Productivity Across Industries and Countries: Time Series Theory and Evidence

Andrew B. Bernard, Charles I. Jones

PRODUCTIVITY ACROSS INDUSTRIES AND COUNTRIES:
TIME SERIES THEORY AND EVIDENCE

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Abstract—We examine whether convergence in aggregate productivity is also occurring at the industry level in 14 OECD countries from 1970–1987. Using both cross-section and time series methods, we find convergence in some sectors, such as services. However, surprisingly, we find that convergence does not hold for the manufacturing sector. Decomposing aggregate convergence into industry productivity gains and changing sectoral shares of output, we find the service sector to be responsible for the bulk of cross-country convergence to the United States. We develop a new result on the asymptotic normality of panel unit root estimators to formally test within sector convergence.

I. Introduction

A KEY issue in understanding the variety of long-run growth paths across countries is whether technology flows primarily between sectors within a nation or across countries within an industry: are there sector-specific or country-specific sources of productivity improvements? To date, the theoretical and empirical literature on convergence and catch-up has focused almost exclusively on aggregate output and productivity. A primary concern has been which branch of growth theory is supported by the empirical evidence: one that predicts convergence to a common, long-run growth path, i.e., a neoclassical growth model, or one that allows countries with similar endowments and preferences to attain different steady state outcomes for growth rates, i.e., a model from the "new growth" literature. Empirical work on convergence has concentrated on testing whether aggregate income is converging or diverging in order to distinguish between competing theories. In this paper, we step back from the debate over neoclassical or "new growth" models to ask whether the focus on aggregate outcomes is masking important variation in sectoral productivity movements. We examine data on six broadly defined sectors and consider the evidence for and against convergence in total factor productivity (TFP) within sectors across countries. In addition to testing for sectoral convergence, we also consider the role of changing sectoral composition in aggregate productivity movements.

We use cross-section and time series techniques to study the movements of productivity levels in 14 OECD countries, employing industry-level data on value-added, capital stocks, and employment to construct total factor productivity by sector from 1970–1987. The results from the sectoral analysis are striking. Using both cross-section and time series techniques, we find evidence of substantial heterogeneity of productivity movements across sectors. Our results show that within sectors across countries there is evidence for convergence for some industries, but not for others. These differences across sectors account for convergence at the national level. Although aggregate total factor productivity appears to be converging across OECD countries, this convergence is driven primarily by the non-manufacturing sectors of the economies, in particular, services. Within manufacturing we find only weak evidence for convergence over the period and find substantial evidence for divergence of productivity levels during the 1980s. Our results indicate that the lack of convergence found in larger samples of countries may be a consequence of varying sectoral composition in countries at different income levels and that sectoral data can be a valuable new resource in the empirical analysis of long-run growth.[1]

The issues and results on sectoral contributions to aggregate productivity convergence are central to the understanding of how countries grow and, in particular, of how followers catch-up to leaders. Theories of convergence along the lines of Gerschenkron (1952) argue that countries that are relatively backward in a field may learn from more advanced producers and increase their productivity more rapidly. Following this, we should see catch-up in all sectors, proportional to the relative backwardness of a country in that sector.

Another approach to understanding the role of industries in aggregate convergence concentrates on the possible influence of international trade. One mechanism for trade and sectors to affect aggregate convergence is through the transmission of knowledge and increased competition. In this framework, rising trade in a sector increases the knowledge flows from leaders to followers, increases competition within the industry, and leads to sectoral convergence. The degree of sectoral convergence should be positively related to the extent of trade in sectoral output. Under these assumptions, we would expect manufacturing to exhibit the greatest degree of convergence. In another scenario, trade leads countries to specialize in particular industries. Those countries that concentrate in relatively higher value-added sectors achieve higher income and productivity levels. Aggregate convergence is possible with specialization only if each country produces a mix of goods, some high productivity products and some low.[2]

[1] See Barro (1991) for evidence against unconditional convergence for large samples of countries.

[2] Dollar and Wolff (1993) argue that this is occuring in OECD countries.

Largely due to a lack of data on labor and capital, almost all previous work on convergence across countries and regions has used real GDP per capita.[3] Using cross-section regressions, Baumol (1986), Barro and Sala-i-Martin (1991, 1992) and Mankiw, Romer and Weil (1992) argue that countries and regions are converging, or catching-up, since initially poorer areas grow faster than their richer counterparts. However, the cross-section evidence is not uniform. Barro (1991) and DeLong (1988) show that the particular sample of countries determines whether catch-up holds. In a series of papers, Quah (1993, 1994) argues that initially rich countries are converging to a common high income growth path, while initially poor countries are converging to a different low steady state. While work on broader samples of countries produces conflicting results, OECD countries in the post-WWII period are generally thought to exhibit convergence.[4]

The neoclassical growth model without technology predicts convergence in output per worker for similar, closed economies based on the accumulation of capital. However, even in the neoclassical model, if the exogenous technology processes follow different long-run paths across countries, then there will be no tendency for output levels to converge. In this paper we examine technological convergence by focussing on TFP. The results indicate that sectoral differences are important for understanding movements in aggregate income and productivity. Further work on longer samples and with more disaggregated data is needed to understand the relation between industry productivity, sectoral shifts, and aggregate productivity.

A. Convergence in Aggregate TFP

The fundamental piece of evidence on cross-national growth in the OECD economies is that productivity and output differences have narrowed during the post-WWII period. Numerous studies have documented convergence for income per capita and labor productivity for these countries, e.g., Baumol (1986). This phenomenon of convergence holds for aggregate TFP during the 1970–87 period as well. TFP movements are shown for 14 OECD countries from 1970–1987 in figure 1.[5] TFP has grown on average at a rate of 1.2% per year but the gap between the most productive country, the United States throughout the sample, and the least productive country declined consistently from 120% in 1970 to 85% in 1987. The overall decline in dispersion across countries can

be seen in figure 2 which plots the cross-sectional standard deviation of TFP. Dispersion has decreased from 17.5% to less than 13.5% during the period. Without the United States, the dispersion is constant at about 13.5% until 1979 and drops steadily through the 1980s to 11% by 1987. The convergence in levels is also captured by regressing the average TFP growth rate for each country on the 1970 levels for TFP, as given in the following equation.[6]

$$\Delta \log TFP_i = \alpha + \quad \beta \ TFP_i^{1970} + \epsilon_i$$

coef   0.1250 -0.0198 $\bar{R}^2$: 0.4726.   (1)

s.e.   (0.0224)   (0.0039)

The coefficient on the initial level is negative and significant, confirming the visual evidence from the cross-section.

B. Definitions of Convergence

Empirical work on convergence has generally used either cross-section or time series techniques. However, the definitions of convergence implied by the two econometric approaches are distinct.[7] Cross-section analyses have focused on the transition to equilibrium growth paths. Convergence is then taken to be a narrowing of initial differences in income levels over some time horizon, poorer regions growing faster than richer ones (β-convergence), or the reduction in cross-region variance of output (σ-convergence), although one does not necessarily imply the other. These interpretations of convergence are associated with the predicted output paths from a neoclassical growth model with different initial levels of capital. β-convergence implies that countries with low initial capital stocks (income levels) accumulate faster than average and grow faster, while countries with higher than average capital stocks grow more slowly. σ-convergence predicts that the cross-region variance in income will be declining during the transition to steady state. Once countries attain their steady state levels of capital there is no further expected reduction in cross-section output variance, and expected growth rates are identical.

Time series studies generally define convergence as transitory deviations from identical long-run trends, either deterministic or stochastic. Carlino and Mills (1993) extend this definition to allow for region-specific compensating differentials which persist. Tests in this framework look for permanent deviations in relative income paths using cointegration or unit root techniques.[8]

Both these approaches have implications within our

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[3] While convergence of labor productivity is predicted by many growth models, the use of output per capita instead of output per worker or per hour potentially confounds changes in participation rates with changes in productivity.

[4] Time series results on longer series for OECD countries also show evidence of common trends but no tendency for convergence in levels (for example, see Bernard and Durlauf (1995)).

[5] The countries in the sample are United States, Canada, Japan, Germany, France, Italy, United Kingdom, Australia, the Netherlands, Belgium, Denmark, Norway, Sweden, and Finland. The United States is denoted by a "o" and Japan is denoted by a "+" in all figures.

[6] Average growth rates here and throughout the paper are constructed as the trend coefficient from the regression of the log of TFP on a constant and a linear trend.

[7] An exception is Carlino and Mills (1993) which embeds cross-section concepts in a time series framework. See Bernard and Durlauf (1996) for a formal distinction between the definitions of convergence and associated empirical tests.

[8] Bernard and Durlauf (1996) relate differing definitions of convergence to empirical methodologies and discuss their potential problems.

FIGURE 1.—LOG (TOTAL FACTOR PRODUCTIVITY) FOR TOTAL INDUSTRY
(U.S. IS LABELLED 'o', JAPAN IS LABELLED '+')
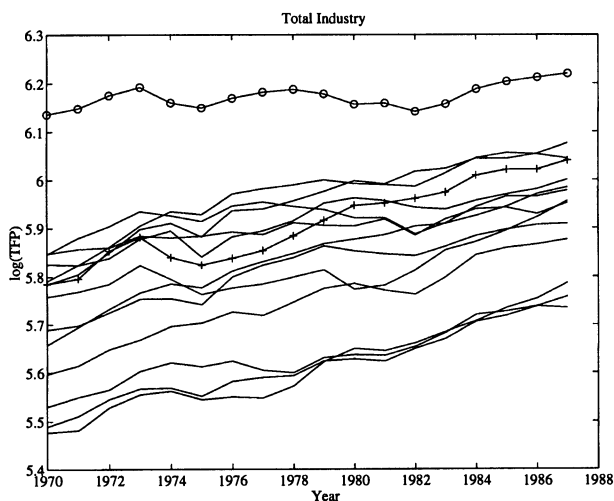
FIGURE 2.—STANDARD DEVIATION OF LOG (TOTAL FACTOR PRODUCTIVITY)
FOR TOTAL INDUSTRY
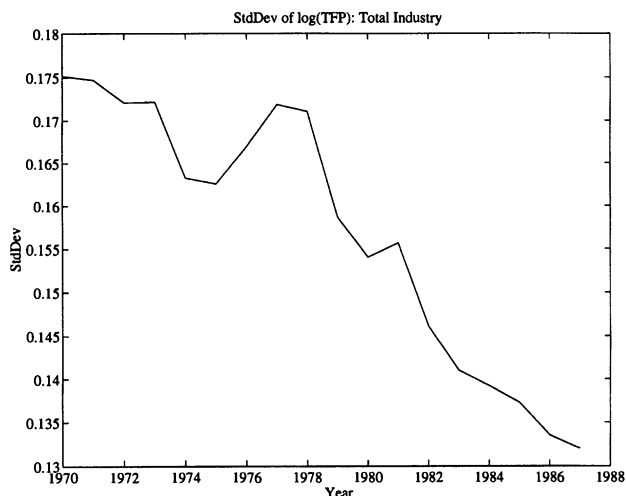
sample of advanced OECD countries from 1970–1987. If these 14 countries are on or near their long-run steady state growth paths as of 1970, then the relevant framework for testing industry level convergence is that of cointegration. To evaluate the appropriateness of the two methodologies for our data, we proceed in several steps. First, we consider evidence on the time paths of the cross-section variance of sectoral TFP levels. Sectors display widely varying behavior; some show no reduction in variance, others substantial reductions. To reconcile these varying patterns across industries with aggregate convergence for total industry, we calculate the contributions of productivity growth within a sector and changes in sectoral composition to aggregate convergence. To determine if sectors with constant cross-section variances represent evidence for or against conver-

gence, we test for transitory versus permanent differences in productivity levels in a time series framework.

The rest of the paper is divided as follows: section II presents a model of catch-up in technology yielding testable time series predictions. Section III describes and displays the data on one digit industries in 14 countries; cross-section standard deviations during the period are also analyzed. Section IV presents a method for decomposing aggregate catch-up into within-sector and between-sector contributions and presents results for countries in the sample. Section V presents a new result on testing for unit roots in panel data and empirical results on industries in the 14 OECD countries. Section VI concludes and discusses possible explanations for the findings.

II. A Basic Model of Technological Catch-up

The neoclassical growth model without technology predicts convergence in output per worker for similar, closed economies based on the accumulation of capital. However, even in the neoclassical model, if the exogenous technology processes follow different long-run paths across countries, then there will be no tendency for output levels to converge. To see this, we construct a simple model of sectoral output in which convergence in output occurs both due to capital deepening and to TFP catch-up. The behavior of TFP in this model is such that relatively backward countries can grow more rapidly by using technologies developed in the leading country.

Assuming a simple Cobb-Douglas production function with constant returns, we can write the log of output in country i, sector j, at time t, $\ln Y_{ijt}$, as

$$\ln Y_{ijt} = \ln A_{ijt} + \alpha \ln K_{ijt} + (1-\alpha) \ln L_{ijt}, \tag{2}$$

where A_{ijt} is an exogenous technology process, K_{ijt} is the capital stock, and L_{ijt} is the number of workers in the sector. We assume A_{ijt} evolves according to

$$\ln A_{ijt} = \gamma_{ij} + \lambda \ln D_{ijt} + \ln A_{ijt-1} + \epsilon_{ijt}, \tag{3}$$

with γ_{ij} being the asymptotic rate of growth of sector j in country i, λ parameterizing the speed of catch-up denoted by D_{ijt}, and ϵ_{ijt} representing an industry and country-specific productivity shock. We allow D_{ijt}, the catch-up variable, to be a function of the productivity differential within sector j in country i from that in country 1, the most productive country,

$$\ln D_{ijt} = \ln \hat{A}_{ijt-1}$$

where a hat indicates a ratio of a variable in country 1 to the same variable in country i, i.e.,

$$\hat{A}_{ijt} = \frac{A_{1jt}}{A_{ijt}}.$$

This formulation of productivity catch-up implies that productivity gaps between countries are a function of the lagged gap in productivity. The choice of the source of

TABLE 1.—PRODUCTIVITY GROWTH RATES BY INDUSTRY AND COUNTRY 1970–1987

	Average Growth Rates of Total Factor Productivity						
Country	Agriculture	Mining	Manufacturing	Utilities	Construction	Service	Total Industry
U.S.	.015	−.037	.016	.006	−.020	.002	.003
Canada	.009	−.060	.008	.006	.014	.006	.004
Japan	−.002	.018	.040	−.012	−.022	.005	.015
W. Germany	.043	−.028	.015	.003	.007	.005	.013
France	.040	−.036	.017	.027	.009	.012	.017
Italy	.020	na	.029	−.017	−.017	na	.010
U.K.	.036	−.015	.012	.007	−.006	.005	.009
Australia	.018	−.008	.014	.019	.008	na	.005
Netherlands	.044	.013	.024	−.012	−.010	na	.013
Belgium	.037	−.012	.035	.028	.011	.005	.016
Denmark	.041	.082	.019	.032	−.011	.010	.014
Norway	.021	.074	.003	.004	−.003	.007	.015
Sweden	.020	−.037	.011	.022	.021	.009	.012
Finland	.022	.019	.026	.014	.015	.013	.017
Average	.026	−.002	.019	.009	−.000	.007	.012

Notes: Average growth rates are calculated by regressing log productivity levels on a constant and a time trend. Source data are from the OECD Intersectoral Database.

catch-up and the simple diffusion process is subject to criticism. Dowrick and Nguyen (1989) allow the catch-up to be determined by labor productivity differentials; however, it seems appropriate to suppose that technological catch-up may be occurring independent of capital deepening.

The model then yields a simple equation for the time path of TFP:

$$\ln \hat{A}_{ijt} = \left(\gamma_{1j} - \gamma_{ij}\right) + \left(1 - \lambda\right)\ln \hat{A}_{ijt-1} + \hat{\varepsilon}_{ijt}. \quad (4)$$

In this framework, values of $\lambda > 0$ provide an impetus for "catch-up": productivity differentials between two countries increase the relative growth rate of the country with lower productivity. However, only if $\lambda > 0$ and if $\gamma_i = \gamma_1$ (i.e., if the asymptotic growth rates of productivity are the same) will countries converge. Alternatively, if $\lambda = 0$, productivity levels will grow at different rates permanently and show no tendency to converge asymptotically.[9] We can estimate equation (4) directly. If $\lambda > 0$, then the difference between the technology levels in the two countries will be stationary. If there is no catch-up ($\lambda = 0$), then the difference of TFP in country i from that in country 1 will contain a unit root. The drift term $\gamma_i - \gamma_1$ will typically be small but nonzero if the countries' technologies are driven by different processes (i.e., under the hypothesis of no convergence). Under the hypothesis of convergence, $\gamma_i = \gamma_1$. In section V, we will focus on the null of no convergence where the expected growth rates of TFP are not equal across countries and shocks to relative productivity are

permanent. The alternative will be that of convergence where relative productivity shocks are transitory and countries have the same long-run expected growth rates of TFP.

III. Cross-section Evidence

A. Data

The empirical work for this paper employs data on total factor productivity for (a maximum of) fourteen OECD countries and six sectors over the period 1970 to 1987. The fourteen countries are Australia, Belgium, Canada, Denmark, Finland, France, Italy, Japan, Netherlands, Norway, Sweden, United Kingdom, United States, and West Germany. The six sectors are Agriculture, Mining, Manufacturing, Utilities (electricity/gas/water), Construction, and Services. The basic data source is an updated version of the OECD Intersectoral Database (ISDB), constructed by Meyer-zu-Schlochtern (1988).[10]

For each country i, sector j, and year t, we construct a measure of the log of total factor productivity (TFP), designated $\ln A_{ij}(t)$. This measure is constructed in the standard way, as a weighted average of capital and labor productivity, where the weights are the factor shares calculated assuming perfect competition and constant returns to scale. Details can be found in Meyer-zu-Schlochtern (1988). Our construction differs from his only in that our labor share is sec-

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[9] Of course, if the country with the lower initial level has a higher γ_i, the countries may appear to be converging in small samples—this case is extremely difficult to rule out in practice.

[10] With the exception of the services aggregate, all the other sectors are taken directly from the ISDB. The services aggregate is constructed by summing Retail Trade, Transportation/Communication, F.I.R.E., and Other Services. Government Services are excluded.

FIGURE 3.—LOG OF TOTAL FACTOR PRODUCTIVITY BY INDUSTRY
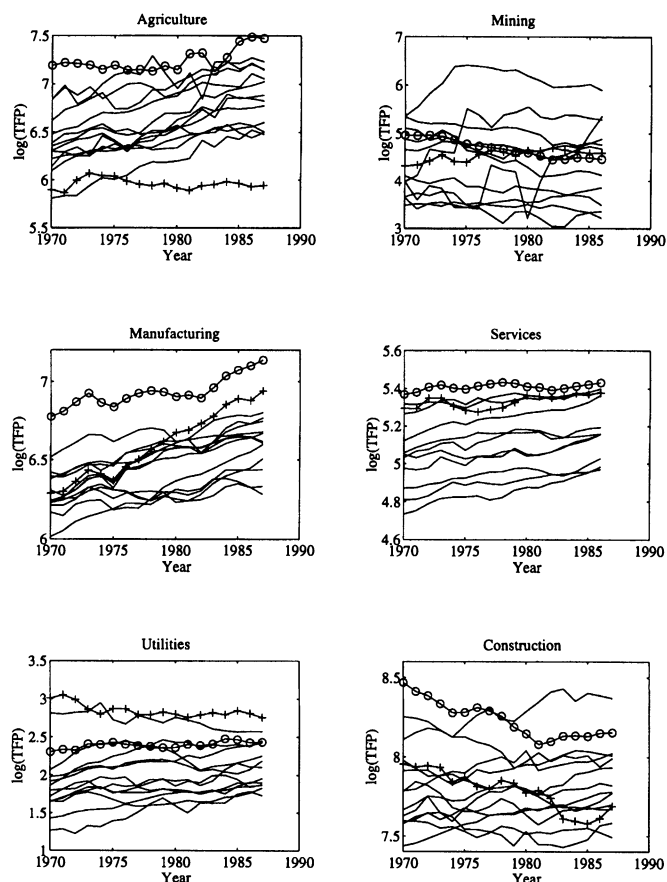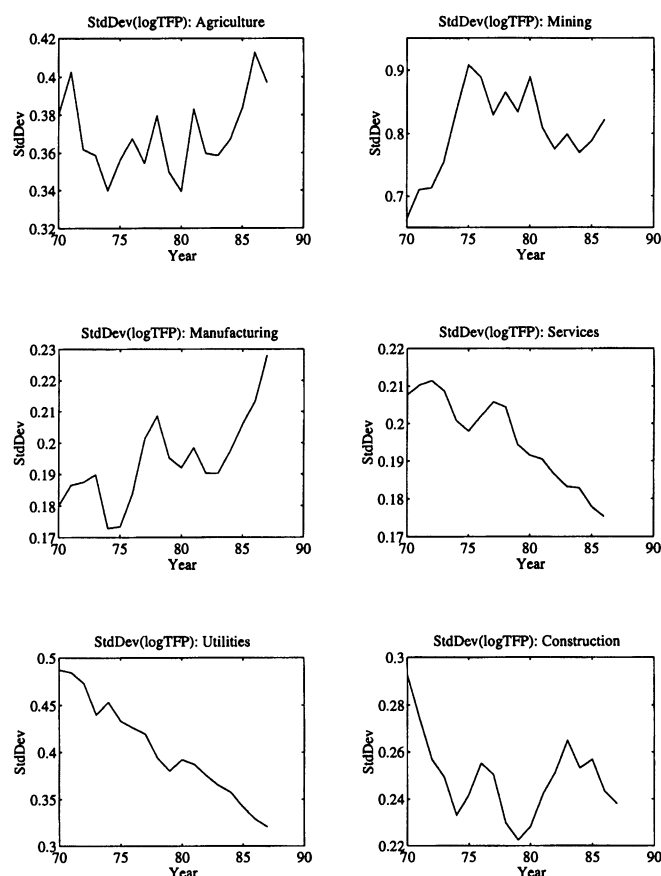(U.S. IS LABELLED 'o', JAPAN IS LABELLED '+')

FIGURE 4.—STANDARD DEVIATION OF LOG (TOTAL FACTOR PRODUCTIVITY) BY
INDUSTRY

tor-specific, i.e., it is calculated as an average over time and country for each sector.[11]

To summarize the data, table 1 reports average annual TFP growth rates by country and sector for the period 1970 to 1987.[12] Similarly, figure 3 plots the log of TFP by sector for each country.

B. Industry TFP

Looking at the TFP levels by sector in figure 3, we can see several immediate differences from the aggregate movements shown earlier. Sectors do not show the same patterns in either trend or dispersion over time, and countries do not perform similarly across sectors. Manufacturing TFP grows on average at 1.9% per year, and there is little change in the overall cross-section dispersion. Within manufacturing there are substantial differences—Japan grows at 4.0% per year

[11] The labor data are potentially problematic since they consider workers, not hours, and are unadjusted for differences in human capital. Direct evidence on hours and schooling by sector is unavailable for these countries over this period. Unpublished data from the ILO for recent years suggest that hours do not vary systematically in a way to overturn the results found here.

[12] For a few sectors, 1986 is taken as the endpoint because of data availability.

and Norway at only 0.3%. On the other hand, utilities has no trend and there is substantial narrowing of the large initial gap between Japan and Norway during the sample.
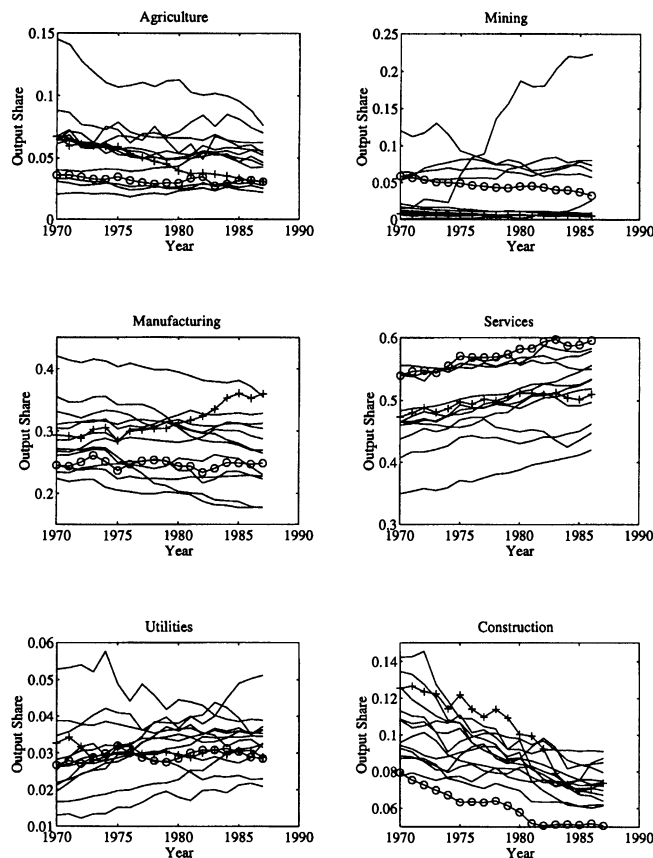
In addition to varying growth rates, we can clearly see large differences in TFP levels across sectors. Manufacturing, construction, and agriculture have substantially higher TFP levels than do services, mining, and utilities. The consequences for convergence are immediate: if the United States is moving out of manufacturing and into services at a faster rate than other countries, then overall U.S. TFP growth will be slower. This pattern will hold even if manufacturing TFP is highest in the United States and that lead does not diminish over time.

The different sectoral contributions to aggregate TFP movements can be seen more clearly in figure 4, which plots the cross-country sectoral standard deviations of TFP against time. Services and utilities show substantial evidence of catch-up, while at the other extreme, manufacturing has an overall increase in cross-country dispersion. Evidence on the other sectors is less clear-cut; construction falls initially and then steadies, mining rises dramatically and then falls back somewhat, while agriculture changes within years but shows little net change. These results do not change if the United

FIGURE 5.—INDUSTRY SHARES IN TOTAL OUTPUT
(U.S. IS LABELLED 'o', JAPAN IS LABELLED '+')

ing share and manufacturing productivity both rise strongly. Services is the only sector to show substantial share growth for most countries, accounting for at least 49% and as much as 64% of total industry output in 1987.

While services is growing as a share of output and manufacturing is declining in most countries, there remain substantial differences in sectoral shares across countries. In particular, there is little tendency for shares to become more similar. Also, since all sectors except manufacturing show convergence in productivity levels and the share of manufacturing is generally declining, the convergence of total industry productivity is less surprising.

A. Sources of TFP Growth

Rather than simply considering the relative growth or decline of industries in total output, we would like a measure of the contributions of industry TFP growth and industry share to aggregate convergence. We perform a decomposition of aggregate growth rates into within-sector and between-sector movements. We begin by decomposing each country's aggregate TFP growth over the period into sectoral contributions. Aggregate output can be written as the sum of sectoral outputs

$$Y = \sum_i Y_i,$$

and aggregate TFP is measured as

$$TFP = \frac{Y}{L^\alpha K^{1-\alpha}}.$$

States is removed from the sample. In fact, the increase in manufacturing TFP dispersion is augmented.

The visual evidence on sectoral differences in TFP growth is dramatic. Sectors differ within and across countries. In particular, manufacturing appears to be leading to divergence in TFP and the aggregate levels converge only because of the substantial narrowing in other sectors, such as services. These results suggest that productivity differences within manufacturing might be permanent rather than transitory and that the time series framework presented in section II is an appropriate methodology.

IV. Changing Sector Shares and Productivity Growth

To reconcile aggregate convergence with the apparent lack of convergence in manufacturing, we examine movements in sectoral shares in total private output. Figure 5 plots the share of total private output for each of the six sectors. The movements over time vary substantially over sectors. Construction, mining and manufacturing show flat or falling shares for most countries, while services generally gains output share. Mining and utilities are basically constant with large exceptions for inaividual countries. The share of manufacturing is flat or declining in almost every country. A notable exception is Japan where the manufactur-

Sectoral TFP is calculated analogously. We can write aggregate TFP as the weighted sum of sectoral TFPs where the weights are ratios of an index of inputs in the sector to an aggregate index of inputs:

$$TFP = \sum_i \frac{Y_i}{L_i^\alpha K_i^{1-\alpha}} \cdot \left(\frac{L_i}{L}\right)^\alpha \left(\frac{K_i}{K}\right)^{1-\alpha} = \sum_i TFP_i \cdot w_i. \quad (5)$$

If we use labor shares as weights, sectoral TFP will not correctly aggregate to total industry TFP, and we systematically underemphasize industries with high capital shares. If we use output shares, we are off by exactly TFP_i / TFP, which is a part of what we are hoping to analyze.

Using this accounting framework, we can decompose the growth of aggregate TFP in each country into between- and within-sector components as follows:

$$\Delta TFP = \sum_j \Delta TFP_j \cdot \overline{\omega_j} + \sum_j \Delta \omega_j \cdot \overline{TFP}_j. \quad (6)$$

Rewriting this decomposition in terms of percentage changes we get

$$\%\Delta TFP = \sum_j \underbrace{\%\Delta\left(TFP_j\right)\left(\frac{TFP_{j,0}}{TFP_0}\right)\left(\overline{\omega_j}\right)}_{\text{Productivity Growth Effect (PGE)}} + \sum_j \underbrace{\Delta\left(\omega_j\right)\frac{\left(\overline{TFP}_j\right)}{TFP_0}}_{\text{Share Effect (SE)}}. \quad (7)$$

TABLE 2.—SOURCES OF AGGREGATE TFP GROWTH

	Average—All countries[a]		
	Growth Effect	Share Effect	Sectoral Total
Agriculture	8%	–8%	–1%
Mining	0%	–1%	–1%
Manufacturing	47%	–23%	25%
Services	38%	40%	78%
Utilities	6%	1%	6%
Construction	–3%	–4%	–7%
Effect Total	96%	4%	100%

[a]Countries included are Belgium, Denmark, France, Japan, Finland, Sweden, the United Kingdom, and the United States. Numbers are averages across countries of contributions of sector–effects (productivity growth (within) and share (between)) to total industry productivity growth rates (or growth rate differentials from the United States). Each country's total industry TFP growth rate is decomposed into

$$\frac{\Delta TFP}{TFP_{t-1}} = \sum_j \Delta(TFP_j)\left(\frac{TFP_{j,t-1}}{TFP_{t-1}}\right)(\bar{\omega}_j) + \sum_j \Delta(\omega_j)\frac{(\overline{TFP_j})}{TFP_{t-1}},$$

where j indexes sectors. The difference between total industry TFP growth in the United States and that in any other country is in turn decomposed into sectoral productivity growth and share effects.

$$\sum_j [PGE_{other} - PGE_{US}] + \sum_j [SE_{other} - SE_{US}].$$

Numbers may not sum to 100% due to rounding.

The first component, which we call the productivity growth effect, captures the contribution of within-sector TFP growth to the aggregate for the country, using the average sectoral weights for the period in question. Sectors with faster TFP growth will have larger contributions. The second term, the share effect, shows the contribution of changing sectoral composition on aggregate TFP growth, where the share changes are weighted by average relative TFP for the sector over the period. Sectors that are declining as a fraction of aggregate output will have negative share effects. The average contributions of sectoral productivity growth and share effects to aggregate TFP growth within countries are given in table 2.[13] Within-sector effects dominate share effects for this sample of countries, accounting for more than 95% of total TFP growth. On average, for these countries, the net effect of changing shares is minimal.

Measuring the importance of individual sectors is less straightforward. The total sectoral effect certainly overstates the importance of growing sectors and underestimates that of declining sectors. One alternative measure is the change in overall TFP growth that would have resulted if a sector had maintained its average level of TFP throughout the period but still gained or lost share. This is equivalent to subtracting the growth effect for an industry from the total. Using this measure, manufacturing is the most important sector, accounting for 47% of total growth, with services a close second at 38%. While services have share growth and

[13] Since these calculations require the presence of data for every sector in every country and since we require summing up of inputs and outputs for each sector, we can only perform them on a subset of the countries in our sample. Countries included are Belgium, Denmark, Finland, France, Japan, Sweden, the United Kingdom and the United States.

thus compensate for the declining share in other sectors, the net effect of changing composition is small.[14]

These averages disguise a large amount of heterogeneity across countries. The United States actually suffers a TFP decline due to share effects as growth in the service sector does not compensate for the declining importance of manufacturing. Considering our alternative measure, if TFP levels in U.S. manufacturing had held constant at their average, the United States would have experienced no overall TFP increase. In France, however, TFP growth within manufacturing represents only 25% of total growth by the same measure.

B. Sources of Catch-up

Since we are also interested in the importance of changing sectoral shares on aggregate catch-up to the United States, we construct a measure of productivity growth for each country relative to the United States.

$$\%\Delta TFP_{other} - \%\Delta TFP_{US} \tag{8}$$

and decompose this into productivity growth and share effects.

$$\sum_j \underbrace{\left[PGE_{other} - PGE_{US}\right]}_{Productivity\,Growth\,Effect} + \sum_j \underbrace{\left[SE_{other} - SE_{US}\right]}_{Share\,Effect}. \tag{9}$$

To understand the relative importance of the two components to overall catch-up, we construct our measures of within- and between-sector catch-up by subtracting the productivity growth and share effects in the United States from those in the other countries for each sector. Since the United States is the most productive country in aggregate in 1970, this means that a positive number is a contribution to catch-up to the United States and a negative number contributes to divergence in aggregate TFP. These numbers are the percentage contributions to aggregate catch-up for each country. Averaging across countries, we compute the average contribution of each "sector-effect" to convergence to U.S. TFP.

Table 3 presents results on the role of sectors and effects in catch-up to the United States. The results largely confirm the within-country analyses. The bulk of convergence is driven by superior within-sector TFP growth rates for countries relative to the United States. However, fully 17% of aggregate convergence is contributed by shifting sectoral shares.

The single most important factor in aggregate convergence is the productivity growth effect for services, contributing more than a third of aggregate convergence in the sample. In other words, without growth in service sector productivity, average catch-up to the United States would have been 34% less. The same number for manufacturing is much smaller, reflecting the good performance of U.S. manufacturing productivity relative to other countries. In

[14] Dropping individual countries did not produce any significant differences in the results.

TABLE 3.—SOURCES OF AGGREGATE CONVERGENCE

	Average—All countries[a]		
	Growth Effect	Share Effect	Sectoral Total
Agriculture	6%	–5%	1%
Mining	12%	4%	16%
Manufacturing	17%	4%	20%
Services	34%	16%	49%
Utilities	5%	2%	7%
Construction	8%	–3%	5%
Effect Total	82%	17%	100%

[a] See notes to table 2.

fact, excluding Japan from the sample the manufacturing contribution falls to a mere 9%. In addition, 5 of the 6 countries not in the sample for data reasons have lower manufacturing TFP growth rates than the United States, suggesting that manufacturing's contribution to aggregate convergence is overstated by this result. Among other sectors, only mining shows up as important, partly because of declining productivity in mining in the United States and partly because of the inclusion of two North Sea countries in the sample.

The results provide direct evidence on the relative importance of changing sectoral composition and industry TFP growth on aggregate convergence. Cross-section evidence on sectors shows little support for convergence in manufacturing and substantial convergence in services. Trends in output shares also differ markedly across sectors, with manufacturing generally declining and services rising as a fraction of total output. Examining the contribution of these two trends from 1970–87, we find that within-sector TFP catch-up, especially in services, provides the largest fraction of aggregate TFP convergence. Share changes play a smaller, yet significant role in the catch-up of other countries to the United Staes.

V. Time Series Evidence

This section uses time series evidence to examine the results found in the previous sections in more detail. First, we extend a recent advance in unit root econometrics by Levin and Lin (1992), and then we apply this technique to test for convergence within sectors across countries.

A. Testing for Unit Roots in Panel Data: Theory

The sectoral data we employ is available for a relatively short time horizon of eighteen years for most countries, 1970–1987. With such a limited sample of years, unit root testing would appear to be out of the question. However, a recent paper by Levin and Lin (1992) illustrates the relatively straightforward technique of testing for unit roots in

panel data. Their basic findings are twofold: (1) that as both N and T go to infinity, the limiting distribution of the unit root estimator is centered and normal,[15] and (2) that the panel setting permits relatively large power improvements.

We consider the following general model:

$$y_{it} = \mu_i + \rho y_{it-1} + \epsilon_{it} \qquad (10)$$

where the $\epsilon_{it} \sim$iid $(0, \sigma_\epsilon^2)$ and $\mu_i \sim$iid $(\bar{\mu}, \sigma_\mu^2)$. We also assume ϵ_{it} has $2 + \delta$ moments for some $\delta > 0$ and that $E\mu_i\epsilon_{it} = 0$ for all i and t. Other standard regularity conditions are assumed to hold.

Let $\hat{\rho}$ and t_ρ be the OLS parameter estimate and t-statistic from a regression of y_{it} on y_{it-1} including country-specific intercepts. Levin and Lin prove the following lemma:

LEMMA 1 (LEVIN-LIN). *When $\bar{\mu} = 0$ and $\sigma_\mu^2 = 0$ (i.e. the unit root processes have no drift), if N and T go to infinity with $\sqrt{N}/T$ going to zero,*

$$T\sqrt{N}\left(\hat{\rho} - \left(1 - \frac{3}{T}\right)\right) \Rightarrow N(0, 10.2)$$

$$\sqrt{1.25}t_\rho + \sqrt{1.875N} \Rightarrow N(0,1).$$

Furthermore, this result holds when a common time trend is included in the regression.

The Levin and Lin (1992) result provides asymptotic normality for the panel unit root tests in some common settings. One setting not considered in that paper is the case in which the data generating process is a unit root with nonzero drifts but time trends are omitted from the regression specification. West (1988) shows that asymptotic normality obtains for the $N=1$ case in this setting.[16] The following proposition extends West's finding to the panel setting.

PROPOSITION 1. *Consider the regression model in equation (10). Under the null hypothesis of a unit root with nonzero drifts ($\sigma_\mu^2 \neq 0$),*

$$\sqrt{N}T^{3/2}(\hat{\rho} - 1) \Rightarrow N\left(0, 12\frac{\sigma_\epsilon^2}{(\sigma_\mu^2 + \bar{\mu}^2)}\right)$$

$$t_\rho \Rightarrow N(0,1). \qquad (11)$$

Proof: See the appendix.

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[15] Quah (1990) first noted this asymptotic normality result using a random fields data structure and rejected convergence of per capita output for a large cross-section of countries over 1960–1985. His estimator does not permit country-specific intercepts.

[16] Park and Phillips (1988) generalize this result and show that asymptotic normality depends critically on the presence of only a single, nonstationary regressor.

TABLE 4.—TIME SERIES TESTS FOR CONVERGENCE

Sector	Deviations from Most Productive Country in 1970					Deviations from Median Productive Country in 1970		
	(1)		(2)			(3)		
	Trend	t-statistic	ρ Biased	t-statistic	ρ Adjusted	ρ Biased	t-statistic	ρ Adjusted
Agriculture	−.0119	−7.54	.633	−7.60[b]	0.787	.722	−5.60	0.871
Mining	.0159	2.84	.739	−6.98[b]	0.863	.812	−4.06	0.923
Manufacturing	−.0033	−3.58	.897	−2.99	1.018	.931	−3.37	0.989
Utilities	−.0226	−14.54	.806	−6.24[b]	0.879	.777	−4.73	0.904
Construction	−.0207	−18.59	.859	−6.84[b]	0.912	.741	−7.65[b]	0.866
Services	−.0059	−16.63	.922	−2.08	1.007	.717	−6.33[b]	0.850
Total Industry	−.0097	−25.35	.915	−4.04[a]	0.970	.821	−5.30[a]	0.957

Notes: Column (1) reports estimates of the average trend in the panel. Columns (2)–(3) report panel unit root tests based on proposition 1 in the paper, i.e., excluding time trends from the specification. All regressions include country-specific intercepts. Lag length was chosen according to the Schwarz information criterion. The bias-adjusted estimate of ρ and the critical values for the t-statistics in columns (2) and (3) are taken from a Monte Carlo simulation with 2500 iterations. For the Monte Carlo simulation for each sector, TFP levels were differenced and then the means and the standard deviations of these first differences were used to generate the data for the appropriate sample size, typically $N=14$ and $T=18$.
[a] Significant at the 10% level.
[b] Significant at the 5% level.

This case differs substantially from that of Levin and Lin (1992). The asymptotic normality of $\hat{\rho}$ occurs as T goes to infinity because the results are driven by the time trends in y_{it}; in contrast, the normality in the Levin-Lin proof is driven by the averaging across N non-normal distributions.

B. Evidence

To examine the convergence hypothesis while taking advantage of the time series aspect of the data, we focus on cross country deviations in TFP levels. Letting country 1 denote the benchmark country, our tests will be based on

$$\ln \hat{A}_{ij}(t) \equiv \ln A_{1j}(t) - \ln A_{ij}(t), \quad i = 2,...,N..$$

Following Bernard and Durlauf (1995), we will say that country i is converging to country 1 if $\ln \hat{A}_{ij}(t)$ is stationary. We do not necessarily require $\ln A_{ij}(t)$ to exhibit a unit root with drift, although pretesting indicated that this null hypothesis could generally not be rejected.[17]

More specifically, the null and alternative hypotheses that we have in mind can be interpreted in the context of the simple model presented earlier. The null hypothesis is $H_0:\lambda=0$ and $\gamma_{ij} \neq \gamma_{1j}$. That is, we are testing the null hypothesis of no convergence, which is defined to mean that the deviation in productivity levels from a benchmark country is a nonstationary process with nonzero drift. The hypothesis has substantial economic content in that, consistent with the "new growth theory," long-run average productivity growth rates of countries that are not converging will be different.

[17] When no time trends are included in the test for a unit root in the panel of TFP levels, the test fails to reject for every sectoral group. With a single common trend, the tests only reject the unit root null for the agricultural sector, but this sector exhibits a significant positive trend.

We include lags of the differenced productivity deviation to control for serial correlation, where the lag length is chosen using the Schwarz information criterion.

The relevant alternative hypothesis is that productivity levels are converging in the sense that deviations in productivity levels across countries are stationary processes. Notice that neither the null hypothesis nor the alternative hypothesis involves a time trend in the specification. In contrast to many unit root tests, we are not testing the null of a difference stationary model against an alternative trend stationary model. Rather, we are testing a nonstationary null against a stationary alternative. The intent is to focus on the relevant aspect of the null hypothesis (i.e., countries that are not converging do not have the same underlying average growth rates), so we may take advantage of a more powerful test.

The disadvantage of the short time horizon in the ISDB data is that we cannot examine the hypothesis that only a subset of the fourteen countries is converging. That is, the panel test focuses on the extremes: we test the null hypothesis that all fourteen countries are converging against the alternative that as a group they are not converging. With the difficulty of constructing longer time series for TFP, we are unlikely to be able to test convergence in smaller groups of countries.

A related issue is how to choose the benchmark country. Asymptotically, of course, this choice should not matter, but in small samples it will be important. We report results when country 1 is chosen in two different ways: as the most productive country at the beginning of the sample, 1970, and as the median country in terms of productivity in 1970. Choosing country 1 as the most productive has the advantage that we can construct a rough test of convergence in terms of the in-sample common trend: if productivity deviations from the most productive country exhibit a positive trend in sample, this would constitute strong evidence against the convergence hypothesis.

The results of our time series tests for convergence are reported in table 4. The first result of note pertains to the simple average trend in the productivity deviations from the most productive country. For total industry and for all sec-

tors except mining, the average trends are negative. The presence of these trends constitutes evidence in favor of the convergence hypothesis for all sectors except mining. Interestingly, however, the trend in the manufacturing sector is the smallest for the six sectors and the least significant of the negative trends. Furthermore, when Japan is excluded from the sample the trend is only slightly negative and is insignificantly different from zero.

Column (2) of table 4 reports the results of the panel unit root tests when no time trends are included in the specification, as in proposition 1. Two estimates of ρ are reported. The first estimate is the unadjusted point estimate from the unit root test. According to proposition 1, this point estimate is asymptotically unbiased under the null hypothesis. However, in practice the point estimates suffer from the small sample Hurwicz-Nickell bias (see Nickell (1981)). In addition, the "speed" with which the asymptotic normality takes over is a function of the magnitude of the drift relative to the variance of the innovation to the unit root process. If the drift is small, the limiting normal distribution may prove to be a poor approximation to the relevant small-sample distribution. In this case, the point estimates and t-statistics will be biased downward (for example, as in the Levin and Lin results).

Because of these two sources of bias, we also report adjusted estimates of ρ and calculate the critical values for the t-statistic from Monte Carlo simulations. The adjusted estimate of ρ is equal to the raw point estimate plus a bias correction term, producing an estimate of ρ that is median-unbiased in the sense of Andrews (1993) under the null hypothesis of a unit root.

This bias adjustment and the associated critical values for the t-statistic are calculated by Monte Carlo simulation as follows. First, for each productivity deviation (e.g., the deviation from the most productive country), we difference the data and estimate the mean and variance for each country. In the notation above, these estimates correspond to μ_i and σ_i^2 (where we allow the variance of growth rates to differ across countries in the Monte Carlo simulations). Next, we generate an $(N-1) \times T$ panel of simulated productivity deviations by randomly drawing from an $N(\mu_i, \sigma_i^2)$ for each country and summing the draws. Third, we run the panel unit root test on this simulated data. Finally, we repeat this experiment 2500 times and then calculate the median bias in the estimate of ρ across our 2500 runs for each sector. This median bias is used as our bias adjustment. Similarly, the critical values for the t-statistic are obtained from these simulations.

Column (2) in table 4 reports the raw and adjusted point estimates of ρ together with the t-test results testing $\rho = 1$. The bias-adjusted point estimates for agriculture, mining, utilities, construction, and total industry are all significantly less than unity, providing evidence against the unit root null in these sectors. In contrast, the results for manufacturing and services fail to reject the null.

Column (3) reports results when the productivity deviations are taken from the country with the median (rather than maximum) level of productivity in 1970. The results highlight the difference between the manufacturing sector and the other sectors, as manufacturing is the only sector with a bias-adjusted point estimate that is both close to unity (0.989) and insignificantly different from one. The adjusted point estimate for total industry is the next highest at 0.954, but the estimate is significantly different from one at the 10% significance level. In terms of the model presented earlier, this estimate implies a speed of convergence of 4.6%, somewhat higher than that found by other authors for labor productivity.[18]

As for other sectors, the point estimate for services is now significantly less than unity, in contrast to the results in column (2). Construction also has a root significantly less than one. Finally, agriculture, mining, and utilities have point estimates that are low (all below 0.92), but are not significantly different from unity because of relatively imprecise estimates.

Overall, these panel/time series results are generally supportive of the results documented using the cross-sectional data. Evidence in favor of convergence appears to be strongest in the non-manufacturing sectors and weakest in the manufacturing sector.

VI. Conclusions and Future Research

In this paper, we have considered the role of sectors in aggregate productivity movements. During the post-WWII period, OECD countries have converged in output, labor productivity and total factor productivity. For our sample of 14 countries from 1970–87, we also find strong evidence for aggregate convergence in TFP with both decreasing cross-country variance and catch-up by lower initial productivity countries.

In contrast, individual sectors display a wide variety of productivity paths. Most striking is the lack of evidence for convergence in the manufacturing sector. Plots of time paths of sectoral productivity show catch-up in services and little or none in manufacturing. We extend existing results on asymptotic normality in panel unit root estimators in panel data and find additional evidence for differences across sectors. Manufacturing again shows the least evidence for convergence, and utilities the most.

To distinguish between the role of changing sectoral shares and widely differing sectoral productivity paths, we decompose total industry TFP growth into within-sector and between-sector components. This decomposition reveals that, on average, total industry TFP growth for a country is dominated by within-sector productivity movements. Shifting sectoral shares play a minor role. Considering individual sectors,

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[18] For example, Barro and Sala-i-Martin (1992) and Mankiw, Romer, and Weil (1992) report convergence rates of about 2% per year for labor productivity.

manufacturing and services are the most important, accounting for three quarters of overall TFP growth during the period.

However, to understand the role of sectors in aggregate convergence, we also decompose TFP convergence into growth and share components. The contribution of changing sectoral shares is almost one-fifth of total catch-up. Services is by far the most significant sector accounting for one-third of aggregate convergence during the period. While there is substantial heterogeneity across countries, the importance of services and the contribution of changing shares are robust to country inclusion.

The surprising results presented here raise several questions. If technological convergence occurs because of transfers from advanced to less advanced countries within a sector, then we would expect to find catch-up in all sectors. In particular, if international trade enhanced the transfer of technology, then manufacturing would be the sector most likely to display convergence. Alternatively, if productivity growth is industry-specific and trade induces specialization by countries, then we might find no convergence, or divergence, in trade-intensive sectors such as manufacturing, and convergence in sectors common to all countries, such as services. In addition, the role of sectors and changing shares in aggregate productivity growth may be more important for countries undergoing rapid structural transformations at lower levels of development. Future research should address these issues and work on assembling data on inputs and output for disaggregated sectors across countries.

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APPENDIX

Proof of Proposition 1

The OLS estimate of $\hat{\rho}$ can be written as

$$\hat{\rho} = \frac{\sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it} - \bar{y}_i)(y_{it-1} - \bar{y}_{i-1})}{\sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it-1} - \bar{y}_{i-1})^2}, \quad (A.1)$$

where $\bar{y}_i$ and $\bar{y}_{i-1}$ denote the mean of $\bar{y}_{it}$ and $\bar{y}_{it-1}$, respectively. Substituting for y_{it} under the null hypothesis and normalizing appropriately, reveals

$$\sqrt{N}T^{3/2}(\hat{\rho}-1) = \frac{\frac{1}{\sqrt{N}} T^{-3/2} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it-1} - \bar{y}_{i-1})\left(\epsilon_{it} - \frac{1}{T}\sum_{t=1}^{T} \epsilon_{it}\right)}{\frac{1}{N} T^{-3} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it-1} - \bar{y}_{i-1})^2}. \quad (A.2)$$

Now, consider the numerator and denominator separately. First, some algebra reveals that

$$y_{it-1} - \bar{y}_{i-1} = \mu_i\left(t - \frac{T+1}{2}\right) + \left(\tilde{y}_{it-1} - T^{-1}\sum_{t=1}^{T} \tilde{y}_{it-1}\right), \quad (A.3)$$

where a tilde is used to denote a variable that is $I(1)$ with zero drift. Define

$$\xi_i = T^{-3/2} \sum_{t=1}^{T} (y_{it-1} - \bar{y}_{i-1})\left(\epsilon_{it} - \frac{1}{T}\sum_{t=1}^{T} \epsilon_{it}\right). \quad (A.4)$$

Arguments such as those below reveal that the terms involving the expression $1/T \sum_{t=1}^{T} \epsilon_{it}$, are $o_p(1)$, so that

$$\xi_i = T^{-3/2} \sum_{t=1}^{T} \mu_i t \epsilon_{it} - \frac{1}{2} \mu_i T^{-1/2} \sum_{t=1}^{T} \epsilon_{it}$$
$$- \frac{1}{2} \mu_i T^{-3/2} \sum_{t=1}^{T} \epsilon_{it}$$
$$+ T^{-1/2}\left(T^{-1}\sum_{t=1}^{T} \tilde{y}_{it-1} \epsilon_{it} - T^{-1}\left(T^{-1}\sum_{t=1}^{T} \tilde{y}_{it-1}\right)\left(\sum_{t=1}^{T} \epsilon_{it}\right)\right) + o_p(1). \quad (A.5)$$

The second to last term of this equation is $T^{-1/2}$ multiplied by a term that is asymptotically the integral of a demeaned Brownian motion, so that term is $o_p(1)$. Also, the third term of the equation involving $T^{-3/2} \sum_{t=1}^{T} \epsilon_{it}$ is obviously $o_p(1)$. Then the results of West (1988) begin to apply:

$$\xi_i = T^{-3/2} \sum_{t=1}^{T} \mu_i t \epsilon_{it} - \frac{1}{2} \mu_i T^{-1/2} \sum_{t=1}^{T} \epsilon_{it} + o_p(1)$$

$$= \mu_i \left(T^{-1} \sum_{t=1}^{T} \left(\frac{t}{T} \right) \left(T^{1/2} \epsilon_{it} \right) - \frac{1}{2T} \sum_{t=1}^{T} T^{1/2} \epsilon_{it} \right) + o_p(1)$$

$$\Rightarrow \mu_i \left(\int_0^1 s\, dW(s) - \frac{1}{2} \int_0^1 dW(s) \right) \tag{A.6}$$

$$= \mu_i \int_0^1 \left(s - \frac{1}{2} \right) dW(s)$$

$$= N\left(0, \frac{1}{12} \mu_i^2 \sigma_\epsilon^2 \right),$$

where σ_ϵ^2 is the variance of ϵ_{it}. Assuming that the μ_i are distributed i.i.d. with mean $\bar{\mu}$ and variance σ_μ^2, the numerator behaves asymptotically like

$$\xi = \frac{1}{\sqrt{N}} \sum_{i=1}^{N} \xi_i \tag{A.7}$$

$$= N\left(0, \frac{1}{12} (\sigma_\mu^2 + \bar{\mu}^2) \sigma_\epsilon^2 \right).$$

Now consider the denominator of equation (A.2):

$$\frac{1}{N} T^{-3} \sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it-1} - \bar{y}_{i-1})^2 = \frac{1}{N} T^{-3} \left(\sum_{i=1}^{N} \sum_{t=1}^{T} (y_{it-1}^2 - T \sum_{i=1}^{N} \bar{y}_{i-1}^2 \right) \tag{A.8}$$

which is naturally thought of as two terms. Once again, the results in West (1988) apply, so that the first term will behave asymptotically like a time trend:

$$T^{-3} \sum_{t=1}^{T} y_{it-1}^2 = \sum_{t=1}^{T} (\mu_i(t-1) + \bar{y}_{it-1})^2$$

$$= T^{-3} \sum_{t=1}^{T} \mu_i^2 (t-1)^2 + 2\mu_i T^{-3} \sum_{t=1}^{T} (t-1) \bar{y}_{it-1} + T^{-3} \sum_{t=1}^{T} \bar{y}_{it-1}^2$$

$$= \mu_i^2 T^{-3} \sum_{t=1}^{T} t^2 + o_p \tag{A.9}$$

$$\Rightarrow \frac{1}{3} \mu_i^2.$$

Similarly, the second term is (ignoring the summation over N for the moment)

$$T^{-3} T \bar{y}_i^2 = T^{-2} \bar{y}_{i-1}^2 = \left(T^{-2} \right) \left(\frac{1}{T} \sum_{t=1}^{T} y_{it-1} \right)^2$$

$$= \left(T^{-2} \sum_{t=1}^{T} y_{it-1} \right)^2$$

$$= \left(T^{-2} \sum_{t=1}^{T} \mu_i(t-1) + \bar{y}_{it-1} \right)^2 \tag{A.10}$$

$$\Rightarrow \frac{1}{4} \mu_i^2.$$

By putting these two terms together, and summing across i the denominator becomes

$$Denom \Rightarrow \frac{1}{12} E\mu_i^2 = \frac{1}{12} \left(\sigma_\mu^2 + \bar{\mu}^2 \right). \tag{A.11}$$

Finally, combine the results for the numerator and denominator, and the convergence of the unit root estimator is proven:

$$\sqrt{N} T^{3/2} (\hat{\rho} - 1) \rightarrow \frac{N\left(0, \frac{1}{12} \left(\sigma_\mu^2 + \bar{\mu}^2 \right) \sigma_\epsilon^2 \right)}{\frac{1}{12} \left(\sigma_\mu^2 + \bar{\mu}^2 \right)}$$

$$= N\left(0, 12 \left(\sigma_\mu^2 + \bar{\mu}^2 \right)^{-1} \sigma_\epsilon^2 \right). \tag{A.12}$$

The convergence of the t-statistic t_ρ to an $N(0,1)$ random variable follows immediately from calculations we have already done once one notes that

$$t_\rho = \frac{\hat{\rho} - 1}{\hat{\sigma}_\epsilon} \sqrt{\sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{t-1} - \bar{y}_{t-1} \right)^2}$$

$$= \frac{1}{\hat{\sigma}_\epsilon} \sqrt{N} T^{3/2} (\hat{\rho} - 1) \sqrt{\frac{1}{N} T^{-3} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{t-1} - \bar{y}_{t-1} \right)^2}. \tag{A.13}$$

Q.E.D.