Performance shocks and misreporting

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Abstract

We propose a parsimonious stochastic model of reported earnings that links misreporting to performance shocks. Our main analytical prediction is that misreporting leads to a negative second-order autocorrelation in the residuals from a regression of current earnings on lagged earnings. We also propose a stylized dynamic model of earnings manipulation and demonstrate that both earnings smoothing and target-beating considerations result in the same predictions of negative second-order autocorrelations. Empirically, we find that the distribution of this measure is asymmetric around zero with 27% of the firms having significantly negative estimates. Using this measure, we specify a methodology to estimate the intensity of misreporting and to create estimates of unmanipulated earnings. Our estimates of unmanipulated earnings are more correlated with contemporaneous returns and have higher volatility than reported earnings. With respect to economic magnitude, we find that, in absolute terms, median misreporting is 0.7% of total assets. Moreover, firms in our sample subject to SEC AAERs have significantly higher estimates of manipulation intensity.

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1. Introduction

A firm’s financial reports are prepared by agents whose interests are not always aligned with the interests of the users of accounting information. This misalignment can lead to misreporting. Broadly, misreporting represents any deviation from the most accurate and informative report of the firm’s performance within the standards of GAAP. Practically, it can range from the misuse of the discretion allowed under GAAP to outright fraud.

One potentially important motive for misreporting is to mitigate the impact of the period’s innovation to earnings (the “performance shock”). Such a motive can arise from incentives to either meet or beat accounting-based performance targets or reduce the volatility of reported earnings. We propose a model of reported earnings that links misreporting to the period’s performance shock. We then use this model to predict and identify serial correlation patterns in reported earnings that arise from misreporting with the goal of masking the period’s performance shock. Specifically, we show that in the presence of such misreporting the residuals from a regression of reported earnings on lagged reported earnings have a negative second-order autocorrelation. Intuitively, this means that if the firm engages in systematic misreporting to mask performance shocks, then a shock to the firm’s reported earnings is likely to partially reverse two periods into the future.
To derive our main prediction, we make two assumptions: (1) earnings are persistent; and (2) the purpose of misreporting is to mask the period’s shock to performance. There are both economic and accounting-based explanations for the persistence in earnings. From the economic perspective, a performance shock to the firm rarely dissipates completely within a single year. Several structural reasons lead to this delayed dissipation. For example, as discussed by Lerner (2002), patent protection of technological innovations typically lasts for multiple years. Other barriers to competition that determine industry structure (trade restrictions, increasing returns to scale, licenses, etc.) are also typically long-lived. Furthermore, commodity prices, interest rates, and wages are highly persistent (for discussions, see Cashin et al., 1999; Neely and Rapach, 2008).

The persistence of economic profits does not, however, necessarily translate into the persistence of accounting earnings. In principle, one could devise a system of financial reporting rules that accounts for the impact of every shock in present-value terms. Namely, all expected effects of the current shock on economic profits could be recognized in the current period, thereby leading to accounting earnings that are uncorrelated across periods. Such fair value accounting is not the financial reporting system currently in place. Instead, financial reporting rules dictate that performance shocks are not immediately recognized to their full extent because of verifiability standards. Such delayed recognition of performance shocks leads to persistence in accounting earnings. Moreover, as discussed by Watts (2003), verifiability standards are asymmetric in the sense that they are stricter for favorable events. These standards can result in asymmetric recognition of positive and negative shocks.1

Regarding our second assumption, a link between misreporting and performance shocks can arise in several settings. Intuitively, if the manager seeks to reduce the variance of reported earnings, then she under-reports earnings when performance shocks are favorable to create a precautionary buffer against future adverse shocks. This buffer produces a negative bias in reported earnings when shocks are positive and subsequently a positive bias when the manager takes advantage of the precautionary buffer. In the context of meeting or beating an earnings target, the same prediction holds. If performance is poor but the target is reasonably close, the manager over-reports to produce the desired number. Again, the bias is in the opposite direction from the performance shock. For both settings, we analytically demonstrate a negative correlation between reporting bias and the period’s performance shock.2 In doing so, we develop a dynamic framework for analyzing earnings manipulation in which the agent can misreport earnings to meet or beat a target or to smooth earnings. The principal, in turn, takes the agent's manipulation into account when rationally setting targets.

Empirically, we demonstrate that negative second-order autocorrelation appears in the data consistent with the analytical predictions.3 We find that approximately 27% of firms with sufficient data on Compustat have significantly negative second-order autoregressive coefficients that are consistent with the systematic manipulation of reported earnings. We next propose a methodology to estimate the intensity of misreporting and to create estimates of unmanipulated earnings. This methodology assumes that discretion is a linear function of the period's performance shock. Based on the linearity assumption, we find that median absolute misreporting for our sample is 0.7% of total assets. This estimate appears more plausible than the discretionary accruals estimates documented in prior research. For example, Hribar and Nichols (2007) report for their sample that median absolute discretionary accruals are 5.2% of total assets, which is similar in magnitude to mean and median ratios of earnings before extraordinary items to total assets (see, for example, Dechow and Dichev, 2002; Wysocki, 2009). Furthermore, correcting the manipulation in reported earnings results in estimates of unmanipulated earnings that are more highly correlated with contemporaneous returns and have higher volatility than reported earnings. Moreover, firms in our sample subject to SEC Accounting and Auditing Enforcement Releases (AAERs) have significantly higher estimates of manipulation intensity.

We contribute to the accounting literature by deriving a testable prediction about the stochastic properties of manipulated earnings, which we use to construct a benchmark of unmanipulated earnings. This benchmark allows us to create measures of systematic earnings manipulation that we use to estimate both the incidence and intensity of earnings manipulation among firms with sufficient data on Compustat. The incidence of earnings manipulation in the economy is relevant for researchers and policy makers who evaluate the efficacy of existing or proposed accounting standards and the reporting incentives arising from compensation and governance mechanisms. Burgstahler and Dichev (1997) and Myers et al. (2007) provide some evidence on the possible incidence of earnings manipulation in the economy and Cohen et al. (2008) compare the incidence of earnings manipulation in the pre- and post-Sarbanes-Oxley periods. Moreover, Leuz et al. (2003), Haw et al. (2004), and Lang et al. (2006) compare the incidence of earnings manipulation across countries.4 Similarly, our measure can be used to compare the incidence of earnings manipulation across countries, industries, and groups of firms. The same way we create the distribution of second-order autocorrelation coefficients for the US, one could create the distribution for any portfolio (country, industry, growth options, etc.). Moreover, our measure can be used on

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1 We later discuss minimal and straightforward modifications to our model that accommodate full and immediate recognition of negative shocks but no recognition of future effects of positive shocks. The downside of these modifications is that they lead to significant data losses in empirical applications.

2 These results are consistent with the findings of Fudenberg and Tirole (1995) and Trueman and Titman (1988), who analyze similar earnings-smoothing settings.

3 For prior work on autocorrelations in earnings, see Dechow (1994) who documents a negative first-order autocorrelation for changes in earnings per share. Within our analytical framework, this result is a natural property of earnings. The negative first-order autocorrelation is driven by the same performance shock entering the two variables in the covariance expression with different signs.

4 For a review of the earnings management literature, see Dechow et al. (2010).
a firm-level basis. For example, one could examine the association of our firm-level measure of manipulation with firm-level characteristics such as corporate governance, transparency, or equity incentives.

While the autocorrelation measure is primarily aimed at evaluating the incidence of earnings manipulation, we show how it can be used to produce estimates of the firm’s performance shocks and to construct a firm-level measure of the intensity of manipulation. The reconstructed performance shocks can be used to estimate the volatility of a firm’s operating environment, which is an important input for many corporate finance and asset pricing models. In addition, such a volatility measure can be used as a control whenever traditional measures of earnings manipulation are used.

There are several caveats to our methodology. First, traditional approaches for identifying earnings manipulation (e.g., those based on Jones, 1991) typically construct a “reasonable” benchmark of accruals based on other information reported by the firm, and then interpret deviations from the benchmark as misreporting. In contrast to such “deterministic” benchmarking, we focus on the manipulation of earnings and are indifferent about whether manipulation occurs through accruals or real activities that manifest in cash flows. This can be either a strength or shortcoming of the model depending on the application. It is a strength in the sense that it allows researchers to identify overall earnings manipulation, both cash and accrual-based. It is a shortcoming in the sense that it does not allow researchers to study accruals manipulation separately from real earnings management.

Second, we use an AR(1) specification to model persistence in unmanipulated earnings. It is important to point out that our AR(1) specification for unmanipulated earnings could be misspecified. Hence, we cannot rule out the possibility that what call misreporting may be driven by the fact that unmanipulated earnings are best described by a stochastic process that is more complex than AR(1). Moreover, our time-series specification imposes several restrictions on the nature of the misreporting that can be identified. Specifically, it has to be a sustainable manipulation strategy (i.e., it does not result in exponentially growing account balances). For example, repeated over-reporting of earnings is not a sustainable manipulation policy, because it requires a progressively growing scale of manipulation that not only has to include the intended increase in current reported earnings but also cover the reversal of past manipulations. To be sustainable (i.e., prevent the scale of manipulation from growing out of proportion), current over-reporting has to be compensated by future under-reporting. In addition, our time-series strategy requires a relatively long series of firm-level observations to estimate our manipulation measure. This restriction has several implications: first, it limits our measure’s applicability; second, it raises the possibility that our results do not generalize to young (or declining) firms; third, it requires that firms have stable time-series properties of earnings. There is a large accounting literature on the time-series properties of annual earnings (for reviews of this literature, see Brown, 1993; Williams, 1995; for examples, see Ball and Watts, 1972; Brooks and Buckmaster, 1976; Albrecht et al., 1977, and Salamon and Smith, 1977). This literature primarily focuses on whether changes in the undeflated level of accounting income or changes in earnings per share are best described by either a random walk or a random walk with a drift. The primary application of this research is to produce estimates of earnings surprises for use in tests of the relation between earnings and stock returns (for applications, see Beaver et al., 1980; Kormendi and Lipe, 1987; Collins and Kothari, 1989). The key difference between this literature and our approach is that we do not take reported earnings at their face value. Instead, we use time-series properties (i.e., persistence and bias reversal) to isolate misreporting.

Third, our notion of misreporting is very specific. We can only identify misreporting that is linked to performance shocks. Although our specification encompasses a wide range of mechanisms that managers can use for misreporting (ranging from variations in recognition policies within GAAP to outright fraud), we are likely to miss other types of misreporting. Foremost, our methodology is unlikely to capture manipulation strategies driven by motives other than masking performance shocks. Such motives can include the manipulation of earnings around major events (i.e., raising capital, avoiding covenant violations, granting employee stock options, managers exercising stock options, managers retiring, and labor negotiations). Moreover, strategies linked to performance shocks may not always result in a negative correlation between the performance shock and the manager’s reporting discretion. A notable example would be a large write down of assets that includes future expenses (“big bath”). Such a write down results in reporting bias that goes in the same direction as the firm’s performance shock.

Because our specification does not capture the above strategies, we are unable to evaluate the relative importance of the different manipulation strategies. We therefore have no basis for claiming that we identify the predominant form of earnings manipulation. However, it is important to point out that misreporting arising due to motives unrelated to performance shocks is statistically similar to measurement error. Because measurement error produces serial correlations of the opposite sign than our prediction, such misreporting would make it more difficult to identify manipulations of the form that we consider.

2. Structural predictions

In this section, we analyze the statistical properties of an earnings specification in which idiosyncratic shocks drive the firm’s performance as well as reporting choices. We then use these properties to specify tests of misreporting.

5 As discussed by Moehrle (2002), such a big bath creates a “cookie jar” (for example, excessive asset impairments or writeoffs of obsolete inventory) that is later used in a manner consistent with our specification: when future shocks are adverse, managers tap into the “reserves” to report satisfactory earnings.
2.1. Statistical properties of unmanipulated earnings

Our underlying measure of unmanipulated earnings is accounting earnings prepared under a “consistent” application of GAAP. To apply GAAP, a manager must make assumptions (e.g., bad debt allowances, inventory writeoffs, and allowances for deferred tax assets). Such assumptions can reflect a manager’s innate sentiment (i.e., pessimism or optimism) or timeliness in the recognition of material events. We assume that the manager applies these assumptions in a “consistent” manner that does not vary with the firm’s performance. Consistency does not imply that the earnings generating process is the same for all firms. It only implies the process does not change over time.

We assume that the firm’s unmanipulated earnings, $y_i_t$, follow a first-order autoregressive process

$$y_i_t = \alpha + \beta y_{i,t-1} + \epsilon_{i,t},$$

in which $\epsilon_{i,t}$ is an i.i.d. shock to the firm’s performance and $0 < \beta < 1$. Throughout our analysis we demonstrate that misreporting introduces new terms into the regression for reported earnings.

This AR(1) specification has been a work horse in prior empirical research on predicting future earnings (for example, see Fairfield et al., 1996; Fama and French, 2000; Dechow and Ge, 2006; Dichev and Tang, 2009). All empirical evidence in support of the AR(1) specification for annual earnings is, however, based on reported earnings. Because unmanipulated earnings are not observed, one cannot rule out the possibility that the true process for unmanipulated earnings is more complex than the process for reported earnings. Therefore, the AR(1) assumption is not completely innocuous. However, the AR(1) assumption is not necessarily restrictive. Our methodology does not require that the underlying stochastic process is exactly AR(1). For example, in a general AR(k) structure our result depends on how the magnitude of the first-lag coefficient compares to the magnitude of the remaining k-1 coefficients. Our result requires that the magnitude of the first-lag coefficient dominates. Empirically, the first lag of a general autoregressive specification is clearly dominant for observed earnings. It is, however, possible that unmanipulated earnings have a more complex time-series structure that completely disappears due to manipulation.

The main advantage of the AR(1) specification is that it is the most parsimonious model that captures two important properties of accounting earnings that are widely accepted in the literature: persistence and mean reversion. Although mean reversion is well established in the literature (see for example, Stigler, 1963; Fama and French, 2000), it is not important for our results. In particular, our analysis would remain unchanged under a random walk specification for unmanipulated earnings. Persistence, on the other hand, is central to our analysis. Under truthful reporting, persistence implies that the period’s performance shock positively affects future earnings (i.e., the delayed recognition of performance shocks). As we discuss below, this persistence is in contrast with the effect of misreporting, which has to reverse in the future.

2.2. Performance shocks

It is important to emphasize the nature of the shocks in our specification of earnings. A common view is that cash flow shocks are the primary source of uncertainty in earnings and that accountants use accruals to offset these shocks (for a discussion, see Dechow, 1994). In contrast, the $\epsilon_t$ shocks in our specification are unrelated to payment uncertainty. Instead, they represent innovations in the firm’s performance (as measured by earnings) driven by both shocks to demand for the firm’s outputs and shocks to supply for the firm’s inputs (i.e., labor, materials, and capital). Some examples are changes in a firm’s costs due to fluctuations in commodity prices and changes in sales due to fluctuations in demand. In our analysis, we do not model the components of earnings (cash flows and accruals). Hence, we make no assumptions about the decomposition of performance shocks into cash flow and accrual components.

2.3. Strategic misreporting

The main concern in estimating Eq. (1) is that none of the variables are observable. They must be inferred from reported values. One potential reason for a discrepancy between the reported values and the actual values is that the manager strategically misreports earnings. Another reason is that earnings can be measured with error. An important distinction
between strategic misreporting and measurement error is that strategic misreporting is likely to be correlated with the performance shock. Our next step is to understand how strategic misreporting factors into Eq. (1).

We assume that the manager can exercise discretion in reporting earnings, \( d_t \). This discretion can represent either willful misreporting or the use of excessively optimistic (or pessimistic) assumptions in the preparation of earnings (for a discussion of the differences and similarities between willful misreporting and excessive optimism, see Schrand and Zechman, 2012). Discretion can be used to either over-report or under-report current earnings. In either case, we assume that misreporting reverses in the subsequent period, so that reported earnings have the following form

\[ \hat{y}_t = y_t + d_t - d_{t-1}. \]

The accounting bias, \( b_t = d_t - d_{t-1} \), is the discrepancy between actual earnings, \( y_t \), and the reported number, \( \hat{y}_t \).

2.4. Statistical properties of reported performance shocks

To formulate the stochastic process of earnings in terms of observable variables, we exploit the link between reported and actual earnings shown in Eq. (2). If we substitute reported for actual earnings, the form of Eq. (1) remains unchanged, but the error term no longer has the interpretation of the current period performance shock. Namely, the structure of reported earnings specified by Eq. (2) implies that Eq. (1) can be written as

\[ \hat{y}_t = \alpha + \beta \hat{y}_{t-1} + \epsilon_t, \]

in which the structural error term, \( \epsilon_t \), takes the following form:

\[ \epsilon_t = \epsilon_t + d_t - (1 + \beta) d_{t-1} + \beta d_{t-2}. \]

The strategy that we propose to identify earnings manipulation relies on the following observation: if reporting discretion is used to mask performance shocks, then \( d_t \) will be correlated with \( \epsilon_t \). This correlation along with persistence and bias reversal leads to autocorrelation patterns in \( \langle \hat{\epsilon}_t \rangle \).

2.4.1. Linear smoothing

It is instructive to consider the special case in which the discretion, \( d_t \), is a linear function of the shock, \( d_t = -\gamma \epsilon_t \), \( 0 < \gamma < 1 \).\(^{12}\) Let \( \text{var}(\epsilon_t) = \sigma^2_{\epsilon} \). In this case

\[ \text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1}) = (1 + \beta) \gamma (1 - (1 + \beta) \gamma) \sigma^2_{\epsilon}, \]

\[ \text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-2}) = -\beta \gamma (1 - \gamma) \sigma^2_{\epsilon}. \]

Note that the second-order autocovariance is always negative and its minimum is achieved at \( \gamma = 1/2 \), which corresponds to half of the current shock being transferred to the future period. In contrast, the sign of the first-order autocovariance depends on how the aggressiveness of smoothing, \( \gamma \), compares to \( 1/(1 + \beta) \). When \( \gamma \) is low, which corresponds to less aggressive smoothing, the first-order autocovariance is positive. As aggressiveness increases \( (i.e. \gamma \text{ increases}) \), the first-order autocovariance decreases. Mathematically, if \( \gamma > 1/(1 + \beta) \), then \( \text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1}) < 0 \), and if \( \gamma < 1/(1 + \beta) \), then \( \text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1}) > 0 \).

2.4.2. Arbitrary monotonic smoothing function

In the next section we analytically show that the linear smoothing rule can arise as an optimal strategy if the manager engages in costly misreporting to avoid a penalty for missing a benchmark. However, linearity per se is not critical to our main prediction. The negative second-order autocorrelation result holds for any arbitrary monotonic smoothing function \( f(\epsilon_t) \), such that \( f'(\epsilon_t) \in (-1, 0) \).\(^{13}\) For such a function, it is straightforward to verify from Eq. (4) that the structural errors in the model with strategic reporting are autocorrelated, with the sign of the first-order autocovariance being ambiguous and the sign of the second-order autocovariance always negative in the presence of strategic reporting.\(^{14}\) The first two autocovariances are

\[ \text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1}) = -(1 + \beta) \text{cov}[\epsilon_{t-1} + f(\epsilon_{t-1}), f(\epsilon_{t-1})] + \beta \text{var}[f(\epsilon_{t-2})], \]

\[ \text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-2}) = \beta \text{cov}[\epsilon_{t-2} + f(\epsilon_{t-2}), f(\epsilon_{t-2})]. \]

\(^{11}\) Mechanical reversal of the manager's discretion does not imply that misreporting is completely undone in the next period. To model reversal over multiple periods, the subsequent period's discretion can be used to carry the current discretion over into the future and can be formalized as \( d_t \) being an increasing function of \( d_{t-1} \).

\(^{12}\) We later examine a model in which linear smoothing is the manager's optimal decision rule when faced with convex costs of manipulation and incentives to smooth.

\(^{13}\) Intuitively, this assumption means that the manager's goal is to mask only a portion of the performance shock, not only on average but also on the margin. This generalization, in particular, arises in our Earnings Smoothing model if one replaces quadratic costs of misreporting with an arbitrary convex cost.

\(^{14}\) For the derivations, see the Appendix.
Once again, the sign of the first-order autocovariance is ambiguous, because the expression includes both a negative and a positive term.\textsuperscript{15} Depending on the functional form of \( f(\epsilon_t) \), either the negative or the positive term can dominate. In contrast, the second-order autocovariance is always negative.

Note also that the smoothing function can be random (i.e., different in every period). In other words, it is not critical for the above result that the smoothing rule is deterministic. It is important, however, that the discretion, \( d_t \), is negatively correlated with the contemporaneous performance shock. We show below that adding pure noise (i.e., terms that are uncorrelated with contemporaneous performance shocks) leads to positive second-order autocovariances. It is therefore critical that earnings manipulation is bias as opposed to noise.

2.5. Discussion

Two assumptions underpin our result. First, we assume that misreporting results in shock-smoothing that reverses in the next period. Second, we assume that the firm's business model is stationary so that the firm's unmanipulated earnings follow an AR(1) process. The first assumption—bias reversal—implies that every period's reported earnings contains the current period's discretion, as well as the previous period's discretion, with the previous period's discretion entering the current report with the opposite sign. The second assumption—the autoregressive nature of earnings—is the reason why the overall autocorrelation structure of the residual becomes complex. The lagged earnings component of the regression contains the reversal of the bias corresponding to the second lag of the performance shock. Intuitively, a negative performance shock in period \( t \) leads to over-reporting, and reversal of this over-reporting leads to under-reporting in period \( t + 1 \), ceteris paribus. Subsequently, when under-reported earnings of period \( t + 1 \) are used as an autoregressive predictor for earnings in period \( t + 2 \), the residual must compensate for the downward bias in the regressor. Therefore, the residual in period \( t + 2 \) must contain a component corresponding to the shock in period \( t \), but of the opposite sign.

An important feature of our analysis is that our main prediction is a correlation as opposed to a deterministic rule. This statistical relationship implies that the model can be extended to incorporate a richer set of accounting regularities and the predictions will remain the same. In particular, we can easily accommodate asymmetric persistence of performance shocks generated by conservative accounting rules. Under conditional conservatism, positive shocks are usually recognized at their current value.\textsuperscript{16} On the other hand, negative shocks are more likely to be recognized at their full value. Namely, firms write down previously recorded assets and recognize contingent liabilities when an adverse event is likely and can be estimated.\textsuperscript{17} These differences in recognition generate asymmetric persistence, which is a statistical marker used to identify conditional conservatism (see, Basu, 1997). Under conditionally conservative accounting rules, the second-order autocorrelation will be driven primarily by persistent positive shocks. Statistically, if the result is driven only by positive shocks, then the power of our tests decreases. Therefore, conservatism works against the results that we document in Section 4.

Similarly, we can allow for misreporting opportunities to be random. As opposed to the setting in which the manager misreports in every period, we could consider a setting in which the opportunity to misreport only arises with some probability. Again, the main predictions survive in this setting, but the statistical power of the tests will be lower, holding the length of the time-series fixed.

2.6. Is shock-smoothing misreporting similar to measurement error?

To further illustrate the link between managerial discretion and the performance shock, we next consider a related setting in which reported earnings contain measurement error, \( \nu_t \). We show that random mismeasurement that is uncorrelated with performance shocks leads to autocorrelation patterns that differ from those predicted by strategic misreporting.

Just as with misreporting, we assume that the measurement error must reverse in the subsequent period, \( \hat{y}_t = y_t + v_t - v_{t-1} \). Under this assumption, the structural error term is

\[
\hat{\epsilon}_t = \epsilon_t + v_t - (1 + \beta)v_{t-1} + \beta v_{t-2}.
\]

While Eq. (6) appears similar to Eq. (4), there is an important difference. Namely, the measurement error is uncorrelated with the performance shock and the expressions for the autocovariances are therefore simpler.\textsuperscript{18}

If we assume that \( \epsilon_t \) is i.i.d. with variance \( \sigma^2_{\epsilon} \), then the first and second-order autocovariances are as follows:

\[
\text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1}) = -(1 + \beta)^2 \sigma^2_{\epsilon},
\]

\[
\text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-2}) = \beta \sigma^2_{\epsilon}.
\]

\textsuperscript{15} To see why the first term is negative, note the following: \( f(\epsilon_t) < -1.0 \), therefore \( \epsilon_t + f(\epsilon_t) \) is increasing in \( \epsilon_t \). Because \( f(\epsilon_t) \) is decreasing in \( \epsilon_t \), the covariance of \( \epsilon_t + f(\epsilon_t) \) and \( f(\epsilon_t) \) is always negative.

\textsuperscript{16} This principle is not universal. For example, positive shocks can be recognized at their full value if a corresponding reserve had been previously created.

\textsuperscript{17} See Roychowdhury and Watts (2007) for a discussion of asymmetric timeliness.

\textsuperscript{18} Zero correlation is implied by \( E[\nu_t | \epsilon_t] = 0 \), which is a typical measurement error assumption. This does not, however, preclude the variance of \( \nu_t \) from being a function of \( \epsilon_t \).
The first-order autocovariance is always negative, while the second-order autocovariance is always positive.\footnote{For the derivations, see the Appendix.}

Both the measurement error and strategic misreporting specifications predict autoregressive patterns in the structural errors. However, the signs of the autocovariances predicted by these two specifications are quite different. In our empirical analysis, we examine which one is supported by the data.

### 3. Analytical foundations

In this section, we show how our predicted autocorrelation patterns in reported earnings can arise out of the manager's optimal reporting choices. To do so, we consider two dynamic models of earnings manipulation. The central feature of these models is that the manager exploits accounting discretion to manipulate reports. As per the previous section, reported earnings, \( y_t \), take the following form:

\[
\hat{y}_t = y_t + d_t - d_{t-1},
\]

in which \( y_t \) is the period's true earnings, \( d_t \) is the discretionary portion of the period's reported earnings, and \( (-d_{t-1}) \) is the reversal of the previous period's discretion.

Apart from the natural incentive to report a higher number, we introduce additional incentives associated with the earnings benchmark, \( y^B_t \). In the first model, the manager's objective is both to report a high earnings number and to not deviate far in either direction from the earnings benchmark. We refer to this case as “Earnings-Smoothing.” In the second model, the manager's objective is to beat the market's expectation—there is a discrete penalty associated with reporting a number below the earnings benchmark. We refer to this case as “Meet-or-Beat.”

For both cases, we assume that market participants have the ability to aggregate into the earnings benchmark all information available prior to the realization of the firm's performance shock. In particular, we assume that the market participants understand the incentives of the manager. We also assume that the earnings benchmark is set at the level of expected reported earnings, taking into account the manager's discretion for the prior period,

\[
y^B_t = E_{t-1}(\hat{y}^B_t) = E_{t-1}(y^*_t + d^*_t - d^*_{t-1}),
\]

Where variables with stars, \( \hat{y}^*_t, y^*_t, d^*_t \), denote the market's conjectures about unobserved variables \( \hat{y}_t, y_t, \) and \( d_t \).

Throughout the analysis we assume that unmanipulated earnings follow an AR(1) process and that there is a quadratic penalty for manipulating earnings.\footnote{A quadratic penalty is the easiest way to capture the convexity of misreporting costs—manipulation becomes increasingly more costly to hide as its scale increases.} In what follows, we show that the second-order autocorrelation of reported performance shocks is negative for both the Earnings-Smoothing and Meet-or-Beat cases.

#### 3.1. Earnings-smoothing

In the first model, we derive the manager's reporting strategy when the penalty for deviating from the earnings benchmark is a symmetric quadratic function. Incentives of this type arise when market participants use reported earnings to update their beliefs about the volatility of the firm performance. Specifically, we assume that the manager's objective function is

\[
\max_{\{d_t\}} \sum_{t=1}^{\infty} \delta E[\hat{y}_t - c_1(\hat{y}_t - y^B_t)^2 - c_2 d^2_t],
\]

in which the reported earnings, \( \hat{y}_t \), are given by Eq. (7), and the earnings benchmark, \( y^B_t \), is given by Eq. (8).\footnote{This objective function can be viewed as a stylized representation of numerous smoothing motives. The manager can be concerned about the company's perceived risk as in Trueman and Titman (1988), retaining his position as in Fudenberg and Tirole (1995), or smoothing consumption when compensation is equity-based as in Zakolyukina (2012).} The manager's objective function linearly increases in reported earnings with a coefficient \( s > 0 \) and the manager discounts future payoffs at the rate \( \delta \).

The manager faces two types of penalties. First, there is a quadratic penalty for deviating from the benchmark with a corresponding coefficient \( c_1 > 0 \). Second, there is a quadratic cost of discretion with a coefficient \( c_2 > 0 \). The solution to the above problem is the discretionary portion of the manager's report, \( \{d_t(y^B_t, \epsilon^t)\}_{t=1}^{\infty} \). In period \( t \), the discretionary portion is a function of the entire history of the performance shocks up to period \( t \), as well as the firm's initial condition, \( y^B_0 \). For expositional simplicity, we assume \( d_0 = 0 \).

To solve for the optimal policy, we use a guess-and-verify technique and conjecture that the manipulation takes a linear form

\[
d_t = d_e + \gamma \epsilon_t,
\]

where \( d_e \) is the expected manipulation, and \( \gamma \) is the sensitivity of the reporting bias to a performance shock. We also conjecture that in every period \( t \) the earnings benchmark is equal to expected reported earnings, \( y^B_t = E(\hat{y}^*_t) \).

19 For the derivations, see the Appendix.
20 A quadratic penalty is the easiest way to capture the convexity of misreporting costs—manipulation becomes increasingly more costly to hide as its scale increases.
21 This objective function can be viewed as a stylized representation of numerous smoothing motives. The manager can be concerned about the company's perceived risk as in Trueman and Titman (1988), retaining his position as in Fudenberg and Tirole (1995), or smoothing consumption when compensation is equity-based as in Zakolyukina (2012).
In the Appendix, we demonstrate that the first-order condition for this problem takes the following form:

$$\delta'(s(1-\delta)-2c_1(y_t^s-y_t^d)-2c_2d_t) = 0. \quad (10)$$

Because the first-order condition (10) must be satisfied for every \( \epsilon_t \), it must be satisfied in expectation. This observation yields the following expression for \( E_{t-1}(d_t) \):

$$E_{t-1}(d_t) = \frac{1-\delta s}{2c_2}. \quad (11)$$

Substituting the above expression for \( E_{t-1}(d_t) \) into Eq. (10), we obtain a closed form solution for the manager's reporting policy

$$d_t = \frac{1-\delta s}{2c_2} \frac{c_1}{c_1 + c_2} - \epsilon_t. \quad (12)$$

The optimal manipulation rule implies that the manager on average over-reports earnings by \((1-\delta s)/2c_2\), and uses discretion to mask a fraction, \(c_1/(c_1 + c_2)\), of the performance shock. This smoothing rule is linear in the performance shock because both the marginal return to manipulation and the marginal cost of manipulation are linear in the manager's discretion. This rule is identical to the linear specification of the manager's discretion that we assumed for the derivation of the negative second-order autocorrelation result presented in Section 2.22

### 3.2. Meet-or-Beat

In the Earnings-Smoothing model, we considered the problem of a manager who faces a penalty for deviating from the benchmark in either direction. In the Meet-or-Beat model, the manager incurs a penalty only if the reported earnings, \( y_t \), are below the benchmark, \( y_t^B \). We assume the penalty is a lump-sum—it does not depend on the gap between the reported earnings and the earnings benchmark. Alternatively, one can think of this discrete jump in the manager's payoff as a lump-sum bonus that the manager receives if he meets the target.

The manager's problem in this case can be written as follows:

$$\max_{d_t} \sum_{t=1}^{\infty} \delta^t E[\delta_t y_t < y_t^B - c_2d_t^2]$$

In this notation, \( \delta_t y_t < y_t^B \) is an indicator function that takes value 1, if \( y_t < y_t^B \), and 0, otherwise.

To avoid statistical complications, we assume that the manager takes all performance benchmarks as given, immune to manipulation. In particular, the manager disregards all possible effects that reported performance may have on future benchmarks. This can be a completely rational view if the benchmarks are not based on the past reports, but instead rely only on external information (e.g., peer performance, macroeconomic news, market surveys).23 Consistent with this assumption, empirical research documents that performance targets do not respond to past performance in a manner consistent with Bayesian updating (see, e.g., Leine and Rock, 2002; Indjejikian and Nanda, 2002, Indjejikian et al., 2012).

Throughout the analysis we assume, as before, that target is set at \( y_t^B = E_{t-1}(\hat{y}_t) \). In the Appendix, we demonstrate that even if the expected reported earnings are estimated with error, \( y_t^B = E_{t-1}(\hat{y}_t) + u_t \), the results still hold.

Two separate situations must be considered to fully characterize the solution to this problem. The first situation arises when meeting the target is not a concern. This occurs either when the actual earnings are so high that the target is not met, or when the actual earnings are so low that exercising discretion to meet the target is too costly. In either of these cases, the first-order condition with respect to \( d_t \) is

$$\delta'(s(1-\delta)-2c_2d_t) = 0. \quad (13)$$

In this case, the only reason for misreporting is the manager's intertemporal rate of substitution. Namely, recognizing one dollar of next period's earnings in the current period generates the marginal benefit of \( s(1-\delta) \) and the marginal cost \( 2c_2d_t \). The manager is therefore always willing to incur the misreporting cost in order to recognize today a portion of the next period's earnings. Let \( d \) denote this level of manipulation, so the first order condition in Eq. (13) leads to

$$d = \frac{s(1-\delta)}{2c_2} - \epsilon_t.$$ 

Note that \( d \) represents the minimum level of manipulation that the manager exercises in equilibrium. Any manipulation arising from the manager's desire to meet the performance target will be added on top of \( d \).24

The second situation arises when the firm's performance is below but sufficiently close to the target. In this case, the manager is willing incur the misreporting cost in order to meet the target. The first-order approach does not apply in this case. Indeed, the objective function is discontinuous in \( d_t \), and the manager compares the cost of missing the target with the incremental cost of manipulation. As long as the cost of missing the target dominates, the manager exercises just enough discretion to meet the target. The point at which the manager's discretion achieves its maximum, \( d^* \), is the where the

22 Given that we are interested in correlations, the constant term does not affect our inferences.

23 Alternatively, this assumption can simply represent the manager's naivete with respect to the benchmarks, which is similar to the price-taking assumption in the analysis of monopolistic competition (for a discussion, see Krugman and Obstfeld, 2008).

24 Because we assume that the manager takes the performance targets as given, “the ratchet effect” consideration does not affect the manager’s report. In other words, the manager does not under-report in order to lower the future targets (for a discussion, see Weitzman, 1980).
manager is indifferent between meeting and missing the target. In the Appendix, we show that $\overline{d}$ can be expressed as Fig. 1 depicts how the manager's discretion varies with the firm's performance shock $\varepsilon_t$. Note that in the Meet-or-Beat case, just as in the Earnings-Smoothing case, the manager's discretion is only a function of the contemporaneous shock. Using the definition of the reported shock, $\hat{\varepsilon}_t = \varepsilon_t + d_t - \beta d_{t-1}$, it is therefore straightforward to obtain the following expression for the second-order autocovariance of $\hat{\varepsilon}_t$:

$$\text{cov}(\hat{\varepsilon}_t, \hat{\varepsilon}_{t+2}) = \beta \text{cov}(\varepsilon_t + d_t, d_t).$$

In the Appendix, we show that the above expression is negative.

### 3.3. Discussion of the analytical results

For both the Earnings-Smoothing and Meet-or-Beat models, we demonstrate that earnings manipulation strategies lead to second-order autocorrelation in reported performance shocks. For the Earnings-Smoothing case, the optimal reporting strategy involves linear smoothing of the performance shocks in combination with systematic over-reporting. In the Meet-or-Beat case, there is a discontinuity in the reporting strategy. Namely, when it is too costly to manipulate earnings to meet the benchmark, the discretion drops from its maximum to its minimum value. The reporting strategy is therefore non-monotonic in the performance shock. As shown in Fig. 1, this reporting strategy resembles the reporting strategy described by Healy (1985). Note, however, that the incentive structure in the Meet-or-Beat model differs from Healy's (1985), who assumes a linear bonus scheme with upper and lower bounds as opposed to a discrete penalty for missing the target.\(^{25}\)

Both models are highly stylized because the scope of our paper does not allow for an exhaustive analytical treatment of earnings manipulation.\(^{26}\) Our primary motivation for the analytical models is to provide structural foundations for our empirical tests and to show how autocorrelation patterns in reported earnings arise out of the manager's optimal reporting choices.

### 4. Empirical tests

In this section, we empirically test our analytical predictions. On a firm-level basis, we regress current earnings on lagged earnings, and then examine the time-series properties of the residuals.

#### 4.1. Sample

To be consistent with prior research, for our measure of reported accounting earnings, we use earnings before extraordinary items $\overline{IB}$ (Compustat $IB$), which includes long term accruals such as depreciation and deferred taxes.\(^{27}\) We find, however, similar results if we include extraordinary items in our measure of earnings. Our sample starts with non-financial US firms that report their financial statements under SFAS 95, which commenced in 1988. We start our sample with the implementation of SFAS 95, so we can benchmark our results using earnings before extraordinary items with prior research that uses earnings before long term accruals. In principle, our approach allows us to extend the sample back further in time. We find similar results when we start the sample at the beginning of Compustat coverage in 1950.

There are 105,428 firm years on the Compustat Fundamental Annual File with non-missing earnings before extraordinary items (Compustat $IB$), total assets (Compustat $AT$), and lagged total assets. We then drop all financial services firms. Finally, we require that each firm in the sample have at least 18 annual observations leading to the final sample of 1346 firms with

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\(^{25}\) Another distinction from Healy (1985) is that our setting does not generate “big bath” incentives. In our setting, future targets are set taking into account the reversal of past manipulation. Therefore, it does not pay to use a “big bath” to meet future targets.

\(^{26}\) In particular, for the Earnings Smoothing model the market can ex post infer unmanipulated earnings. Although this feature is common in theoretical models of earnings manipulation that focus on contracting inefficiency, it might viewed as unrealistic because it suppresses the informational motive for misreporting.

\(^{27}\) An alternative approach would be to create and use a measure based on earnings before long term accruals. We do not use earnings before long term accruals, because such a measure does not typically show up in either analyst forecasts or managerial performance evaluations. Nevertheless, we find quantitatively and qualitatively similar empirical results when we use earnings before long term accruals as the measure of reported earnings.
Table 1
Coefficient estimates for income before extraordinary items.
This table presents coefficient estimates from the firm-level regressions of Eq. (14) in which the \( \hat{ib}_t \) represents income before extraordinary items:

\[
\hat{ib}_t = a + \hat{\beta} \hat{ib}_{t-1} + \hat{\epsilon}_t,
\]

We estimate these firm-level regressions using Stata's ARIMA function allowing the residuals to follow an AR(2) process

\[
\hat{\epsilon}_t = \epsilon_t + \rho_1 \epsilon_{t-1} + \rho_2 \epsilon_{t-2}
\]

in which \( \epsilon_t \) represents an idiosyncratic shock to the firm's performance. We use robust standard errors to address heteroskedasticity and classify a coefficient as statistically significant if its two-sided p value is less than 5%.

<table>
<thead>
<tr>
<th>( \hat{ib}_t )</th>
<th>( \hat{\beta} )</th>
<th>( \hat{\rho}_1 )</th>
<th>( \hat{\rho}_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.0079</td>
<td>0.3662</td>
<td>0.1443</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.1026</td>
<td>0.5185</td>
<td>0.5065</td>
</tr>
<tr>
<td>25th Percentile</td>
<td>0.0014</td>
<td>-0.0472</td>
<td>-0.2189</td>
</tr>
<tr>
<td>Median</td>
<td>0.0178</td>
<td>0.4523</td>
<td>0.0998</td>
</tr>
<tr>
<td>75th Percentile</td>
<td>0.0438</td>
<td>0.7767</td>
<td>0.4858</td>
</tr>
<tr>
<td>Significantly positive (%)</td>
<td>32.8</td>
<td>49.7</td>
<td>25.2</td>
</tr>
<tr>
<td>Significantly negative (%)</td>
<td>5.1</td>
<td>10.8</td>
<td>14.2</td>
</tr>
</tbody>
</table>

27,718 firm years. We deflate earnings before extraordinary items \( \hat{ib}_t \) by lagged total assets. The distributions of our empirical measures are similar to those presented in both Dechow and Dichev (2002) and Wysocki (2009).

4.2. Estimation

For each firm in the sample, we estimate the following specification using Stata’s ARIMA function:

\[
\hat{ib}_t = a + \beta \hat{ib}_{t-1} + \epsilon_t
\]

in which the structural error term follows \( \epsilon_t \) an AR(2) process

\[
\epsilon_t = \eta_t + \rho_1 \eta_{t-1} + \rho_2 \eta_{t-2}
\]

with \( \eta_t \) representing an idiosyncratic shock to the firm's performance. We allow the structural error terms to follow an AR(2) process because our empirical predictions are about the second-order autocovariances of the structural errors, not the coefficients on the idiosyncratic performance shocks. Of the 1346 firms in the sample, nine do not converge. Table 1 presents the coefficient estimates for Eq. (14). Mean and median estimates on prior earnings are 0.366 and 0.452.29

We next examine the model’s predictions about the autocorrelations of the residuals. Columns (3) and (4) of Table 1 present estimates of the first- and second-order coefficients of the residuals from Eq. (14). These coefficients represent the partial autocorrelations of the structural residuals, which purge the estimates of the indirect correlations in the process. The mean and median first-order coefficient estimates are positive (0.144 and 0.100). In contrast, consistent with the analytical predictions, the mean and median estimates for the second-order coefficients are negative (−0.191 and −0.194). We find a similar divergence when we compare significantly positive to significantly negative coefficient estimates. For the first lag, 25% of the estimates are significantly positive while 14% are significantly negative. In contrast, 27% of the second lag coefficient estimates are significantly negative while less than 4% are significantly positive. Depending on the notion of misreporting, previous studies find estimates ranging from 5% for corporate fraud (Dyck et al., 2011) to 63% for earnings smoothing (Zakolyukina, 2012). Fig. 2 presents histograms of the firm-level first and second lag autocorrelation estimates. It graphically summarizes this study’s main message—strategic bias can be identified in the autoregressive patterns of the earnings regression residuals.

As a benchmark to gauge significance of the second-order coefficients, we simulated earnings processes that involve no reporting bias and zero autocorrelation in the residuals. For the simulations, we find that approximately 5% of the coefficients were negative and statistically significant. Although we attribute the difference between the 27% in our sample and the 5% under the zero bias simulations to manipulation, it does not imply that only 22% of the firms in the economy

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28 This restriction limits our sample to relatively large firms that do not experience significant growth or decline. Although we find similar results if we require a minimum of 12 annual observations, our empirical results may not be generalizable to the full sample of public firms. This lack of generalizability may be especially relevant for the latter part of our sample period during which survival rates dropped (see, Fama and French, 2004).

29 Negative coefficients on prior earnings are difficult to interpret economically and inconsistent with our specification. They can arise, statistically, because of survivorship bias (i.e., negative earnings must reverse in order for the firm to continue operating over a long sample period). They can also arise because of asymmetric treatment of positive and negative shocks. Removing firms with negative coefficients on prior earnings does not affect the distribution of second-lag autocorrelations.
manipulate earnings. The 22% estimate may be a lower bound for our sample.\textsuperscript{30} As we discuss in Section 2, there are many reasons to believe that in the real data the residuals should be positively autocorrelated. In the simulations we assume zero autocorrelation in the residuals. Positive autocorrelation in the residuals in the absence of strategic manipulation would lead to even fewer having significant negative second-order autocorrelation.

We carried out several additional tests to rule out alternative interpretations and verify the implications of our model. First, it is possible that in general there are asymmetries in the distributions of the high order autocorrelations. In additional tests, we therefore examined whether there are asymmetries for the third and fourth lag estimates. For these lags, we found no such asymmetries. Second, misreporting implies that bias in response to negative performance shocks should be greater than or equal to bias in response to positive performance shocks. We therefore examined whether the second-order autocovariance differed based on the sign of the period’s shock. Consistent with our misreporting story, we found that the second-order autocovariances are more negative conditional on the current period’s shock being negative, implying that firms introduce greater bias when they experience negative performance shocks. Third, it could be the case that the negative coefficients are driven by small sample bias (for a discussion, see Kendall, 1954). Nevertheless, even after taking into account small sample bias, the distributions of second-order coefficient estimates are still shifted in the negative direction. For example, assuming zero autocorrelations, bias is \(-1/n\), which is \(-0.048\) for the median firm in the sample.

A convenient feature of our specification is that it allows us to examine earnings manipulation that is unrelated to accruals management. Namely, we can re-specify Eq. (14) in terms of cash flows from operations. Earnings manipulation operationalized through the intertemporal shifting of cash flows (commonly referred to as “real earnings management”) has similar mechanics to accruals-based earnings management and results in the same autocorrelation patterns that we predict for earnings. In Table 2 we present estimates for cash flow from operations. Consistent with earnings manipulation through real activities, we find that the mean and median second-order autocorrelations are negative (\(-0.1545\) and \(-0.1630\)) and that approximately 23% of the firms have significantly negative second-order autocorrelations.\textsuperscript{31}

5. Unmanipulated earnings

A useful feature of our time-series approach is that it allows researchers to create estimates of unmanipulated earnings that have informational value incremental to that of other accounting variables. The traditional approach to constructing estimates of unmanipulated earnings is based on linear projections of earnings on other observable variables (e.g., Jones, 1991 or Dechow and Dichev, 2002). Such an approach implies that unmanipulated earnings provide no additional information beyond what is already contained in the other observable variables. In contrast, our approach filters out from earnings information contained in past reports, thereby preserving all new information.

\textsuperscript{30} Interestingly, when we require only a minimum of 12 annual observations, the proportion of firms with negative and significant coefficients increases from 27% to 29%. This increase may be due to the small sample bias that we discuss further.

\textsuperscript{31} These findings are consistent with real earnings management. Our specification does not allow us to estimate the relative intensity of misreporting (i.e., accruals-based versus real activities). An extreme example would be that, for an individual firm, 95% of misreporting could be done through accruals and 5% through real activities even though the second-order coefficients are negative and significant for both earnings and cash flows.
In what follows, we describe a procedure to create estimates of unmanipulated earnings. First, we assume that discretion is a linear function of current performance shock, \( q_t = -\gamma \epsilon_t \), so that

\[
\hat{\epsilon}_t = \epsilon_t + \phi_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} = (1-\gamma) \epsilon_t + (1-\beta) \gamma \epsilon_{t-1} - \beta_2 \epsilon_{t-2}.
\]  

(15)

In our estimation procedure we impose the following structure on \( \hat{\epsilon}_t \):

\[
\hat{\epsilon}_t = \hat{\epsilon}_t + \phi_1 \hat{\epsilon}_{t-1} + \rho_2 \hat{\epsilon}_{t-2}.
\]  

(16)

and obtain estimates of \( \rho_1 \) and \( \rho_2 \). In addition, our estimation procedure provides an estimate of \( \beta \). We use these estimates to reconstruct the series of unmanipulated performance shocks and then earnings. Comparing Eqs. (15) and (16) leads to the following expressions: \( \hat{u}_t = (1-\gamma) \epsilon_t \) and \( \hat{u}_t = -\beta \gamma / (1-\gamma) \). Using the estimates for \( \rho_2 \) and \( \beta \), we express \( \gamma \) as the positive root of a quadratic equation

\[
\gamma = \frac{1}{2} \left( \sqrt{1-4 \rho_2^2 / \beta^2} - 1 \right).
\]

To derive our construct for the estimated series of unmanipulated performance shocks \( \langle \epsilon_t \rangle \), we pull the sequence \( \langle \hat{u}_t \rangle \) and divide it by \( (1-\gamma) \). Further, to construct a series of unmanipulated earnings we purge \( \hat{y}_t \) of the reporting bias

\[
y_t = y_t - \phi_1 \epsilon_t + \phi_1 \epsilon_{t-1} = \hat{y}_t + \gamma (\epsilon_{t-1} - \epsilon_{t-1}).
\]  

(17)

In Table 3 we present descriptive statistics for reported earnings, our estimates of unmanipulated earnings, and our estimates of the linear smoothing parameter, \( \gamma \). Consistent with the smoothing intuition, the volatility of unmanipulated earnings is approximately 25% higher than for reported earnings. With respect to the smoothing parameter, \( \gamma \), the mean firm smooths approximately 32% of the period's performance shock. In addition, median absolute misreporting as a percentage of total assets is 0.7% and the first and third quartiles are 0.07% and 3.7% of total assets. These estimates appear more plausible than the discretionary accruals estimates documented in prior research. For example, Hribar and Nichols (2007) report for their sample that median absolute discretionary accruals are 5.2% of total assets, which is similar in magnitude to mean and median ratios of earnings before extraordinary items to total assets (see, for example, Dechow and Dichev, 2002; Wysocki, 2009).
As an example of an application of our methodology, we evaluate earnings timeliness. Following the methodology of Ball et al. (2000), we estimate at the firm-level the relations between annual returns and contemporaneous earnings. We use two measures of earnings. The first measure is the earnings reported by the firm, $ib$. The second measure of earnings is our estimate of unmanipulated earnings as shown in Eq. (17). To compare the performance of each measure, we use the adjusted $R^2$s from firm-level regressions of contemporaneous returns on the two measures of earnings. As shown in Table 4, we find that our estimates of unmanipulated earnings better explain the variation in annual returns ($\text{mean} = 0.0802$, $\text{median} = 0.0257$) than do reported earnings ($\text{mean} = 0.0514$, $\text{median} = 0.0011$), with differences for the mean and median significant at the 0.0001 level.

To further apply our methodology, we exploit the fact that $\gamma$ represents a firm-level measure of manipulation intensity. We therefore test whether $\gamma$s are higher for sample firms subject to SEC AAERs. Consistent with this conjecture, mean $\gamma$s are significantly higher for sample firms subject to AAERs during the estimation period. The mean and median $\gamma$ for sample firms subject to AAERs during our sample period are $0.4004$ and $0.3262$, which are significantly higher ($\text{mean}, \text{median} = 0.0618; \text{median} = 0.0609$) than the mean and median $\gamma$s for sample firms not subject to AAERs ($0.3145$ and $0.2659$).

### 6. Conclusion

The traditional approach for identifying earnings manipulation has been to construct a prediction model for earnings (or its components) and then to treat deviations from these predictions as evidence of either deliberate misrepresentation or low reporting quality. Two classic examples of this approach are Jones (1991) and Dechow and Dichev (2002). Jones (1991) develops a measure of discretionary accruals based on the assumption that non-discretionary accruals are a deterministic, linear function of the change in sales and the level of property, plant, and equipment, implying that anything unexplained by the model represents discretionary accruals. In a similar fashion, Dechow and Dichev (2002) specify a deterministic intertemporal decomposition of cash flows and then measure accruals quality as the estimation error in a regression of changes in working capital on past, current, and future cash flows from operations. This approach is, however, limited in its ability to separate earnings manipulation from operating volatility (for a discussion, see Dechow et al., 2010). Essentially, the deterministic benchmarking approach classifies innovations to firm performance as misreporting and therefore potentially leads to the excessive identification of earnings manipulation (for discussions, see Kasznik, 1999; Nichols, 2002).

To separate misreporting from operating volatility, we explore the time-series properties of reported earnings. We demonstrate that several forms of misreporting produce serial correlation patterns in earnings that are difficult to rationalize in the absence of misreporting. Specifically, we show that the residuals from a regression of reported earnings on lagged reported earnings will have a negative second-order autocorrelation in the presence of misreporting. Empirically, we find that the distribution of the second-order autocorrelation measure is asymmetric around zero with 74% of the observations being negative and 27% being significantly negative. Assuming that misreporting is linear in the performance shock, we find that firms in our sample subject to SEC AAERs have significantly higher estimates of manipulation intensity and that our estimates of unmanipulated earnings are more highly correlated with contemporaneous returns and have higher volatility than reported earnings.

There are, however, several important caveats to our methodology. First, we focus on the manipulation of earnings and are indifferent about whether manipulation occurs through accruals or real activities that manifest in cash flows. Second, we use an AR(1) specification to model persistence in unmanipulated earnings. It is important to point out that our AR(1) specification for unmanipulated earnings could be misspecified. Third, our time-series specification imposes several restrictions on the nature of the misreporting that can be identified. Specifically, it has to be a sustainable manipulation strategy (i.e., it does not result in exponentially growing account balances). Fourth, our time-series strategy requires a relatively long and stable series of firm-level observations to estimate our manipulation measure. Fifth, our methodology is unlikely to capture manipulation strategies driven by motives other than masking performance shocks.

Overall, we do not claim that we identify the predominant form of earnings manipulation. However, it is important to point out that misreporting arising due to motives unrelated to performance shocks is statistically similar to measurement...
error. Because measurement error produces serial correlations of the opposite sign than our prediction, such misreporting would make it more difficult to identify manipulations of the form that we consider.

Appendix A

A.1. Derivation of autocovariances for an arbitrary monotonic smoothing function

First-order autocovariance

\[
\text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1}) = \text{cov}(v_t + f(\epsilon_t) - (1 + \beta)f(\epsilon_{t-1}) + \beta f(\epsilon_{t-2}),
\]
\[
= \text{cov}(-v_t \epsilon_{t-1} + f(\epsilon_{t-1}) - (1 + \beta)f(\epsilon_{t-1}) - (1 + \beta)f(\epsilon_{t-2}))
\]
\[
= \text{cov}(-v_t \epsilon_{t-1} + f(\epsilon_{t-1}) + \text{cov}(f(\epsilon_{t-2}), -(1 + \beta)f(\epsilon_{t-2}))
\]
\[
= -(1 + \beta)\text{cov}(f(\epsilon_{t-1}), \epsilon_t) + \beta \text{cov}(f(\epsilon_{t-2}))
\]

Second-order autocovariance

\[
\text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-2}) = \text{cov}(v_t + f(\epsilon_t) - (1 + \beta)f(\epsilon_{t-1}) + \beta f(\epsilon_{t-2}),
\]
\[
= \text{cov}(-v_t \epsilon_{t-1} + \epsilon_t - f(\epsilon_{t-1}) \epsilon_{t-2} + \beta f(\epsilon_{t-4})]
\]
\[
= \beta \text{cov}(f(\epsilon_{t-2}), \epsilon_{t-2} + f(\epsilon_{t-2}))
\]

A.2. Derivation of autocovariances for earnings measured with noise

Let \( \nu_t \) denote the measurement error in period \( t \). If the actual earnings follow an AR(1) process, \( y_t = \alpha_t + \beta y_{t-1} + \epsilon_t \), then reported earnings can be expressed as

\[
\hat{y}_t = y_t + \nu_t - v_{t-1}
\]
\[
= \alpha + \beta y_{t-1} + \epsilon_t + \nu_t - v_{t-1}
\]
\[
= \alpha + \beta(\hat{y}_{t-1} - v_{t-1} - v_{t-2}) + \epsilon_t + v_t - v_{t-1}
\]
\[
= \alpha + \beta \hat{y}_{t-1} + \epsilon_t + v_t - (1 + \beta)v_{t-1} + \beta v_{t-2}.
\]

The structural error term \( \hat{\epsilon}_t \), therefore takes the form

\[
\hat{\epsilon}_t = \epsilon_t + v_t - (1 + \beta)v_{t-1} + \beta v_{t-2}.
\]

If we assume that \( \nu_t \) is i.i.d. with variance \( \sigma^2_{\nu} \), then the first- and second-order autocovariances are as follows:

\[
\text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-1}) = \text{cov}(\nu_t + f(\epsilon_t) - (1 + \beta)f(\epsilon_{t-1}) + \beta f(\epsilon_{t-2}), \nu_{t-1} + f(\epsilon_{t-1}) - (1 + \beta)f(\epsilon_{t-2} + \beta f(\epsilon_{t-3}))
\]
\[
= \text{cov}(-v_t \epsilon_{t-1} + f(\epsilon_{t-1}) - (1 + \beta)f(\epsilon_{t-1}) - (1 + \beta)f(\epsilon_{t-2} + \beta f(\epsilon_{t-3}))
\]
\[
= -(1 + \beta)(1 + \beta)\text{cov}(f(\epsilon_{t-2}), \epsilon_{t-2} + f(\epsilon_{t-2}))
\]

\[
\text{cov}(\hat{\epsilon}_t, \hat{\epsilon}_{t-2}) = \text{cov}(\epsilon_t + f(\epsilon_t) - (1 + \beta)f(\epsilon_{t-1}) + \beta f(\epsilon_{t-4}), \epsilon_{t-2} + f(\epsilon_{t-1}) - (1 + \beta)f(\epsilon_{t-2} + \beta f(\epsilon_{t-4}))
\]
\[
= \beta \text{cov}(f(\epsilon_{t-2}), \epsilon_{t-2} + f(\epsilon_{t-2}))
\]

A.3. Derivation of the first-order condition for the Earnings-Smoothing model

Let \( \gamma \) denote the market’s belief about the actual earnings in period \( t \), \( y_t \). Similarly, let \( d_t \) denote the market’s belief about the reporting bias in period \( t \), \( d_t \). Suppose, up to period \( t-1 \) the market’s beliefs were correct, \( y^*_t = y_t \), \( d^*_t = d_t \), \( r = 1, \ldots, t-1 \), and the conjecture (9) is correct. In this case, the benchmark

\[
y^*_t = a + b y^*_{t-1} + d_t - d^*_{t-1}
\]

is equal to the expected reported earnings in period \( t \), \( y^*_t = E(\hat{y}^*_{t}) \). The firm’s reported earnings in period \( t \) can thus be expressed as

\[
\hat{y}_t = E(\hat{y}^*_{t}) + \gamma \epsilon_t + \epsilon_t.
\]

Combining (18) and (19), the market can infer the firm’s performance shock as follows:

\[
\epsilon^*_t = \frac{\hat{y}_t - y^*_t}{1 + \gamma}.
\]

With this information, Eq. (18) can be used to produce the firm’s performance benchmark for period \( t+1 \)

\[
y^*_{t+1} = a + b y^*_{t} - \gamma \epsilon^*_t
\]
\[
= a + b(\hat{y}_t - d_t + d^*_{t-1}) - \gamma \epsilon^*_t
\]
\[
= a + b(\hat{y}_t - \gamma(1 + \beta)\epsilon^*_t - \gamma \epsilon^*_{t-1})
\]
\[
= a + b(\hat{y}_t - \gamma(1 + \beta)\epsilon^*_t) + \frac{\hat{y}_{t-1} - y^*_{t-1}}{1 + \gamma}.
\]
Therefore, the earnings benchmark of the subsequent period will be affected by today’s reporting bias
\[
\frac{\partial y_{t+1}^s}{\partial d_t} = \frac{\partial y_{t+1}^H}{\partial y_t} \frac{\partial y_t}{\partial d_t} = \frac{\partial y_{t+1}^H}{\partial y_t} = b - \gamma(1 + \beta) > 0.
\]
Due to recursive formulation of the performance target, the reporting bias of period \( t \) affects performance targets in all future periods. However, in the first-order condition of the manager’s problems, these effects are multiplied by expected deviation of the reported earnings from the performance benchmark:
\[
\delta^i[(1 - \beta)s - 2c_1(\hat{y}_t - \gamma_t) - 2c_2d_t] + \sum_{i=1}^{\infty} \beta^{i-1}2c_1E_t(\hat{y}_{t+i} - \gamma_{t+i})\frac{\partial y_{t+i}^H}{\partial d_t} = 0.
\]
According to our conjecture and the law of iterated expectations, \( E_t(\hat{y}_{t+i} - \gamma_{t+i}) = 0 \) for every \( i \). This expression yields the following first-order condition:
\[
\delta^i[(1 - \beta)s - 2c_1(\hat{y}_t - \gamma_t) - 2c_2d_t] = 0.
\] (20)

A.4. Derivation of maximum discretion for the Meet-or-Beat model

The maximum discretion \( \bar{d} \) can be characterized by the following indifference condition:
\[
sd^2 - c_2d^2 = sd - c_1 - c_2d^2.
\]
The left-hand side represents the manager’s payoff when the target is met. If the manager meets the target, he receives the benefit of the higher report \( s(\bar{d} - d) \), but the cost of misreporting increases by \( c_2(\bar{d} - d)^2 \). The right-hand side represents the manager’s payoff when the manager misses the target. In this case, he incurs a penalty of \( c_1 \). Solving for \( \bar{d} \), we obtain
\[
\bar{d} = -c_1/c_2. \quad \text{32}
\]
Let \( F \) denote the interval of \( \varepsilon_t \) in which the reporting strategy is “flat” (i.e., for any performance shock in this interval the manager reports the same earnings number, \( \hat{y}_t(\varepsilon_t) = \gamma_t^H \)). Using the levels of discretion at the end points of this interval, \( \bar{d} \) and \( d \), it is straightforward to show that \( F = [E_{t-1}(d_t) - \bar{d}, E_{t-1}(d_t) - d] \). In this interval, the manager’s discretion is such that the benchmark is exactly met, \( d_t = E_{t-1}(d_t - \varepsilon_t) \). In all other cases, the manager’s discretion is set at the minimum level, \( d_t = d \).

A.5. Negative second order autocorrelation for the Meet-or-Beat model

Claim.
\[
\text{cov}((\varepsilon_t + d_t, d_t)) \leq 0.
\]
Proof. This expression can be decomposed as
\[
\text{cov}((\varepsilon_t + d_t, d_t)) = \text{cov}(\varepsilon_t, d_t) + \text{var}(d_t) = E(\varepsilon_t d_t) - E(\varepsilon_t)E(d_t) + E(d_t^2) - [E(d_t)]^2. \quad (21)
\]
To determine the sign of \( \text{cov}(\varepsilon_t + d_t, d_t) \), we further decompose its terms as follows. First, we define \( \tau(\varepsilon_t) = E_{t-1}(d_t, \varepsilon_t) \), so that \( d_t = d + \tau(\varepsilon_t) \). Then \( E[\varepsilon_t d_t] \) can be expressed as
\[
E[\varepsilon_t d_t] = E[\varepsilon_t (d + \tau(\varepsilon_t))] = E[\varepsilon_t \tau(\varepsilon_t)] = -E[(\varepsilon_t)^2] - dE[\tau(\varepsilon_t)]. \quad (22)
\]
Second, \( E[d_t^2] \) can be expressed as
\[
E[d_t^2] = E[(d + \tau(\varepsilon_t))^2] = d^2 + 2dE[\tau(\varepsilon_t)] + E[\tau(\varepsilon_t)^2]. \quad (23)
\]
Finally, \( E[d_t]^2 \) can be expressed as
\[
E[d_t]^2 = E[(d + \tau(\varepsilon_t))^2] = d^2 + 2dE[\tau(\varepsilon_t)] + E[\tau(\varepsilon_t)^2]. \quad (24)
\]
Substituting the expressions for \( E[\varepsilon_t d_t], E[d_t^2], \text{and } E[d_t]^2 \) given by Eqs. (22)–(24) into Eq. (21) yields the following expression for \( \text{cov}(\varepsilon_t + d_t, d_t) \):
\[
\text{cov}(\varepsilon_t + d_t, d_t) = -dE[\tau(\varepsilon_t)] - E[\tau(\varepsilon_t)^2]. \quad (25)
\]
The right-hand side of Eq. (25) is negative, because \( \tau(\varepsilon_t) \) is non-negative everywhere and strictly positive on \( F \). Therefore, \( E[\tau(\varepsilon_t)] > 0 \) and \( \text{cov}(\varepsilon_t + d_t, d_t) < 0 \), showing that the second-order autocorrelation of reported performance shocks is negative, \( \text{cov}(\varepsilon_t, \varepsilon_{t+2}) < 0. \quad \square \)

32 The smaller root of the quadratic equation is discarded because \( \bar{d} \) must be greater than \( d \) in order for reported earnings to meet the target.
A.6. Measurement noise in the benchmark for the Meet-or-Beat model

Suppose expected earnings are measured with noise for the purpose of benchmarking, and let $\kappa_t$ denote the measurement error in period $t$, so that $E(\epsilon_t) = 0$. This implies two changes in the above proof. First, the pooling interval will be $F = [E_{t-1}(d_1) + \kappa_t - \bar{d}, E_{t-1}(d_1) + \kappa_t - \underline{d}]$, and the function $t(\epsilon_t)$ will change to $t(\epsilon_t) = [E_{t-1}(d_1) + \kappa_t - \tau, \eta_{\epsilon_t}]$. None of the steps in the above proof are affected by $\kappa_t$, and the result remains the same.

References

Cashin, P., Liang, H., McDermott, C., 1999. How Persistent are Shocks to World Commodity Prices? Staff Report, International Monetary Fund.