condition (4.1) and hence they also define an upper bound on F such that diversifiers do not want to switch to a specialist strategy. Thus for each value of $n_D^* > 0$ there is an upper bound on F: if the fixed cost of diversifying is too high, then firms will not be willing to diversify. Condition (5.2) and (5.4) define a lower bound on F such that specialized firms do not want to diversify.

5.2. Main Results

We are now ready to state the main results on the level of diversification. Results depend on whether diversification increases competitiveness ($k > 1$) or decreases it ($k < 1$). With $k > 1$, the characterization is straightforward.[7]

Proposition 5.3. *Consider the case where diversification enhances competitiveness ($k > 1$). (i) Higher levels of industry relatedness are associated with higher levels of diversification. (ii)*

[7] A formal statement of propositions 5.3, 5.4 and 6.2 are contained in the appendix.

For any set of parameter values, there is a unique equilibrium level of diversification (except at boundaries). (iii) Any level of diversification, from no firms diversifying ($n_D^* = 0$) up to and including all firms diversifying ($n_D^* = n$), can be an equilibrium.

At the core of the results is the fact that while relatedness varies continuously, diversification is a discrete event. Thus, a given level of diversification is supported by a range of F and k values. Figure 5.1 shows the set of F and k values supporting each level of diversification for the case of $n = 4$. For example, parameter values falling between lines b and c are associated with diversification by two firms. With $k > 1$, there is a unique level of diversification except for parameter combinations that fall on one of these boundary lines. Increases in relatedness result in higher k and lower F. Referring to Figure 5.1, an increase in relatedness involves a shift from the upper left towards the lower right, which if sufficiently large, leads to an increase in the equilibrium level of diversification.

Despite our assumption that all firms are *ex ante* identical and have equal access to the diversification opportunity, we find that diversification need not be an all or nothing phenomenon. Rather only a subset of firms may choose to diversify. The reason is that there is a fixed cost to diversifying and the returns to diversification fall as more firms diversify. Thus, for intermediate levels of fixed costs it is economical for only a subset of firms to diversify.

To what extent do these results hold when diversification decreases competitiveness?

Proposition 5.4. *Consider the case where diversification erodes competitiveness ($k < 1$). (i) Higher levels of industry relatedness are associated with higher levels of diversification. (ii) There are regions of the parameter space where there are multiple equilibrium levels of diversification. (iii) Firms diversify in pairs (except for the case of the first firm to diversify when the total number of firms is an odd number).*

As in the case of $k > 1$, we have that increases in industry relatedness continue to be associated with increases in diversification. However, while with $k < 1$ there is a unique equilibrium level of diversification for every values of F and k (except at boundaries), we now have regions of the parameter space where there are multiple equilibrium levels of diversification. In addition, it is no longer the case that every level of diversification can be supported in equilibirum. Both of these differences arise because the case of falling competitiveness introduces greater strategic interdependence.

With $k > 1$ diversification by a rival firm had a preemptive effect and lowered the incentive of all other firms to diversify. In contrast, when diversification lowers competitiveness, a firm's decision to diversify from market A into market B *increases* the incentive for firms specialized in B to diversify into A. Why? With $k < 1$, the average level of competitiveness of firms active in market A is now lower, which increases the growth opportunity from entry into market A (i.e., higher G). Simultaneously, because the diversifier's entry into market B makes that market less attractive, B specialists are more willing to diversify and suffer the degradation of their competitiveness at home (i.e., H is less negative). For both reasons, the incentive to diversify increases (i.e., higher $G + H$).

An implication of this kind of strategic interdependence is that firms diversify in pairs, one from each industry. The exception is when no firm has yet diversified and there are an unequal number of firms specialized in each market, in which case initial diversification is pursued by a single firm.

The increasing incentive to diversify when $k < 1$ explains the existence of multiple equilibria. Consider a pair of firms. One possibility is that each firm expects the other to diversify. Another is that each firm expects the other to specialize. Because a firm's incentive to diversify (specialize) increases if its rival is expected to diversify (specialize), both sets of expectations can be self-fulfilling, which results in multiple equilibrium levels of diversification. The possibility of multiple equilibria raises the issue of coordination among firms in related industries.

The case of $k < 1$ is illustrated in Figure 5.2 for the case of $n = 4$. Recall that when $k > 1$, the regions between a and b is where the unique equilibrium involves one firm diversifying. For $k < 1$ the lines cross over and there is no longer a region where one firm diversifies. Rather, there is now a region of multiple equilibria in which either 0 or 2 firms diversify. This is a region in which coordination matters. There is also a region where either 2 or 4 firms diversify (between lines d and c) and even a small region where either 0, 2 or 4 firms diversify (between lines d and a).

Despite some significant differences between the cases of $k > 1$ and $k < 1$, there is still a positive association between the level of diversification and the level of relatedness (i.e., n_D^*

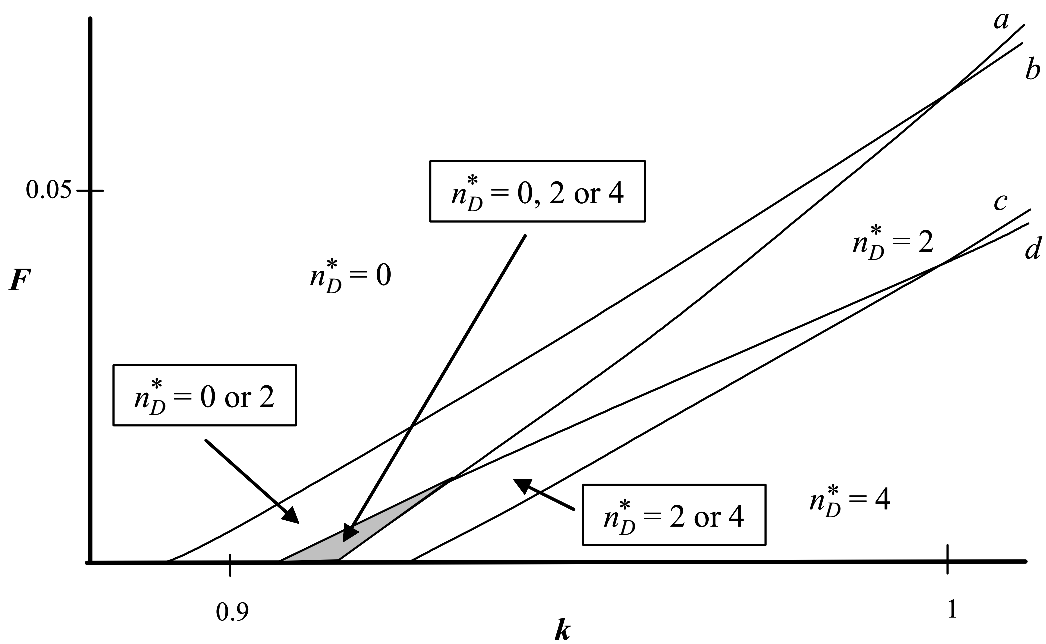

Figure 5.2: Parameter regions that support different equilibirum levels of diversification (n_D^*) for the case of $n = 4$. Relatedness increases from the upper left to the lower right.

increases with shifts from the upper left of Figure 5.2 towards the lower right).

6. Diversification and Profits

When the number of diversified firms is fixed, greater industry relatedness increases the profits of diversified firms while decreasing the profits of specialized firms, as shown in Proposition 3.1. However, greater industry relatedness can also trigger an increase in the number of diversified firms, as shown in Section 5. Therefore, to fully characterize the effect of relatedness on profits, we need to characterize the effect of increasing levels of diversification on profits.

Proposition 6.1. *(i) For $k > 1$, diversification by one firm decreases the profits of all other firms. (ii) For $k \leq 1$, diversification by one firm decreases the profits of firms that are already diversified (except possibly in the case $n_A = n_B = n_D = 1$), decreases the profits of firms that are specialized in the target market, and weakly increases the profits of firms specialized in the diversifier's home market.*

Diversification impacts the profits of other firms in two ways. First, it increases the number of competitors in the target market. For firms specialized in this market, this is the only effect and they are necessarily worse off. The second impact comes from the changed competitiveness of the diversified firm. For firms specialized in the home market of the diversifier, this is the only effect and their profits decrease in the case of $k > 1$ and increase in the case of $k < 1$.[8]

The effect of additional diversification on the profits of existing diversifiers incorporates both the impact of increased competition in the target market and the change in the new diversifier's competitiveness. For $k > 1$, both effects go in the same direction and existing diversifiers are unambiguously worse off. For $k < 1$, the effects go in opposite direction but the effect of increased competition usually dominates so that the profits of existing diversifiers are still falling.[9]

In summary, there are two effects of increased relatedness. First, diversifiers experience an increase in competitiveness and a decrease in fixed costs. Second, market structure deteriorates because more firms choose to diversify and hence there are more firms active in each market. For specialized firms, both effects undermine profitability. For diversified firms, this raises an important question: does the deterioration in market structure offset the benefits of greater relatedness?

Figure 6.1 illustrates the net effect of increasing relatedness when $n = 4$ and F is large enough such that diversification only occurs when $k > 1$. The dashed lines show the falling profits of the specialists as the number of diversifiers increases from $n_D^* = 0$ up to $n_D^* = 3$.[10] The solid lines show the profits of the diversifiers, which are highly non-monotonic.

Figure 6.2 shows the relationship between profits and relatedness for a case where F is low enough that diversification occurs when $k < 1$. Here, because firms diversify in pairs, there are only three possible equilibrium levels of diversification ($n_D^* = 0, 2, 4$). Note that in both Figure 6.1 and 6.2 there is a strong net negative impact on industry profits as relatedness increases

[8] In the case of $k < 1$, we know from Proposition 5.4 that diversification happens in pairs. Hence even a firm that benefits from the reduced competitiveness of a rival that diversifies out of the home market will also face new competition from a firm diversifying into the home market. This increase in competition can more than offset any gains in relative competitiveness for the remaining specialists such that profits decline for all firms. See Figure 6.2 for an illustration.

[9] The only case where the effect of increased competition might not dominate is when $n = 3$ and there is one firm of each type (i.e., $n_A = n_B = n_D = 1$).

[10] Note that when $n_D^* = 1$ there are two different levels of profits for specialized firms depending on whether they are in the market with one or two specialists.

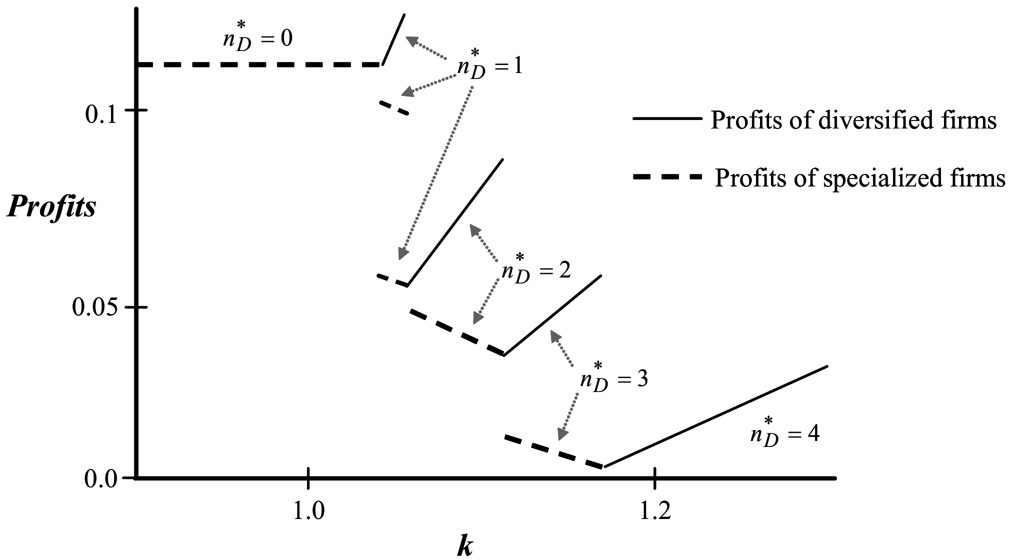

Figure 6.1: The effect of relatedness (k) on the equilibrium profits of specialized firms (dashed lines) and diversified firms (solid lines) when $F = .1$ and $n = 4$.

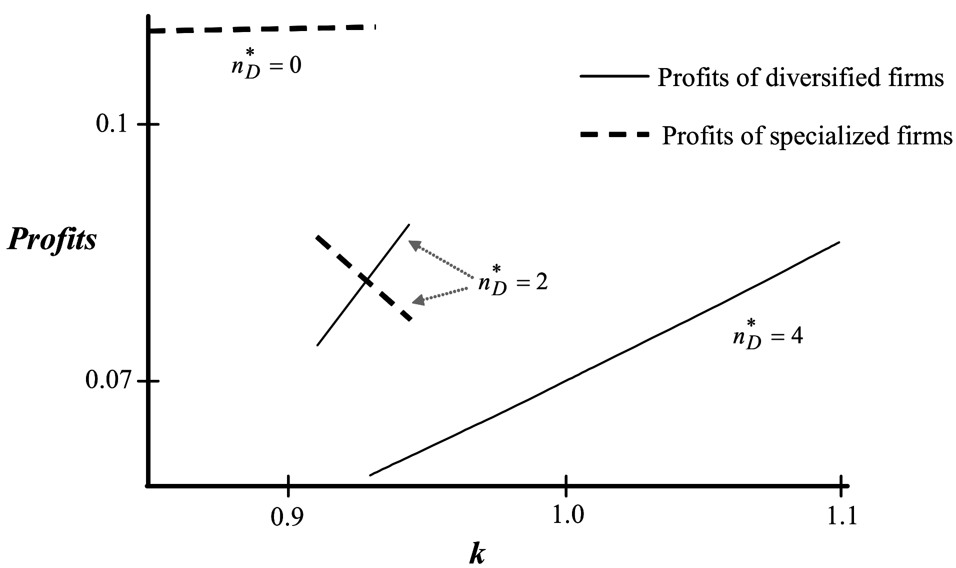

Figure 6.2: The effect of relatedness (k) on the equilibrium profits of specialized firms (dashed lines) and diversified firms (solid lines) when $F = .01$ and $n = 4$.

and the level of diversification increases. This is a general property of the model, regardless of whether the increase in relatedness affects competitiveness (as shown in Figures 6.1 and 6.2) or fixed costs.

Proposition 6.2. *An increase in industry relatedness involving either k or F such that there is a shift from minimal diversification ($n_D^* = 0$) to a point where maximal diversification ($n_D^* = n$) is just possible, reduces the profits of all firms.*

Our theory applies to diversification across industries as well as to diversification across market segments within an industry. For example, the move by the major auto manufacturers to expand across geographies (e.g., US, Europe, South America, Asia) and across product lines (e.g., light trucks, sport utility vehicles, luxury sedans) might correspond to increases in relatedness among the segments and, consistent with our theory, is associated with declining industry attractiveness due to increased rivalry.

We now consider the extent to which there is a clear prediction from the theory regarding the relative profitability of specialized and diversified firms. Note that in Figure 6.2 there is no clear ordering of these profits (i.e. the profits of specialists may be greater or less than the profits of diversified firms when $n_D^* = 2$, depending on the value of k). In contrast, in Figure 6.1, where the values of k are all greater than 1, we have that the profits of diversified firms are greater than those of specialized firms for any given n_D^*. This is a general result when $k \geq 1$:

Proposition 6.3. *Suppose that $k \geq 1$ and diversified and specialized firms coexist (i.e., $0 < n_D^* < n$). The profits of diversified firms are strictly greater than the profits of the specialized firms with which they compete.*

Our result that diversifiers have higher profits than specialists when $k \geq 1$ is related to the existence of positive profits in IO models of entry with fixed costs (see, for example, Sutton, 1991). In such models, the equilibrium level of entry is one where profits are at least as great as the fixed cost of entry, but where they would fall below this level were an additional firm to enter. This allows entrants to have positive profits, which are greater than the zero profits of non-entrants. In our model, diversifiers correspond to the entrants and specialists correspond to the non-entrants.

A major difference between our model and IO entry models is that we allow diversification to affect a firm's competitiveness in its home market. When this effect is negative, we show in Proposition 5.4 that firms diversify in pairs. The diversifying firms impose a negative externality on each other. This additional effect, not present in traditional entry models, is what causes the profits of diversifiers to sometimes fall below those of specialists when $k < 1$.

Our results linking diversification and profits have implications for the existence of a diversification discount or premium. An implication of Proposition 6.3 is that among firms competing in a given industry pair, there is a diversification premium as long as $k > 1$. On the other hand, when $k < 1$, one can get either a diversification discount or premium. Somewhat counter intuitively, in our model a diversification discount can only occur when the fixed costs of diversification (F) are low such that firms diversify when $k < 1$.

Thus far we have focused on comparisons of specialists and diversifiers *within* a given industry pair. What are the implications for cross sectional data that pool observations *across* many different industry pairs?

7. Simulated Cross-Sectional Data

We now present a simple exercise designed to explore how the linkages that we have identified among industry relatedness, market structure, diversification decisions, and profits might impact empirical inferences about the relationship between diversification and performance. To this end we generate cross-sectional data from the model and then consider the results generated by different regression specifications. The regressions have firm profits as the dependent variable and vary in the controls and interactions that they consider.

7.1. Data Generation

We construct the data set as follows. We consider fifty industry pairs that vary in their degree of relatedness. There are four firms competing in each industry pair ($n = 4$). The restriction to $n = 4$ is to simplifying the coding of the data generation; the underlying theory holds for any number of firms. For each industry pair, we generate the equilibrium scope strategies of the firms and record the resulting profits, output quantities, the degree of relatedness for the industry pair, and the firm scope strategies themselves.

The range of industry-pair relatedness is determined as follows. From the theory, for any given level of fixed costs there is a lower bound on the level of competitiveness (k) required for at least one firm to diversify, and an upper bound beyond which all firms diversify. We set the value of fixed costs at $F = 0.1$ as in Figure 6.1 and then identify the associated lower bound ($k = 1.044$) and upper bound ($k = 1.173$). We construct our sample to begin and end at k values so as to extend this range by 30% above and below these critical cutoffs.[11] We make our observations at fifty levels of k uniformly distributed along this range (i.e., $k = 1.005$, $1.010, \ldots, 1.215, 1.220$).[12]

In our setting, industries only vary along two dimensions: relatedness and market structure. We do not include controls for the underlying attractiveness of a given industry since, by construction, all industries are the same in this regard (e.g. the same size parameter S and the same demand function). One can interpret this as a situation where the empiricist has effectively controlled for differences in the underlying industry attractiveness.

We construct a market structure variable using the weighted average of the Herfindahl indexes corresponding to the industries in which a firm competes. Specifically, the market structure variable associated with firm i is

$$H_i = \frac{q_{Ai}}{q_{Ai} + q_{Bi}} \sum_{j=1}^{n} \left(\frac{q_{Aj}}{Q_A}\right)^2 + \frac{q_{Bi}}{q_{Ai} + q_{Bi}} \sum_{j} \left(\frac{q_{Bj}}{Q_B}\right)^2$$

where Q_A and Q_B are the total output in each market and q_{Ai} and q_{Bi} are firm i's output in each market.[13]

Our data thus consist of profit levels (π_i) for firms pursuing strategies of diversification ($D_i = 1$) and specialization ($D_i = 0$) in industries that vary in their level of relatedness (k_i), where relatedness varies in terms of its effect on competitiveness rather than on fixed costs, and where we have used a weighted Herfindahl index (H_i) to capture the market structure of

[11] We also constructed data sets with k ranges 10%, 20% and 40% above and below the critical cutoffs. The qualitative results were robust in all cases.

[12] For the level of fixed costs that we focus on, diversification only occurs for values of $k > 1$. Hence, the construction of the data is simplified by the fact that there is a unique equilibrium outcome (as long as one avoids boundaries), as highlighted in Proposition 5.3.

[13] Firm outputs are taken from the Cournot model detailed in Appendix I.

Variable	Model 1	Model 2	Model 3	Model 4	Model 5
Constant	0.0705	0.7128	0.9178	-0.0946	0.1861
Diversification (D_i)	$-.0286$	0.0231	-0.3208	-0.0430	-0.5664
Relatedness (k_i)	–	-0.604	–	–	–
Structure (H_i)	–	–	–	–	–
$D_i \times k_i$	–	–	-0.4831	–	0.1680
$(1 - D_i) \times k_i$	–	–	-0.7967	–	-0.2275
$D_i \times H_i$	–	–	–	0.6102	0.7791
$(1 - D_i) \times H_i$	–	–	–	0.4093	0.3133
R^2	0.1588	0.7960	0.8366	.9767	0.990
Average Premium	-2.86%	2.31%	2.33%	2.54%	2.63%

Table 7.1: OLS regressions for firm profits using simulated data and the implied diversification premium

the industries in which a firm competes.

Figure 7.1 plots the data. Note that diversifiers always have higher profits than specialists within a given industry pair (i.e., corresponding to a given level of relatedness). That is, there is a diversification premium in the data.

7.2. Data Analysis

We use OLS regression to examine the relationship between firm profits and diversification status. We consider a series of increasingly well specified models – well specified with regards to the theory used to generate the data – to explore the effects of omitted variables and interactions on the inferences that can be drawn. The results are reported in Table 1.

In the baseline case, Model 1, we regress firm profits on diversification status alone, $\pi_i = a + \beta_1 D_i$. Diversification is found to have a negative effect on profits, with an average diversification discount of 2.86%. With only one degree of freedom, the fit to the data is low ($R^2 = 0.159$), despite the absence of noise in the data. Figure 7.2 plots the predicted profits of diversified and specialized firms against the data.

We know that the finding of a diversification discount is spurious and does not correspond to the underlying process that generated the data. The problem is not that diversification *per se* lowers performance but that diversifiers tend to be in more related industry pairs and face higher levels of competition.

Model 2 adds a control for relatedness, under the assumption that k_i is observable by the

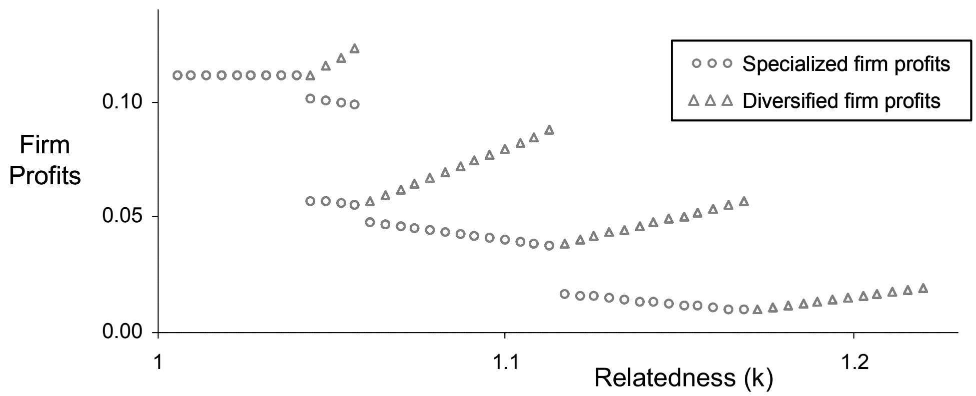

Figure 7.1: The simulated data set

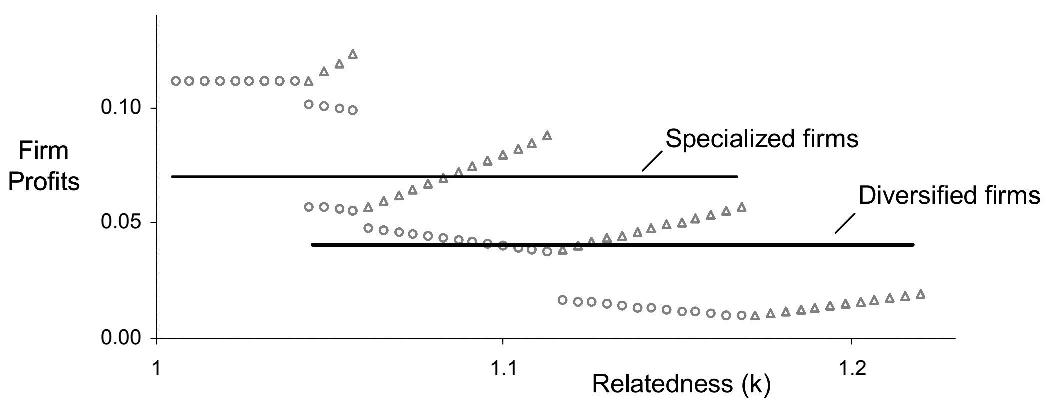

Figure 7.2: Estimated firm profits based on Model 1

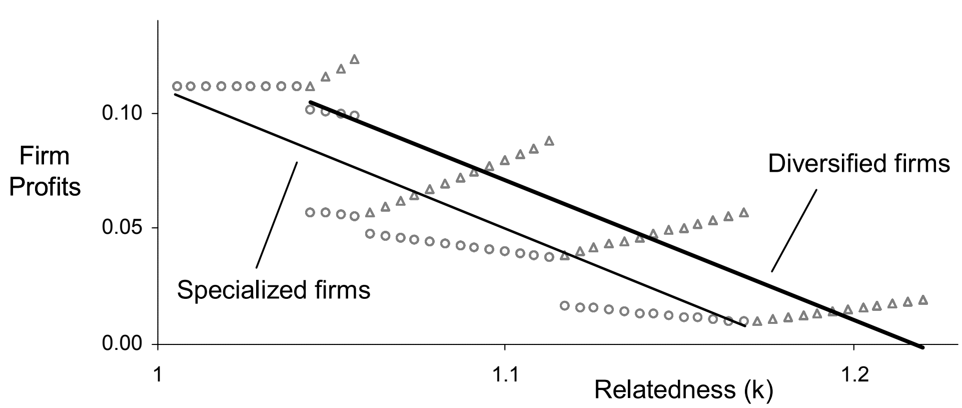

Figure 7.3: Estimated firm profits based on Model 2

empiricist:

$$\pi_i = a + \beta_1 D_i + \beta_2 k_i.$$

Diversification is now found to have a positive effect on profits, with a diversification premium of 2.31%. Relatedness is found to have a strongly negative effect on profits ($\beta_2 = -0.604$). The fit to the data is much higher ($R^2 = 0.796$). Figure 7.3 plots the predicted profits of diversified and specialized firms against the data. With the control for relatedness, the regression uncovers the underlying diversification premium. This specification, however, overlooks the fact that relatedness should have a differential impact on the performance of diversifiers and specialists.

Model 3 adds an interaction between relatedness and diversification status:

$$\pi_i = a + \beta_1 D_i + \beta_2 k_i D_i + \beta_3 k_i (1 - D_i).$$

The diversification premium increases to 2.81%.[14] Relatedness is found to have a strongly negative effects on the profits of all firms, and the effect is more negative for specialists ($\beta_3 = -0.797$) than for diversifiers ($\beta_2 = -0.483$). The fit to the data increases marginally ($R^2 = 0.837$). Figure 7.4 plots the predicted profits. The reason for the negative effect of relatedness on the profits of diversified firms is that relatedness captures both the increasing competitiveness of diversified firms and the erosion of market structure that comes from increased levels of diversification.

Model 5 presents a complete specification in which the concentration measure is interacted with diversification status:

$$\pi_i = a + \beta_1 D_i + \beta_2 k_i D_i + \beta_3 k_i (1 - D_i) + \beta_4 H_i D_i + \beta_5 H_i (1 - D_i).$$

There is a diversification premium of 2.63%.[15] The effect of relatedness is now positive for diversifiers ($\beta_2 = 0.168$) and negative for specialists ($\beta_3 = -0.228$). The effect of market concentration is strongly positive for both diversifiers ($\beta_4 = 0.780$) and for specialists ($\beta_5 =$

[14] Given the interaction term we calculate the diversification premium as $\beta_1 + (\beta_2 - \beta_3)\bar{k}$ where $\bar{k} = 1.097$ is the average relatedness in the data set.

[15] We calculate the diversification premium as $\beta_1 + (\beta_2 - \beta_3)\bar{k} + (\beta_4 - \beta_5)\bar{H}$ where $\bar{k} = 1.097$ and $\bar{H} = 0.341$ is the average weighted Herfindahl in the data set.

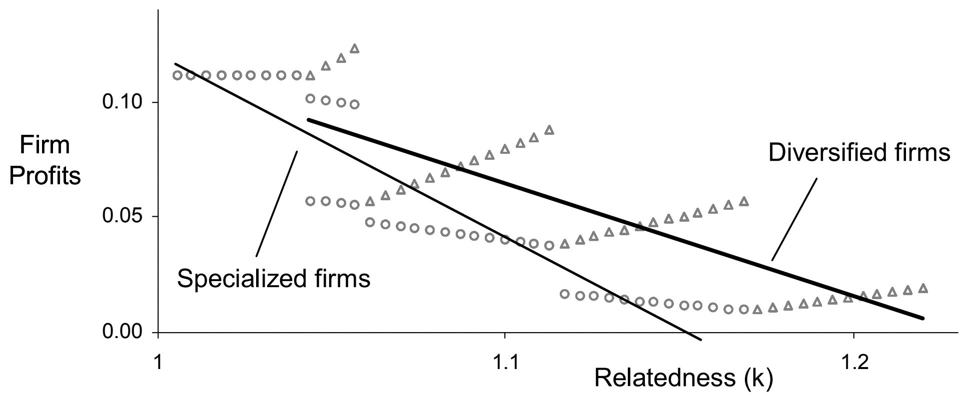

Figure 7.4: Estimated profits based on Model 3

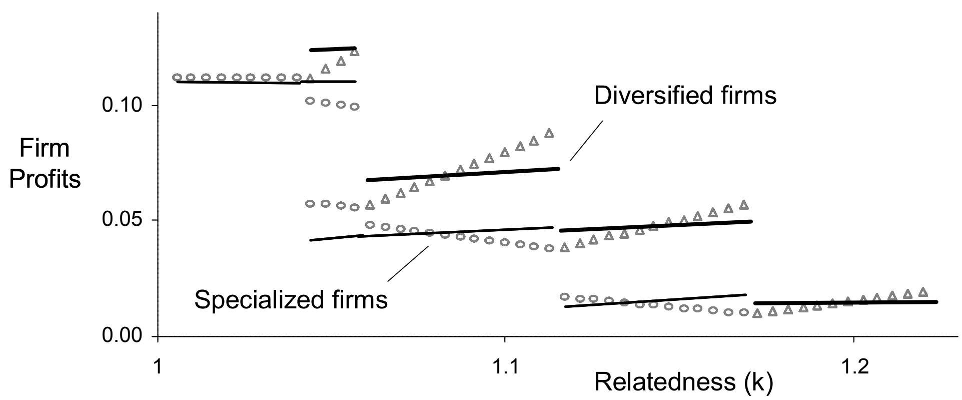

Figure 7.5: Estimated profits based on Model 4

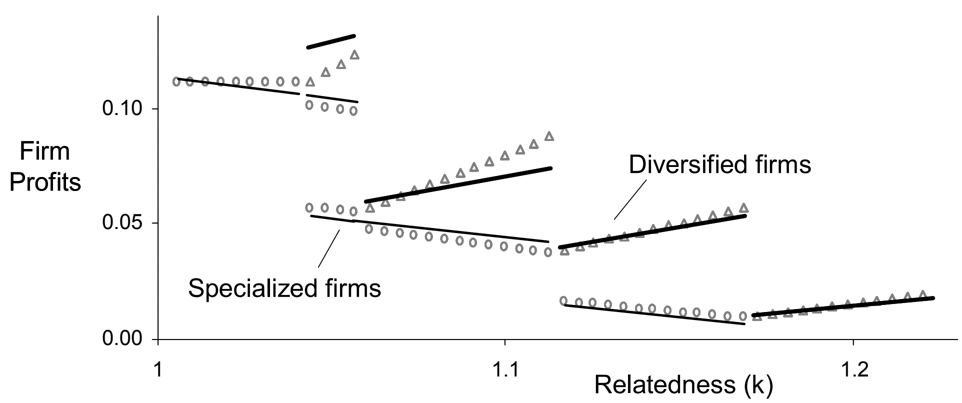

Figure 7.6: Estimated profits based on Model 5

0.313). The fit is now almost perfect ($R^2 = 0.990$). Figure 7.5 plots the predicted profits.

The final exercise is to consider the inferences when the empiricist can observe concentration but not relatedness, which leads to Model 4:

$$\pi_i = a + \beta_1 D_i + \beta_2 H_i D_i + \beta_3 H_i (1 - D_i).$$

There is a diversification premium of 2.54%.[16]. The effect of market concentration is strongly positive for both diversifiers ($\beta_2 = 0.610$) and for specialists ($\beta_3 = 0.410$). The fit is very high ($R^2 = 0.980$). Figure 7.6 plots the predicted profits. Note that the fitted lines vary with relatedness even though it is not controlled for, because concentration varies with relatedness. Controls for either market relatedness or market structure are sufficient to uncover the underlying diversification premium in this data.[17]

There are two ways to interpret the models with omitted variables. The first is that the variable is literally omitted and the second, is that the operationalization of the variable does not correspond to the underlying theoretical construct. The operationalization of relatedness and market structure in empirical studies has proven to be challenging and has generated considerable debate.

Rumelt's (1974) original measures of relatedness relied on a partially subjective characterization of the potential for shared activities and resources across businesses given the industries in which the firm competes. Subsequent authors sought to develop less subjective and more systematic measures by using Standard Industry Classification (SIC) codes for a firm's industries to construct concentric-based (e.g., Montgomery and Wernerfelt, 1988) and entropy-based (e.g., Davis and Duhaime, 1992) measures of relatedness. SIC codes, however, have been criticized as not corresponding to the possibilities for sharing activities and resources (Rumelt, 1982; Robins and Wiersema, 1995, 2003; Villalonga, 2004a). More recent approaches include using technology flows across markets (Robins and Wiersema, 1995; Bowen and Wiersema, 2005), patent citations (Kim and Kogut, 1996), and the actual patterns of diversification (Teece et. al., 1994; Bryce and Winter, 2004) to measure relatedness. However a consensus

[16] We calculate the diversification premium as $\beta_1 + (\beta_2 - \beta_3)\bar{H}$ where $\bar{H} = 0.341$.

[17] We also ran variations on models 4 and 5 in which the variable H_i is included in the specification but not interacted with D_i. In both cases, there is a diversification premium and a marginal decline in the R^2.

does not yet seem to have emerged.

Market structure has been operationalized using the standard concentration measures such as the Herfindahl index. Since these measures are usually constructed using data based on SIC codes, they are subject to similar critiques as SIC-based relatedness measures. Bryce and Winter (2004:19), for example, note that although most analysts would agree that the "Paving Mixtures and Blocks" industry and the "Concrete, Ready-Mixed" industry are highly related, the SIC coding structure treats them as highly unrelated, assigning them to different single-digit classifications.

The simple exercise in this section elucidates some of what is at stake in the efforts to develop effective control variables. For example, comparing Model 3 and Model 5 shows that the lack of effective controls for market structure can bias downward estimates on the effects of relatedness on performance.

8. Conclusion

We address two key debates in the extensive empirical literature on corporate diversification: the existence of a diversification discount and the relative importance of relatedness and market structure for the performance of diversified firms. We develop a formal theory in which the decision to diversify is endogenous and affects the both market structure and firms' competitiveness. Our key exogenous variable is the extent of industry relatedness.

We decompose the decision to diversify into three key components – the fixed costs of entry, the growth opportunity and the effect on home-market competitiveness – that all depend on the degree of industry relatedness. Although we start with homogeneous firms, heterogeneity naturally emerges as firms make different diversification decisions due to decreasing returns. As more firms diversify, the increased number of rivals in each industry reduce the growth opportunities available to additional diversifiers. This emergent heterogeneity in firm scope strategies leads to heterogeneity in market shares and profits.

Market structure depends on the relative number of firms that choose to specialize versus the number that choose to diversify and compete in multiple industries. Because these diversification decisions depend on the level of relatedness, we find that relatedness and market structure are not distinct constructs in our theory. Within our formal model we make precise the negative impacts of relatedness on market structure.

We find a non-monotonic effect of relatedness on the performance of diversified firms. Although greater relatedness increases the competitiveness of diversified firms, it can also spur additional diversification and thereby erode market structure and performance. As we show in the data simulation, overlooking these effects in empirical specifications can give rise to spurious inferences of a diversification discount. The emerging literature on the formal foundations of strategy has tended to emphasize business-level rather than corporate-level issues. In this paper, we have sought to expand the scope of inquiry.

Like any model, ours contains many simplifying assumptions such as there being only two, symmetric markets and initially homogeneous firms. Our objective, however, is not to reproduce reality, but rather to elucidate some of the drivers of diversification patterns that one might see in real markets. Nonetheless, relaxing some of the simplifying assumptions could form the basis of future research. The simplicity of our model, especially when specialized to $n = 2$, 3 or 4 firms, suggests that it could serve as a tractable platform.

Given the importance of resource heterogeneity in the literature, a natural extension would be to endow some firms in our model with valuable resources. One could then explore whether resourced or unresourced firms have a greater incentive to diversify and how the specificity of the resources affects these incentives. By considering more than two markets, one could address the extent to which a firm has diversified and possibly elucidate the construction of measures of relatedness in a firm's portfolio of businesses. Finally, one could endogenize the number of firms in order to address entry and exit dynamics, possibly involving merger and acquisition activity.

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9. Appendix I: Cournot Competition

This Appendix fully specifies the Cournot model that generates the profits given by (2.2) and derives the bounds on k given by (2.1) such that this profit expression is valid for any choice of scope strategy by the n firms.

We begin by formally defining v_{ij}, the index of firm i's value creation in market j. Each firm i has a constant marginal cost of production in market j given by $c_{ij} \geq 0$. There is a continuum of consumers in market j that have a willingness to pay for firm i's product given by $w_{ij} - \varepsilon$. While the w_{ij} can vary across products but are the same for all consumers, the ε are the same for all products but vary across consumers. In particular, ε is uniformly distributed with density S between 0 and some arbitrarily large upper bound. The assumption of a uniform distribution generates linear demand for each firm with an intercept of w_{ij} and a slope of $1/S$. We then define $v_{ij} = w_{ij} - c_{ij}$. This is an index of marginal value creation; the actual marginal value creation varies across consumers according to $w_{ij} - c_{ij} - \varepsilon$.

With Cournot competition, each of the N firms that have entered market j choose an output $q_{ij} \geq 0$. Prices are such that markets clear, which in our context yields the price $p_{ij} = w_{ij} - \sum_{m=1}^{N} q_{mj}/S$ for firm i in market j. Firms seek to maximize their profits from the market, which are $\pi_{ij} = (p_{ij} - c_{ij})q_{ij}$. At an interior equilibrium where $q_{ij} > 0$ for all i, we have that the first order conditions

$$w_{ij} - c_{ij} - \frac{1}{S}\left(\sum_{m \neq i} q_{mj} + 2q_{ij}\right) = 0 \tag{9.1}$$

are satisfied for all i. Note we can substitute $v_{ij} = w_{ij} - c_{ij}$ into (9.1). There is then a unique Nash equilibrium satisfying these conditions given by

$$\begin{aligned} q_{ij}^{*} &= S\left(\frac{v_{ij} + \sum_{m \neq i}(v_{ij} - v_{mj})}{N+1}\right), \\ \pi_{ij}^{*} &= \frac{1}{S}\left(q_{ij}^{*}\right)^{2}, \end{aligned}$$

which establishes the Cournot profit expression (2.2).

The textbook Cournot model with homogenous products (e.g. Tirole, 1988) is a special case of this model with $w_{ij} = D$ so that $v_{ij} + \sum_{m \neq i}(v_{ij} - v_{mj}) = D - \sum_{m \neq i}(c_{ij} - c_{mj})$. The generalization of the Cournot model that we are using has been used in the IO literature to study network externalities (Katz and Shapiro, 1985) and disruptive technologies (Adner and Zemsky, 2005). For a more extensive and general treatment of Cournot competition with differentiated products see Vives (1999).

For (2.2) to hold, we need to restrict k such that firm outputs q_{ij}^{*} and profits are positive for all values of n, n_A, n_B and n_D. For specialized firms to have positive output, it must be that $v_S + n_D(v_S - v_D) > 0$ which we can rewrite as

$$\frac{n_D + 1}{n_D} > \frac{v_D}{v_S}.$$

Substituting $k = v_D/v_S$ and setting $n_D = n - 1$, which minimizes $(n_D + 1)/n_D$ subject to their being at least one specialized firm, yields the following upper bound on k

$$\frac{n}{n-1} > k.$$

For diversified firms to have positive output in both markets it must be that $v_D + \bar{n}_S(v_D - v_S) > 0$ where $\bar{n}_S = \max\{n_A, n_B\}$, which we can rewrite as

$$\frac{v_D}{v_S} > \frac{\bar{n}_S}{\bar{n}_S + 1}.$$

We want to find the highest value for $\bar{n}_S/(\bar{n}_S+1)$ subject to $n_D > 0$. If n is odd, then the highest value occurs at $\bar{n}_S = (n-1)/2$ which yields

$$k > \frac{n-1}{n+1},$$

while if n is even the highest value occurs at $\bar{n}_S = (n+1)/2$ which yields

$$k > \frac{n+1}{n+3}.$$

Noting that $(n+1)/(n+3) > (n-1)/(n+1)$, we have that expression (2.1) is sufficient to guarantee that firm outputs are positive and that (2.2) holds.

A possible limitation of Cournot models is that the choice variables (quantities) are strategic substitutes rather than strategic complements. In other plausible models of competition, especially when firms choose prices, choice variables are strategic complements. Such quantity competition in the second stage can drive results in two-stage models because it means that firms want to take aggressive actions in the first stage in order to commit themselves to high output levels; see for example the literature on strategic incentives (Fershtman and Judd 1987). However, our theory of diversification is not fundamentally driven by the desire of firms to commit to aggressive actions and hence we do not expect that the assumption of quantity rather than price competition is an important driver of the results.

10. Appendix II: Proofs

Proof of Lemmas 5.1 and 5.2

Necessary and sufficient conditions for (n_A, n_B, n_D) to constitute a pure strategy Nash equilibrium are that no firm has a profitable deviation:

$$\begin{aligned}
\pi_S(n_A, n_D) &\geq \max\{\pi_D(n_D+1, n_A-1, n_B), \pi_S(n_B+1, n_D)\}, && (10.1)\\
\pi_S(n_B, n_D) &\geq \max\{\pi_D(n_D+1, n_A, n_B-1), \pi_S(n_A+1, n_D)\}, && (10.2)\\
\pi_D(n_D, n_A, n_B) &\geq \max\{\pi_S(n_A+1, n_D-1), \pi_S(n_B+1, n_D-1)\}, && (10.3)
\end{aligned}$$

where one drops any condition where there are no firms pursuing the strategy, e.g. if $n_A = 0$ then condition (10.1) does not apply.

Suppose we have $n_A > n_B + 1$. Note that the profits of specialized firms are falling in the number of other specialized firms. This and the symmetry in the two markets implies that $\pi_S(n_A, n_D) < \pi_B(n_B + 1, n_D)$, which contradicts this being a PSNE. Similarly, we can not have $n_B > n_A + 1$. This establishes Lemma 5.1

We henceforth restrict attention to $|n_A - n_B| \leq 1$, which assures that $\pi_S(n_A, n_D) \geq \pi_S(n_B + 1, n_D)$ and $\pi_S(n_B, n_D) \geq \pi_S(n_A + 1, n_D)$. Suppose that $n_A \geq n_B$. Then $\pi_S(n_B + 1, n_D - 1) \geq \pi_S(n_A + 1, n_D - 1)$ and (5.1) is sufficient for (10.3). It remains to show that condition (5.2) implies $\pi_S(n_B, n_D) \geq \pi_D(n_D + 1, n_A, n_B - 1)$. To do this define $\Phi = \pi_S(n_B, n_D) - \pi_D(n_D + 1, n_A, n_B - 1) - (\pi_S(n_A, n_D) - \pi_D(n_D + 1, n_A - 1, n_B))$ and then it remains to show that $\Phi \geq 0$. If $n_A = n_B$ we have $\Phi = 0$. Suppose $n_A = n_B + 1$. If $k = 1$ then $\Phi = Sv_S^2(2n_D + 5 + 2n_B)(n_D + n_B + 2)^{-2}(n_D + n_B + 3)^{-2} > 0$. Given that Φ is a convex function of k, it remains to show that $\Phi \geq 0$ for extreme values of k. From (2.1) we have that the upper bound on k is $n/(n-1)$ and the lower bound on k is $(n+1)/(n+3)$ where $n = n_D + 2n_B + 1$. With some tedious but straightforward calculations, we have that $\Phi > 0$

at these values. *Q.E.D.*

We now restate formally Proposition 5.3 and Proposition 5.4 as well as introducing a new proposition for the case of $k = 1$, which we include for completeness. We then prove these propositions.

Proposition 5.3 (restated) *There exist critical values* $0 = F_{n+1} < F_n < F_{n-1} < \ldots < F_1 < F_0 = \infty$ *such that* n_D^* *is an equilibrium level of diversification if and only if* $F_{n_D^*+1} \leq F \leq F_{n_D^*}$. *These critical values* F_j *are increasing in* k *for* $1 \leq j \leq n$.

Proposition 5.4 (restated) *(i) There exists values* $F_{n_D^*+1} < F_{n_D^*}$ *such that* $n_D^* > 0$ *firms diversify if and only if* $n - n_D^*$ *is even and* $F_{n_D^*+1} \leq F \leq F_{n_D^*}$. *(ii) The interval* $[F_{n_D^*+1}, F_{n_D^*}]$ *in which* n_D^* *is an equilibrium overlaps non trivially with the interval* $[F_{n_D^*+2}, F_{n_D^*+3}]$ *where* $n_D^* + 2$ *is an equilibrium; specifically,* $F_{n_D^*+1} < F_{n_D^*+2} < F_{n_D^*} < F_{n_D^*+3}$. *(iii) For* $1 \leq j \leq n$, *the critical values* F_j *are increasing in* k *with* $F_j > 0$ *for* k *sufficiently close to* 1.

Proposition 10.1. *Suppose* $k = 1$. *There exists values* $0 = F_{n+1} < F_n \leq F_{n-1} \leq \ldots \leq F_1 \leq F_0 = \infty$ *such that* n_D^* *is an equilibrium if and only if* $F_{n_D^*+1} \leq F \leq F_{n_D^*}$. *If* $n - n_D^*$ *is even, then* $F_{n_D^*+1} < F_{n_D^*}$. *If* $n - n_D^*$ *is odd, then* $F_{n_D^*+1} = F_{n_D^*}$ *for* $n_D^* > 0$.

Proof of Propositions 5.3, 5.4 and 10.1

Based on Lemma 5.1 and the symmetry of the markets, when considering any equilibrium level of diversification we can restrict attention to $n_A \in \{n_B, n_B + 1\}$. The first step in the proof is to define a couple of critical F values. Condition (5.1), which is a necessary condition for any n_D satisfying $0 < n_D \leq n$ to be an equilibrium, can be rewritten as

$$F \leq F_H(n_D) \equiv \frac{(k(n_A+1) - n_A)^2}{(n_D + n_A + 1)^2} + \frac{(k(n_B+1) - n_B)^2}{(n_D + n_B + 1)^2} - \frac{(n_D - k(n_D-1))^2}{(n_D + n_B + 1)^2}.$$

Condition (5.2), which is a necessary condition for any n_D satisfying $0 \leq n_D < n$ to be an equilibrium, can be rewritten as

$$F \geq F_L(n_D) \equiv \frac{(kn_A - n_A + 1)^2}{(n_D + n_A + 1)^2} + \frac{(k(n_B+1) - n_B)^2}{(n_D + n_B + 2)^2} - \frac{(n_D + 1 - kn_D)^2}{(n_D + n_A + 1)^2}.$$

Next we establish two lemmas about properties of these critical F values.

Lemma 10.2. *(i)* $F_H(n_D) = F_L(n_D - 1)$ *for* $0 < n_D \leq n$. *(ii)* $F_H(n_D)$ *and* $F_L(n_D)$ *are both increasing in* k.

Proof (i) Suppose first that $n_A = n_B + 1$. $F_H(n_D)$ is defined by $\pi_D(n_D, n_A, n_B) = \pi_S(n_B + 1, n_D - 1)$. $F_L(n_D - 1)$ is defined by $\pi_S(n_A, n_D - 1) = \pi_D(n_D, n_A - 1, n_B + 1)$ where the reduction in the number of diversified firms from n_D to $n_D - 1$ involves an increase in the number of firms specialized in B from n_B to $n_B + 1$. We have $\pi_D(n_D, n_A, n_B) = \pi_D(n_D, n_A - 1, n_B + 1)$ and $\pi_S(n_B + 1, n_D - 1) = \pi_S(n_A, n_D - 1)$ since $n_A = n_B + 1$. Hence $F_H(n_D) = F_L(n_D - 1)$ in this case.

Suppose now that $n_A = n_B$. $F_H(n_D)$ is defined by $\pi_D(n_D, n_A, n_B) = \pi_S(n_B + 1, n_D - 1)$. $F_L(n_D - 1)$ is defined by $\pi_S(n_A + 1, n_D - 1) = \pi_D(n_D, n_A, n_B)$, where the reduction in the number of diversified firms from n_D to $n_D - 1$ involves an increase in the number of firms specialized in A from n_A to $n_A + 1$. We have that $\pi_S(n_B + 1, n_D - 1) = \pi_S(n_A + 1, n_D - 1)$ since $n_A = n_B$. Hence $F_H(n_D + 1) = F_L(n_D)$ in this case as well.

(ii) Follows directly from inspection of the expressions for $F_H(n_D)$ and $F_L(n_D)$. *Q.E.D.*

Lemma 10.3. *Suppose that $0 < n_D < n$. (i) For $n_A = n_B$, $F_H(n_D) > F_L(n_D)$. (ii) For $n_A = n_B + 1$, $F_H(n_D) > F_L(n_D)$ if $k > 1$, $F_H(n_D) = F_L(n_D)$ if $k = 1$, $F_H(n_D) < F_L(n_D)$ if $k < 1$.*[18]

Proof Suppose that $0 < n_D < n$ and define $\Delta(k) = F_H(n_D) - F_L(n_D)$, which is a quadratic function of k.

(i) Suppose that $n_A = n_B$. The coefficient on k^2 in $\Delta(k)$ is positive while the coefficient on k is negative. Hence, the value of k that minimizes $\Delta(k)$ satisfies $\Delta'(k_L) = 0$, which yields

$$k_L = 1 - \frac{(n_B + 1)(2n_B + 2n_D + 3)}{16n_B + 10n_D + 20n_Bn_D + 15n_B^2 + 4n_B^3 + 8n_D^2 + 2n_D^3 + 6n_Bn_D^2 + 8n_B^2n_D + 3}.$$

At this value we have $\Delta(k_L) = 2(2n_B + 2n_D + 3)(n_B + n_D)/((n_B + n_D + 1)^2$ $(16n_B + 10n_D + 20n_Bn_D + 15n_B^2 + 4n_B^3 + 8n_D^2 + 2n_D^3 + 6n_Bn_D^2 + 8n_B^2n_D + 3)) > 0$.

(ii) Suppose $n_A = n_B + 1$. We can write $\Delta(k) = \left(a_1 - (a_1 + a_2)r + a_2r^2\right)/((n_B + n_D + 2)^2$ $(n_B + n_D + 1)^2)$ where $a_2 = 12n_B + 12n_D + 20n_Bn_D + 13n_B^2 + 4n_B^3 + 7n_D^2 +$ $4n_Bn_D^2 + 8n_B^2n_D + 2$ and $a_1 = 6n_B + 6n_D + 12n_Bn_D + 9n_B^2 + 4n_B^3 + 3n_D^2 + 4n_Bn_D^2 + 8n_B^2n_D + 2$. Note that $a_2 > a_1 > 0$. Hence, $\Delta(1) = 0$, $\Delta'(1) > 0$,which implies that $\Delta(k) > 0$ for $k > 1$. It remains to show that $\Delta(k) < 0$ for $k < 1$. Given that $\Delta(k)$ is quadratic in k, it suffices to show the result for a lower bound on k, for which we use $k > n_A/(n_A+1) = (n_B+1)/(n_B+2)$. At this lower bound we have $\Delta((n_B + 1)/(n_B + 2) = \left(2 - n_B^2 - n_D^2 - 2n_Bn_D\right)(n_B + 2)^{-2}$ which is negative unless $n_D = 1$ and $n_B = 0$. For $n_B = 0$ and $n_D = 1$ we have $F_H(1) = (25k^2 - 16k - 5)/36$, which is positive if and only if $k > (3\sqrt{21} + 8)/25$. At this lower bound we have $\Delta((3\sqrt{21} + 8)/25) < 0$. *Q.E.D.*

We now construct the critical values of F used in the propositions. Suppose that $0 < j \leq n$. Note that by Lemma 10.2 (i) $F_H(j) = F_L(j - 1)$. By Lemma 10.2 (ii), F_j is increasing in k. Let $F_j = F_H(j) = F_L(j - 1)$. Define $F_{n+1} = 0$ and $F_0 = \infty$ and now let j satisfy $0 \leq j \leq n$.

By Lemma 5.2 and the definition of $F_H(n_D) = F_L(n_D)$, a value n_D such that $0 \leq n_D \leq n$ is a PSNE level of diversification if and only if $F_{n_D+1} \leq F \leq F_{n_D}$.

Suppose that $k > 1$. By Lemma 10.3, we have $F_j < F_{j-1}$ for $0 < j \leq n$. For Proposition 5.3 it only remains to show that $F_n > F_{n+1} = 0$. We have $F_n = \left(2k^2 - (n - k(n - 1))^2\right)(n + 1)^{-2}$, which is negative for k sufficiently large and $n > 2$. However, at the upper bound $k = n/(n-1)$, we have $F_n = \left(n^2 - 2n + 3\right) n/(n - 1) > 0$. This establishes Proposition 5.3.

Suppose that $k = 1$. By Lemma 10.3, we have for $0 < n_D < n$ that $F_{n_D+1} < F_{n_D}$ when $n - n_D$ is even and that $F_{n_D+1} = F_{n_D}$,when $n - n_D$ is odd. With $k = 1$, we have that $F_n = \lim_{k\to1}\left(2k^2 - (n - k(n - 1))^2\right)/(n + 1)^2 = 1/(n + 1)^2 > F_{n+1} = 0$. This establishes Proposition 10.1.

Suppose that $k < 1$ and $n_D > 0$. By Lemma 10.3 (ii), for $n_A = n_B + 1$ either $F_{n_D} < F_{n_D+1}$ or $F_{n_D+1} < 0$ and it is not possible for $F_{n_D+1} \leq F \leq F_{n_D}$. Hence, $n_A = n_B$ is a necessary

[18] *For $n = 2$ and $m < 1$, we have that $F_H(n_D) < F_L(n_D)$ as long as m is large enough that $F_H(n_D) \geq 0$.*

condition for n_D to be a PSNE level of diversification and it is only possible for $n_A = n_B$ if $n - n_D$ is even. By Lemma 10.3 (i), if $n - n_D$ is even so that $n_A = n_B$, then $F_{n_D+1} < F_{n_D}$. This establishes part (i) of Proposition 5.4.

Maintaining $k < 1$, suppose in addition that $n_D + 2 < n$ and $n - n_D$ is even. From Lemma 10.3 (i) we have that $F_{n_D+1} < F_{n_D}$ and $F_{n_D+3} < F_{n_D+2}$. From Lemma 10.3 (ii) we have that $F_{n_D+1} < F_{n_D+2}$. Let $\Delta_1(k) = F_{n_D} - F_{n_D+2}$, which is a quadratic function of k with a positive coefficient on k^2 and a negative coefficient on k. Let k_1 satisfy $\Delta'(k_1) = 0$. At this minimum value we have $\Delta_1(k_1) = 2\left(4n_A + 4n_D + 4n_An_D + 2n_A^2 + 2n_D^2 + 1\right)/$
$((2n_A + 2n_D + 3)(n_A + n_D + 2)^2 (n_A + n_D + 1)^2 (22n_A + 16n_D + 32n_An_D$
$+24n_A^2 + 8n_A^3 + 11n_D^2 + 2n_D^3 + 10n_An_D^2 + 16n_A^2n_D + 5)) > 0$. Hence $F_{n_D} > F_{n_D+2}$. This establishes part (ii) of Proposition 5.4.

We have that F_j is a continuous, increasing function of k and above we showed that $F_j > 0$ for $k = 1$ and $1 \le j \le n$. Hence, for k sufficiently close to 1 we also have $F_j > 0$ for $1 \le j \le n$. This establishes part (iii) of Proposition 5.4. *Q.E.D.*

Proof of Proposition 6.1 Fix values for n_D, n_A and n_B with $n_A \in \{n_B, n_B+1\}$ and consider what happens when one of the firms specialized in market A diversifies. (i) Suppose that $k > 1$. The change in profits for the firms specialized in market B are $\pi_S(n_B, n_D+1) - \pi_S(n_B, n_D) < 0$. The change in profits for firms specialized in market A are $\pi_S(n_A - 1, n_D + 1) - \pi_S(n_A, n_D) = -(k-1)\frac{2n_D(1-k)+3-k}{(n_A+n_D+1)^2} < 0$. The change in profit for already diversified firms is $\pi_D(n_D+1, n_A - 1, n_B) - \pi_D(n_D, n_A, n_B) = -(k-1)\frac{2n_A(k-1)+k+1}{(n_D+n_A+1)^2} - (kn_B + k - n_B)^2 \frac{2n_B+2n_D+3}{(n_D+n_B+2)^2(n_B+n_D+1)^2} < 0$.

(ii) Suppose that $k \le 1$. The impact on firms specialized in market B is $\pi_S(n_B, n_D) - \pi_S(n_B, n_D + 1)$, which is positive since $\partial\pi_S(n_D, n_B)/\partial n_D = -2(k(n_A + 1) - n_A)$
$(n_D(1-k) + 1)/(n_B + n_D + 1)^3 < 0$ given that $k > n_A/(n_A + 1)$. The impact on firms specialized in market A is negative since $\pi_S(n_A, n_D) - \pi_S(n_A - 1, n_D + 1) = -(1-k)(2n_D(1-k) + 3 - k)/(n_A + n_D + 1)^2 \le 0$. It remains to show that the profits of the n_D diversified firms fall except for $n_A = n_B = n_D = 1$. We first show the result for $n_A = n_B$ and then show that this implies the result for $n_A = n_B + 1$.

Suppose that $n_A = n_B$. Let $g = (\pi_D(n_D, n_A, n_A) - \pi_D(n_D + 1, n_A - 1, n_A)$ and we want to establish that $g > 0$. We have that g is quadratic in k with a positive coefficient on k^2 and that for $k = 1$ we have $g = (2n_A + 2n_D + 3)(n_A + n_D + 2)^{-2}(n_A + n_D + 1)^{-2} > 0$ and $\partial g/\partial k = 2(9n_A + 6n_D + 4n_An_D + 3n_A^2 + n_D^2 + 7)(n_A + n_D + 2)^{-2}(n_A + n_D + 1)^{-2} > 0$. Hence, if $g > 0$ for some value of k, it is positive for all higher values ass well. The minimum r such that n_D is an equilibrium occurs for $F = 0$ and is $k = (\sqrt{2}n_A + n_D)/(\sqrt{2}n_A + n_D + \sqrt{2} - 1)$. Evaluating g at $k = k$ yields an expression with the same sign as $h(n_A, n_B) = 4\sqrt{2} - 6 - 2(n_D + n_A) + n_An_D\left(8 - 6\sqrt{2}\right) + (n_A^3 + n_D^3)\left(2 - \sqrt{2}\right) +$
$(n_A^2 + n_D^2)\left(4 - 3\sqrt{2}\right) + (n_An_D^2 + n_Dn_A^2)\left(6 - 3\sqrt{2}\right)$. Note that h is symmetric in its arguments and is increasing in each since $\partial h(n_A, 1)/\partial n_A > 0$ and $\partial^2 h(n_A, n_D)/\partial n_A \partial n_D > 0$. We have that $h(1,1) = -0.627\,42 < 0$ and $h(1,2) = h(1,2) = 7.\,289\,3 > 0$.

Now suppose that $n_A = n_B + 1$. Let $y = \pi_D(n_D, n_A, n_A - 1) - \pi_D(n_D + 1, n_A - 1, n_A - 1)$. Let $f(n_A, n_D) = (k(n_A + 1) - n_A)^2/(n_A + n_D + 1)^2$. Then $y > g$ is equivalent to $f(n_D, n_A - 1) - f(n_D + 1, n_A - 1) > f(n_D, n_A) - f(n_D + 1, n_A)$ and this follows from $\partial^2 f(n_A, n_D)/\partial n_A \partial n_D > 0$. Hence, fixing n_A and n_D if the profits of diversified firms are falling for $n_B = n_A$ then they are also falling for $n_B = n_A - 1$. It remains then to show the result for $n_D = 1$, $n_A = 1$ and $n_B = 0$, in which case we have that $y = \left(17k^2 - 16k + 4\right)/36 > 0$. *Q.E.D.*

We now restate formally Proposition 6.2 and then provide a proof.

Proposition 6.2 (restated) *(i) Let $F_1 > F_n > 0$ be such that $n_D^* = 0$ is an equilibrium iff $F \geq F_1$ and $n_D^* = n$ is an equilibrium iff $F \leq F_n$. Decreasing F from $F \geq F_1$ down to $F = F_n$ and thereby shifting from minimal to maximal diversification causes a fall in the profits of all firms.*
(ii) Let $k_1 < k_n < n/(n-1)$ be such that $n_D^* = 0$ is an equilibrium iff $k \leq k_1$ and $n_D^* = n$ is an equilibrium iff $k \geq k_n$. Increasing k from $k \leq k_1$ up to $k = k_n$ and thereby shifting from minimal to maximal diversification causes a fall in the profits of all firms.

Proof of Proposition 6.2 (i) Suppose that $F_n = F_H(n) \geq 0$. For F large we have that the profits of all firms are $\pi_S(n/2, 0)$ if n is even and $\pi_S((n-1)/2, 0)$ and $\pi_S((n+1)/2, 0)$ if n is odd. We have $\pi_S((n-1)/2, 0) > \pi_S(n/2, 0) > \pi_S((n+1)/2, 0) = 4/(n+3)^2$ and we focus on the lower bound. The profits when $n_D = n$ are $\pi_D(n, 0, 0) = 2k^2/(n+1)^2 - F$. The largest F such that $n_D = n$ is an equilibrium is $F_H(n) = 2k^2/(n+1)^2 - (n-k(n-1))^2/(n+1)^2$ and at this F we have $\pi_D(n, 0, 0) = (n-k(n-1))^2/(n+1)^2$, which is falling in k. The lowest k such that $F_H(n) \geq 0$ is $k = \left(n - 1 - \sqrt{2}\right) n/(n^2 - 1 - 2n)$ and at this lower limit we have $\pi_S(\frac{n+1}{2}, 0) - \pi_D(n, 0, 0) = (4/(n+3)^2 - (n-k(n-1))^2/$
$(n+1)^2) = 2\left(n - \sqrt{2}\right)\left(n^2 + 3\sqrt{2} + 5n\left(\sqrt{2} - 1\right) - 4\right)(n-1)(n+1)^{-2}(n+3)^{-2}$
$\left(n + \sqrt{2} - 1\right)^{-2} > 0$.

(ii) Suppose that k is sufficiently low that all firms are specialized. Profits are at least $\pi_S((n+1)/2, 0) = 4/(n+3)^2$. Profits of the firms when $n_D = n$ are $\pi_D(n, 0, 0)$. We have $k = k_n$, which is defined by $\pi_D(n, 0, 0) = \pi_S(1, n-1) = (n-k(n-1))^2/(n+1)^2$ and hence the difference in profits are $4/(n+3)^2 - (n-k(n-1))^2/(n+1)^2$ which is increasing in k. How low can k_n be? This occurs for $F = 0$ in which case k_n satisfies $2k^2/(n+1)^2 = (n-k(n-1))^2/(n+1)^2$, which yields $k_n \geq \left(n - 1 - \sqrt{2}\right) n/(n^2 - 2n - 1)$ and at this lower bound we have that $4/(n+3)^2 - (n-k(n-1))^2/(n+1)^2 = 2\left(n + \sqrt{2} - 1\right)^{-2}(n+3)^{-2}$
$(n+1)^{-2}\left(n - \sqrt{2}\right)\left(n^2 - 4 + 3\sqrt{2} + 5n\left(\sqrt{2} - 1\right)\right)(n-1) > 0$. *Q.E.D.*

Proof of Proposition 6.3

Suppose we have an equilibrium where $n_A \in \{n_B, n_B + 1\}$ and $0 < n_D < n$. From (5.1) we have that $\pi_D(n_D, n_A, n_B) \geq \pi_S(n_B + 1, n_D - 1)$. For $k \geq 1$, we have that $\pi_S(n_B + 1, n_D - 1) > \pi_S(n_B, n_D)$. Since $n_A \geq n_B$ we have that $\pi_S(n_B, n_D) \geq \pi_S(n_A, n_D)$. Hence the profit of diversified firms, $\pi_D(n_D, n_A, n_B)$, is strictly greater than the profits specialized firms, $\pi_S(n_A, n_D)$ and $\pi_S(n_B, n_D)$. *Q.E.D.*