

MULTIPRODUCT FIRMS AND TRADE LIBERALIZATION*

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This article develops a general equilibrium model of multiple-product, multiple-destination firms, which allows for heterogeneity in ability across firms and in product attributes within firms. Firms make endogenous entry and exit decisions and each surviving firm chooses optimally the range of products to supply to each market. We show that the resulting selection, across and within firms, provides a natural explanation for a number of features of trade across firms, products, and countries. Using both time-series changes in trade policy and cross-section variation in trade, we provide empirical evidence in support of the predictions of the model. *JEL* Codes: F12, F13, L1.

I. INTRODUCTION

While trade is dominated by firms that export more than one product to more than one destination, comparatively little research examines their production and export decisions or how these choices are affected by globalization. This article develops a general equilibrium model of multiple-product, multiple-destination firms in which the ability to produce a particular product depends on both firm and product attributes. Based on these attributes, firms choose whether to serve export destinations and which products to supply to those export destinations. As a result, the model features selection both across firms within an industry and across products within firms. We use the model to guide our empirical analysis and find support for many of its implications in U.S. transactions-level trade data.

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Modelling multiple-product firms is useful for several reasons. First, we are able to account for a number of features of disaggregated trade data that standard models do not consider. These features include the skewness in export sales across products within firms and a positive relationship between the number of products exported, the number of destinations served, and sales of a particular product to a given destination. Second, allowing firms to choose the number of products to export and destinations to serve permits product and destination composition to influence firm characteristics. In our model, for example, declining trade costs raise firm productivity by causing firms to drop their least attractive products. Finally, expanding the range of activities firms may undertake sheds light on the mechanisms driving aggregate economic relationships. We show, for example, that the well-known negative relationship between aggregate trade and distance is driven entirely by firm and product entry.

Our approach is a natural generalization of Melitz's (2003) single-product, heterogeneous-firm model of trade in horizontally differentiated products. To enter, firms incur a sunk entry cost, which reveals their profitability. Firms then choose among a continuum of products and many export markets. Firm profitability depends on the interaction of a firm attribute, "ability," and product attributes, which are idiosyncratic across products and possibly also across export destinations within the firm. Though we model "ability" as firm productivity and product attributes as "consumer taste" for the firm's products, they can be interpreted more broadly. Indeed, under our assumptions of constant elasticity of substitution (CES) preferences and monopolistic competition, both productivity and consumer tastes enter equilibrium firm revenue in exactly the same way. All that matters for our results is that there are firm- and product-specific components of profitability, where the firm component generates selection across firms and the product component generates selection within firms.

Firms face fixed costs in serving each market and in supplying each product to each market. Higher ability firms can generate sufficient variable profits to cover the product fixed cost at a lower value of product attributes and, therefore, supply a wider range of products to each market. For sufficiently low values of firm ability, the excess of variable profits over product fixed costs in the small range of profitable products does not cover the fixed cost of serving the market, and therefore the firm does not supply the market.

The lowest-ability firms exit, intermediate-ability firms serve only the domestic market, and the highest-ability firms export. Within exporters, products with the worst attributes are supplied only to the domestic market, whereas products with the best attributes are exported to the largest number of markets.

We characterize the equilibrium of the model analytically for symmetric countries using general continuous distributions of firm ability and product attributes. We also analyze the equilibrium for asymmetric countries assuming Pareto distributions for firm ability and product attributes. We compare the model's predictions for two formulations of product attributes: a "common-product-attributes" specification, where product attributes vary across products within firms but are the same across countries (e.g., technology), and a "country-specific-product attributes" specification, where product attributes vary across both products and countries within firms (e.g., demand for product characteristics).

The model yields three sets of core implications, which we examine in our empirical analysis. The first is that trade liberalization causes firms to drop their least-successful products, which induces compositional changes within firms. Analysis of U.S. microdata reveals that U.S. firms more exposed to tariff reductions under the Canada–U.S. Free Trade Agreement reduce the number of products they produce relative to firms less exposed to these tariff reductions. The second implication is that higher variable trade costs reduce the number of exporting firms, the number of products exported by each firm, and exports of a given product by a given firm, but have an ambiguous effect on average exports per firm and product. We find strong support for these relationships in estimated gravity equations derived from the model under the assumption of Pareto distributions. The third implication is that firms exporting many products also serve many export destinations and export more of a given product to a given destination. Again we find support for this relationship in U.S. export data, as well as for several other features of the model.

The remainder of the article is structured as follows. Section II reviews the existing theoretical and empirical literature on multiple-product firms. Section III develops the model. Section IV characterizes the symmetric-country equilibrium for general continuous distributions of firm ability and product attributes. Section V characterizes the asymmetric-country equilibrium under the assumption of Pareto distributions. Section VI presents empirical evidence on the model's predictions. Section VII

concludes. An Online Appendix contains the technical derivations of expressions in each section and the proofs of propositions.

II. RELATED LITERATURE ON MULTIPRODUCT FIRMS

Our article is related to existing theoretical research on multiple-product firms in the industrial organization and international trade literatures. As noted in the survey by [Bailey and Friedlaender \(1982\)](#), early research on multiple-product firms in industrial organization emphasizes supply-side economies of scope. Subsequent analyses, by [Brander and Eaton \(1984\)](#), [Shaked and Sutton \(1990\)](#), [Eaton and Schmidt \(1994\)](#), and [Johnson and Myatt \(2003\)](#), focus on demand-side forces favoring the production of multiple goods as well as analyses of strategic interaction among firms. More recent models by [Klette and Kortum \(2004\)](#) and [Lentz and Mortensen \(2005\)](#) examine the role of innovation in firms' decisions to produce multiple products.

Recent theoretical contributions to the international trade literature consider several approaches to modeling the production of multiple goods. While early contributions such as [Ottaviano and Thisse \(1999\)](#) and [Allanson and Montagna \(2005\)](#) modeled firms and products symmetrically, more recent research has explored the idea that firms have core competences. [Eckel and Neary \(2010\)](#) consider a model of flexible manufacturing where each firm faces rising marginal costs in producing products further from its core competence. Firms are large relative to the market and hence face a cannibalization effect, where introducing additional products diminishes the demand for the firm's existing products, as also considered in [Feenstra and Ma \(2008\)](#) and [Dhingra \(2010\)](#). Though our analysis assumes that firms are small relative to the market and abstracts from cannibalization effects to focus on selection, we allow for a rich range of asymmetries across firms, within firms, and across countries. Firms and products within firms are heterogeneous in terms of their productivity/demand and participation in international trade. Countries can differ in terms of their size, productivity, and bilateral trade costs.

Three other recent papers have developed monopolistically competitive models of multiple-product firms without cannibalization effects. In [Mayer, Melitz, and Ottaviano \(2010\)](#), firms face a product ladder, where productivity/quality declines discretely for each additional variety produced. Together with variable mark-ups, this generates the prediction that firm sales are more

skewed toward core competences in more competitive markets. In [Arkolakis and Muendler \(2010\)](#), firms face declining productivity for each additional variety supplied to a market and market entry costs that are increasing in the number of varieties supplied to a market, which generates a positive relationship between firms' extensive and intensive margins. In [Nocke and Yeaple \(2006\)](#), products are symmetric within firms, but firms differ in terms of organizational capability, which determines the rate at which the common marginal cost for each product rises with the number of products produced.¹ Firms with higher organizational capability produce more products, and hence in equilibrium have higher marginal costs, which generates a negative relationship between firms' extensive and intensive margins.

In contrast, in our framework, firms draw a distribution of profitabilities across products and countries, which has a firm component, a product component and possibly a country component. The sets of firms active in each market and the products supplied by those firms reflect endogenous selection based on this distribution of profitabilities. The key theoretical contributions of our article are to demonstrate the implications of this selection within firms for the impact of trade liberalization on productivity, the relationship between aggregate trade and variable trade costs, and patterns of disaggregate trade. The key empirical contributions show that each of these theoretical implications of selection within firms receives strong empirical support in U.S. trade data.

One limitation of our framework is the assumption of CES preferences, which implies that mark-ups are constant, and hence differences in competition across markets do not affect the skewness of firm sales across products in common to those markets, unlike in [Mayer, Melitz, and Ottaviano \(2010\)](#). On the other hand, by making this simplifying assumption, we are able to develop a general equilibrium model with no outside sector, in which asymmetries across countries feed back to influence wages and demand for each firm and product. Our analysis of selection within firms reveals the role played by export composition and the functional form of the export sales distribution in shaping the relationship between firms' extensive and intensive margins.

In contrast to the closed economy model of [Bernard, Redding, and Schott \(2010\)](#), our theoretical analysis examines the

1. See [Agur \(2006\)](#) and [Baldwin and Gu \(2009\)](#) for other heterogeneous firm models in which products are symmetric within firms.

implications of selection within firms for an economy's response to trade liberalization and for the pattern of trade in the open economy. While some descriptive evidence on trade flows across and within firms is presented in [Bernard, Jensen, and Schott \(2009\)](#) and [Bernard et al. \(2007, 2009\)](#), our empirical analysis tests the theoretical implications of our model and estimates empirical relationships implied by the model.²

III. THE MODEL

We consider a world consisting of many countries and many products. Firms decide whether to enter, what products to produce, and where to supply these products. Products are imperfect substitutes in demand and, within each product, firms supply horizontally differentiated varieties of the product.³ We allow countries to be asymmetric in terms of their bilateral trade costs (geography), size (labor endowment), and productivity.

III.A. Preferences and Endowments

Countries are indexed by $i \in \{1, \dots, J\}$ and are endowed with L_i units of labor that are supplied inelastically with zero disutility. The representative consumer in each country derives utility from the consumption of a continuum of symmetric products that we normalize to the interval $[0, 1]$. There is a constant elasticity of substitution across products so that the utility function for the representative consumer in country j takes the standard [Dixit-Stiglitz \(1977\)](#) form:

$$(1) \quad U_j = \left[\int_0^1 C_{jk}^\nu dk \right]^{\frac{1}{\nu}}, \quad 0 < \nu < 1,$$

where k indexes products. Within each product, a continuum of firms supply horizontally differentiated varieties of the product. Hence C_{jk} is a consumption index, which also takes the CES form,

2. For empirical evidence on multiproduct firms in a developing-country context, see [Goldberg et al. \(2010a, 2010b\)](#).

3. Our model focuses on firms that supply multiple products for final consumption. While vertical integration provides another reason firms can produce multiple products (intermediate and final), many firms supply multiple products for final consumption.

and depends on varieties consumed from each country in the world:

$$(2) \quad C_{jk} = \left[\sum_{i=1}^J \int_{\omega \in \Omega_{ijk}} [\lambda_{ijk}(\omega) c_{ijk}(\omega)]^\rho d\omega \right]^{\frac{1}{\rho}}, \quad 0 < \rho < 1,$$

where i and j index countries, ω indexes varieties of product k supplied from country i to country j , and Ω_{ijk} denotes the endogenous set of these varieties. The parameter $\lambda_{ijk}(\omega) \geq 0$ captures what we term “product attribute”. Although modeled here as the strength of consumer tastes in country j for a variety of product k supplied by firm ω in country i , we discuss below an alternative and equivalent supply-side formulation.

We make the natural assumption that the elasticity of substitution across varieties within products is greater than the elasticity of substitution across products: $\sigma \equiv \frac{1}{1-\rho} > \kappa \equiv \frac{1}{1-\nu} > 1$.⁴ We also assume for simplicity that the elasticity of substitution across varieties within products, $\sigma \equiv \frac{1}{1-\rho}$, is the same for all products. The corresponding price index dual to Equation 2 is:

$$P_{jk} = \left[\sum_{i=1}^J \int_{\omega \in \Omega_{ijk}} \left(\frac{p_{ijk}(\omega)}{\lambda_{ijk}(\omega)} \right)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}.$$

III.B. Production Technology

The specification of entry and production follows Melitz (2003). However, we augment that model to allow firms to supply multiple products and to allow for heterogeneity across products within firms as well as across firms. There is an unbounded measure of potential firms who are identical prior to entry. To enter, firms must incur a sunk entry cost of $f_{ei} > 0$ units of labor in country i . Incurring the sunk entry cost creates a firm brand, which can be used to supply one horizontally differentiated variety of each of the continuum of products. We assume that varieties are differentiated from one another by their brand, which implies that

4. While we distinguish between two elasticities of substitution, one across products and another across firm varieties within products, the elasticity of substitution across products could in practice depend on whether the products are supplied by the same firm.

a given brand cannot be used to supply more than one differentiated variety of each product.⁵

Following existing models of industry dynamics, we take a technological approach to the boundaries of the firm, such that a firm is defined by its production technology and product attributes. Both production technology and product characteristics are uncertain prior to entry and are only revealed once the sunk entry cost has been incurred. There are two components of production technology and product characteristics that influence firm profitability: one is common across products and countries (“ability” captured by φ), and the other is idiosyncratic to products and possibly countries (“product attributes” captured by λ). Although we model φ as firm productivity and λ as consumer tastes, under our assumptions of CES preferences and monopolistic competition, both productivity and consumer tastes enter equilibrium firm revenue in exactly the same way. All that matters for our analysis is that there are common and idiosyncratic components of firm profitability, where the common component generates heterogeneity across firms and the idiosyncratic component generates heterogeneity within firms. For this reason, we refer to φ as “firm ability” and λ as “product attributes” to emphasize that each could refer to either a component of demand or productivity.⁶

We consider two possible specifications for the idiosyncratic component of firm profitability, λ . In the first of these specifications—the “common product attributes” formulation—product attributes vary across products but are the same across countries (e.g., a firm may have specialized expertise in the production of some products that is relevant for all countries). In the second of these specifications—the “country-specific product attributes” formulation—product attributes vary across both products and countries (e.g., product attributes may be perceived more favorably in some countries than in others). We discuss the predictions

5. This formulation is a natural generalization of the single-product model of Melitz (2003), in which incurring the sunk entry cost creates a firm brand that can be used to produce one horizontally differentiated variety. While our framework could be extended to allow firms to produce a measure of horizontally differentiated varieties of each product, such an extension would imply that firms were no longer of measure zero within each product and would introduce strategic interaction within and across firms.

6. Though our formulation captures heterogeneity within firms in an intuitive and tractable way, one could also generate such heterogeneity from interactions between firm, product, and country characteristics (e.g., firm ability could have a greater impact on profitability for some products and/or countries).

of the model under each of these specifications and present evidence on the extent to which heterogeneity across products is common across countries.⁷

Once the sunk entry cost has been incurred, the firm observes its ability, φ , and its product attributes for each country j and product k , λ_{jk} .⁸ To capture cross-country differences in productivity, we allow the firm ability distribution to vary across countries. Firm ability, $\varphi \in [0, \infty)$, is drawn from a continuous distribution $g_i(\varphi)$ in country i , with cumulative distribution function $G_i(\varphi)$. Product attributes, $\lambda \in [0, \infty)$, are drawn from a continuous distribution $z(\lambda)$ with cumulative distribution function $Z(\lambda)$. Both the range of products $k \in [0, 1]$ and the distribution of product attributes are the same for all countries. In the common-product-attributes specification, there is a single realized value for product attributes for a given product across all countries: $\lambda_{jk} = \lambda_k$ for product k for all countries j . In contrast, in the country-specific-product-attributes specification, there are different realizations for product attributes for a given product for each country: in general, $\lambda_{ik} \neq \lambda_{jk}$ for product k for countries i and $j \neq i$.

To make use of law of large numbers results, we assume that the firm ability and product attributes distributions are independent across firms. For the same reasons, we assume that the firm ability and product attributes distributions are independent of one another and that the product attributes distributions are independent across products. In the country-specific-product-attributes specification, we make the further assumption that the product attributes distributions are independent across countries. Despite these simplifying assumptions, firm profitability is correlated across products and countries within firms, because firm ability is common across both products and countries. In the common-product-attributes specification, there is a further source of correlation in profitability within firms, because product attributes take the same value across countries for a given product.⁹

7. One can also consider a hybrid case, in which product attributes have both a common and country-specific component, as discussed in the Online Appendix.

8. As the focus of our analysis is the cross-section distribution of exports, we develop a static model that abstracts from stochastic variation over time in firm ability and product attributes, and hence from steady-state adding and dropping of products and countries. However, the model can be extended to incorporate these dynamics, as shown in the Online Appendix.

9. Additionally, one could allow for a component of product attributes that is common across related products or explicitly allow the realizations of product attributes to be correlated. While these extensions would complicate the analysis,

Once the sunk cost has been incurred, and firm ability and product attributes have been observed, a firm decides whether to enter and what products and countries to supply. Labor is the sole factor of production.¹⁰ We assume that firms based in country i face a fixed cost of supplying country j of $F_{ij} > 0$ units of labor. These market-specific fixed costs capture, among other things, the costs of building distribution networks. In addition, we assume that firms based in country i face fixed costs of supplying each product to country j of $f_{ij} > 0$ units of labor. These product-specific fixed costs capture the costs of market research, advertising, and conforming to foreign regulatory standards for each product. As more products are supplied to a market, total fixed costs rise, but average fixed costs fall, because the fixed cost of serving each market is spread over a larger number of products. These fixed costs affect a firm's decision whether to supply a market, but do not affect sales conditional on supplying that market. Although all fixed costs are assumed to be incurred in the source country, it is straightforward to consider instead the case where they are incurred in the destination market.

In addition to the fixed costs, there is also a constant marginal cost of production for each product that depends on firm ability, such that $q_{ijk}(\varphi, \lambda_{jk})/\varphi$ units of labor are employed in country i to supply $q_{ijk}(\varphi, \lambda_{jk})$ units of output of product k to market j . Finally, we allow for variable costs of trade, such as transportation costs, which take the standard "iceberg" form. A fraction $\tau_{ij} > 1$ of a variety must be shipped from country i for one unit to arrive in country j , where $\tau_{ii} = 1$.¹¹

III.C. Firm-Product Profitability

Demand for each variety of a product depends on the own-variety price, the price index for the product, the price indices for all other products, and aggregate expenditure. If a firm is active

they would not alter the model's central mechanism of selection within firms, which is driven by heterogeneity in profitability within firms.

10. In the Online Appendix, we consider a multi-industry version of the model with multiple factors of production, which gives rise to Heckscher-Ohlin-based comparative advantage, as in the single-product heterogeneous-firm model of Bernard, Redding, and Schott (2007).

11. For evidence on the magnitude of overall trade costs, see Anderson and van Wincoop (2004) and Hummels (2001). For evidence on the fixed costs of exporting, see Roberts and Tybout (1997); Bernard and Jensen (2004), and Eaton, Kortum, and Kramarz (2008).

in a product market, it supplies one of a continuum of varieties, and hence the firm is unable to influence the price index for any product. Therefore, the firm's profit maximization problem reduces to choosing the price of each product variety separately to maximize the profits derived from that product variety.¹² This optimization problem yields the standard result that the equilibrium price of a product variety is a constant mark-up over marginal cost:

$$(3) \quad p_{ij}(\varphi, \lambda) = \tau_{ij} \frac{1}{\rho} \frac{w_i}{\varphi}.$$

Because the production technology and elasticity of substitution across varieties are the same for each product, all products with productivity φ have the same price. Therefore we suppress the implicit dependence on product, k , from now on.

Using the above pricing rule, the equilibrium revenue received by a firm in country i from supplying a product to country j is:

$$(4) \quad r_{ij}(\varphi, \lambda) = (w_i \tau_{ij})^{1-\sigma} w_j L_j (\rho P_j \varphi \lambda)^{\sigma-1}.$$

The corresponding equilibrium profits from supplying the product to that market are:

$$(5) \quad \pi_{ij}(\varphi, \lambda) = \frac{r_{ij}(\varphi, \lambda)}{\sigma} - w_i f_{ij}.$$

From these last two expressions, firm ability enters equilibrium revenue (4) and profit (5) in exactly the same way as product attributes, since CES preferences and monopolistic competition imply that prices are a constant mark-up over marginal costs. These properties imply that the relative revenues (4) of any two varieties of a given product in a given market depend solely on relative firm abilities and product attributes:

$$r_{ij}(\varphi'', \lambda'') = (\varphi''/\varphi')^{\sigma-1} (\lambda''/\lambda')^{\sigma-1} r_{ij}(\varphi', \lambda').$$

The same properties also imply that the relative revenues (4) of varieties of products with the same $\varphi\lambda$ in any two markets depend solely on relative variable trade costs and market characteristics:

12. The structure of our model eliminates strategic interaction within or between firms. This choice of model structure enables us to isolate the implications of introducing selection within firms into a model of firm heterogeneity without introducing additional considerations associated with strategic interaction. Exploring the implications of strategic interaction is an interesting area for further research.

$$r_{ij}(\varphi, \lambda) / r_{ih}(\varphi, \lambda) = (\tau_{ij} / \tau_{ih})^{1-\sigma} (w_j L_j / w_h L_h) (P_j / P_h)^{\sigma-1}.$$

A firm with a given ability φ decides whether to supply a product with attributes λ to a market based on a comparison of variable profits and fixed costs for the product. For each firm ability φ , there is a *zero-profit cutoff for product attributes*, $\lambda_{ij}^*(\varphi)$, for each source country and destination market, such that the firm only supplies the product if it draws a value of λ equal to or greater than $\lambda_{ij}^*(\varphi)$. This product cutoff is defined by the following zero-profit condition:

$$(6) \quad r_{ij}(\varphi, \lambda_{ij}^*(\varphi)) = \sigma w_i f_{ij}.$$

Using this product cutoff for each firm ability (6) together with relative variety revenues within the same market, $\lambda_{ij}^*(\varphi)$ can be expressed relative to its value for the lowest ability firm from source country i supplying destination market j , $\lambda_{ij}^*(\varphi_{ij}^*)$:

$$(7) \quad \lambda_{ij}^*(\varphi) = (\varphi_{ij}^* / \varphi) \lambda_{ij}^*(\varphi_{ij}^*).$$

Higher ability firms have lower product cutoffs (7), because their higher ability generates sufficient variable profits to cover product fixed costs at lower values of product attributes. In contrast, markets with higher values of φ_{ij}^* or $\lambda_{ij}^*(\varphi_{ij}^*)$ have higher product cutoffs, because rival firms' products are more attractive in these markets, which implies that a higher value for product attributes is required to generate sufficient variable profits to cover product fixed costs.

Because product attributes are independently distributed across the unit continuum of symmetric products, the fraction of products supplied by a firm with a given ability φ from source country i to destination market j is simply equal to the probability of drawing a value for product attributes above $\lambda_{ij}^*(\varphi)$: $\left[1 - Z\left(\lambda_{ij}^*(\varphi)\right)\right]$.

From the relative revenues of varieties in two different markets, the zero-profit cutoffs for product attributes for any two markets are related as follows:

$$(8) \quad \lambda_{ij}^*(\varphi) = \frac{\tau_{ij} P_h}{\tau_{ih} P_j} \left(\frac{f_{ij} w_h L_h}{f_{ih} w_j L_j} \right)^{\frac{1}{\sigma-1}} \lambda_{ih}^*(\varphi).$$

For sufficiently high fixed and variable trade costs in Equation 8, the product cutoff in each export market $j \neq i$ lies above the product cutoff in the domestic market i , which implies product

selection into export markets. In the common-product-attributes specification, this implies that no product is exported without also being supplied domestically. In contrast, in the country-specific-product-attributes specification, each product is less likely to be exported than supplied domestically, but with different realizations of product attributes across countries, a given product can be exported but not supplied domestically.¹³

III.D. Firm Profitability

Since product attributes are independently distributed across the unit continuum of symmetric products, the law of large numbers implies that a firm’s expected revenue across the unit continuum of products equals its expected revenue for each product. Expected revenue for each product is a function of firm ability φ and equals the probability of drawing a value for product attributes above the zero-profit cutoff times expected revenue conditional on supplying the product. Therefore total firm revenue across the unit continuum of products in each market is:

$$r_{ij}(\varphi) = \int_{\lambda_{ij}^*(\varphi)}^{\infty} r_{ij}(\varphi, \lambda) z(\lambda) d\lambda.$$

Similarly, total firm profits in each market equal expected profits from each product minus the market fixed costs:

$$(9) \quad \pi_{ij}(\varphi) = \int_{\lambda_{ij}^*(\varphi)}^{\infty} \left(\frac{r_{ij}(\varphi, \lambda)}{\sigma} - w_i f_{ij} \right) z(\lambda) d\lambda - w_i F_{ij}.$$

The lower a firm’s ability, φ , the higher its product cutoff, $\lambda_{ij}^*(\varphi)$, and the lower its probability of drawing a value for product attributes high enough to profitably supply the product to the market $\left[1 - Z\left(\lambda_{ij}^*(\varphi)\right) \right]$. As a result, firms with lower abilities supply a smaller fraction of products to a given market and have lower expected profits from each product. For sufficiently low values of firm ability, the excess of variable profits over product fixed costs in the small range of profitable products falls short of the fixed cost of supplying the market, $w_i F_{ij}$. Therefore, there is a zero-profit cutoff for firm ability, φ_{ij}^* , such that a firm only supplies a market if it draws a value of φ equal to or greater than φ_{ij}^* . This

13. For empirical evidence that some products are exported but not supplied domestically, see for example [Iacovone and Javorcik \(2010\)](#).

firm cutoff is defined by the zero-profit condition:

$$(10) \quad \pi(\varphi_{ij}^*) = 0.$$

Combining the firm cutoff (10), the product cutoff (6), total firm profits (9), and relative variety revenues within the same market, we can determine the product cutoff for the lowest ability firm in source country i that supplies destination market j , $\lambda_{ij}^*(\varphi_{ij}^*)$,

$$(11) \quad \int_{\lambda_{ij}^*(\varphi_{ij}^*)}^{\infty} \left[\left(\frac{\lambda}{\lambda_{ij}^*(\varphi_{ij}^*)} \right)^{\sigma-1} - 1 \right] f_{ij} z(\lambda) d\lambda = F_{ij},$$

where terms in wages have cancelled from the above expression.

The zero-profit condition (11) implies that the model has a recursive structure, where $\lambda_{ij}^*(\varphi_{ij}^*)$ is determined for each source country and destination market independently of wages, price indices, and labor endowments as a function of fixed trade costs and other parameters. This feature of the model follows from the properties of CES demand, which implies that the relative revenues of any two varieties within the same market depend solely on relative firm abilities and product attributes, while the revenue of the least-profitable variety is proportional to fixed costs.

Combining relative revenues for a given product in two different markets, relative revenues for different varieties of a given product within the same market, and the product cutoff (6), we obtain the following relationship between the firm cutoffs in country i for two different markets j and h :

$$(12) \quad \varphi_{ij}^* = \Gamma_{ijh} \varphi_{ih}^*, \quad \Gamma_{ijh} \equiv \frac{\tau_{ij} P_h}{\tau_{ih} P_j} \left(\frac{f_{ij} w_h L_h}{f_{ih} w_j L_j} \right)^{\frac{1}{\sigma-1}} \frac{\lambda_{ih}^*(\varphi_{ih}^*)}{\lambda_{ij}^*(\varphi_{ij}^*)}.$$

For sufficiently high fixed and variable trade costs in Equation 12, the firm cut-off in each export market $j \neq i$ lies above the firm cutoff in the domestic market i ($\Gamma_{iji} > 1$), which implies firm selection into export markets. Consistent with a large empirical literature, we focus on parameter values for which such firm selection into export markets occurs.¹⁴ As discussed above, in the country-specific-product-attributes specification, different realizations of product attributes across markets can induce a firm

14. See for example Bernard and Jensen (1995, 1999) and Roberts and Tybout (1997).

to export a product and yet not supply the product domestically. But the law of large numbers implies that these idiosyncratic differences in product attributes average out across the unit continuum of products. As a result, for parameter values for which $\Gamma_{iji} > 1$, no firm exports without also serving the domestic market.

III.E. Free Entry

Firms decide whether to enter based on a comparison of the expected value of entry and the sunk entry cost. The expected value of entry, V_i , equals the ex ante probability of successful entry times expected firm profits conditional on entry, $\bar{\pi}_i$.¹⁵ The free entry condition therefore takes the following form:

$$(13) \quad V_i = [1 - G_i(\varphi_{ii}^*)] \bar{\pi}_i = w_i f_{ei},$$

where the ex ante probability of successful entry is $[1 - G_i(\varphi_{ii}^*)]$. Expected firm profits conditional on entry, $\bar{\pi}_i$, equal the sum across markets of the probability of supplying a market conditional on entry times expected profits conditional on supplying the market:

$$\bar{\pi}_i = \sum_{j=1}^J \frac{1 - G_i(\varphi_{ij}^*)}{1 - G_i(\varphi_{ii}^*)} \int_{\varphi_{ij}^*}^{\infty} \pi_{ij}(\varphi) \frac{g_i(\varphi)}{1 - G_i(\varphi_{ij}^*)} d\varphi.$$

Using total firm profits (9), product profits (5), and the relationship between relative variety revenues, the free entry condition can be written in terms of the zero-profit cutoffs for firm ability and product attributes, and parameters:

$$(14) \quad V_i = \sum_{j=1}^J \int_{\varphi_{ij}^*}^{\infty} \left[f_{ij} \int_{\lambda_{ij}^*(\varphi)}^{\infty} \left[\left(\frac{\lambda}{\lambda_{ij}^*(\varphi)} \right)^{\sigma-1} - 1 \right] z(\lambda) d\lambda - F_{ij} \right] g_i(\varphi) d\varphi = f_{ei},$$

where terms in wages have again canceled from the above expression.

III.F. Goods and Labor Markets

In goods markets, the mass of firms producing in each country is a constant fraction of the mass of entrants (M_{ei}), which depends

15. For simplicity, we abstract from the constant exogenous probability of firm death in Melitz (2003). Introducing this feature is straightforward, but complicates the model without affecting any of our results.

on the probability of successful entry:

$$(15) \quad M_i = [1 - G_i(\varphi_{ii}^*)] M_{ei}.$$

Of the mass of firms producing, a constant fraction supply each market, which depends on the probability of supplying a market conditional on producing:

$$(16) \quad M_{ij} = \frac{1 - G_i(\varphi_{ij}^*)}{1 - G_i(\varphi_{ii}^*)} M_i.$$

To determine the mass of firms supplying each product to each market (m_{ij}), we use the property that a fraction $[1 - Z(\lambda_{ij}^*(\varphi))]$ of the mass of firms with ability φ in country i supply a product to country j :

$$(17) \quad m_{ij} = \left[\int_{\varphi_{ij}^*}^{\infty} [1 - Z(\lambda_{ij}^*(\varphi))] \left(\frac{g_i(\varphi)}{1 - G_i(\varphi_{ij}^*)} \right) d\varphi \right] M_{ij}.$$

Using the equilibrium pricing rule (3), the price index for each product in country j can be expressed in terms of the masses of firms supplying the product and the prices charged by a firm with a weighted average productivity in each country i :

$$(18) \quad P_j = \left[\sum_{i=1}^J m_{ij} \tau_{ij}^{1-\sigma} p(\tilde{\varphi}_{ij})^{1-\sigma} \right]^{\frac{1}{1-\sigma}},$$

where weighted average productivity ($\tilde{\varphi}_{ij}$) depends on the firm cutoff (φ_{ij}^*) and a weighted average of product attributes for each firm ability ($\tilde{\lambda}_{ij}(\varphi)$), as defined in the Online Appendix.

Aggregate revenue can be determined from the relationship linking the masses of firms and entrants (15) and the free entry condition (13), which together imply that total payments to labor used in entry equal total profits. On the other hand, total payments to labor used in production equal total revenue minus total profits, from which it follows that aggregate revenue equals total payments to labor and the labor market clears: $R_i = w_i L_i$.

Aggregate revenue is also equal to total expenditure on goods produced in a country, which implies that equilibrium wages are implicitly defined by the following relationship:

$$(19) \quad w_i L_i = \sum_{j=1}^J \alpha_{ij} w_j L_j, \quad \alpha_{ij} = \frac{1 - G_i(\varphi_{ij}^*)}{1 - G_i(\varphi_{ii}^*)} M_i \bar{r}_{ij} / w_j L_j,$$

where α_{ij} is the share of country j 's expenditure on goods supplied by country i , which equals the mass of firms from country i supplying country j , $M_i \left[1 - G_i(\varphi_{ij}^*) \right] / \left[1 - G_i(\varphi_{ii}^*) \right]$, times the average sales of each supplier, \bar{r}_{ij} , divided by aggregate income in country j .

III.G. Revenue-Based Productivity

As in Melitz (2003), firms supply horizontally differentiated varieties, which implies that productivity can be measured either using the quantity aggregator derived from CES preferences or using a revenue-based aggregator. We focus our analysis on revenue-based measures of productivity because these correspond to those used in empirical work, and we discuss their relationship to productivity measures derived from the CES quantity aggregator.

Standard revenue-based measures of productivity deflate a firm's revenue from a given product and market by a price index to obtain a "revenue production function," as in Klette and Griliches (1996), Levinsohn and Melitz (2006), De Loecker (2007), and Foster, Haltiwanger, and Syverson (2008). Deflating firm revenue by the CES price index (P_j), using the definition of firm revenue ($r_{ij}(\varphi, \lambda) = p_{ij}(\varphi, \lambda) q_{ij}(\varphi, \lambda)$), substituting for prices using the inverse CES demand curve, and substituting for quantities using the production technology, we obtain:

$$\log r_{ij}(\varphi, \lambda) - \log P_j = \frac{\sigma - 1}{\sigma} \left[\log(\varphi \lambda) + \log l_{ij}^{\text{var}}(\varphi, \lambda) \right] + \frac{1}{\sigma} \log \left(\frac{w_j L_j}{P_j} \right).$$

In principle, this revenue production function can be used to estimate revenue-based productivity for each firm, product and market ($\theta_{ij}(\varphi, \lambda) \equiv \varphi \lambda$) if data on revenue and variable labor input by firm, product, and market are available, if appropriate instruments for variable labor input ($l_{ij}^{\text{var}}(\varphi, \lambda)$) can be found, and if product market conditions ($w_j L_j / P_j$) are controlled for. In practice, obtaining data on output and factor inputs at the appropriate level of aggregation is a key empirical challenge in the literature on productivity measurement.

Following standard empirical methods for productivity aggregation, revenue-based productivity for the firm is the revenue-share weighted average of revenue-based productivity for each product and market:

$$\theta_i(\varphi) \equiv \sum_{j=1}^J \left[\int_{\lambda_{ij}^*(\varphi)}^{\infty} \theta_{ij}(\varphi, \lambda) \frac{I_{ij}(\varphi) r_{ij}(\varphi, \lambda) z(\lambda)}{r_i(\varphi)} d\lambda \right],$$

where $I_{ij}(\varphi) = 1$ if $\varphi \geq \varphi_{ij}^*$ and a market is served or 0 otherwise.

Similarly, revenue-based productivity for the industry is the revenue-share weighted average of revenue-based productivity for each firm:

$$\Theta_i \equiv \int_{\varphi_{ii}^*}^{\infty} \frac{r_i(\varphi)}{R} \theta_i(\varphi) \frac{g(\varphi)}{1 - G(\varphi_{ii}^*)} d\varphi.$$

These revenue-based measures of productivity are closely related to those derived from the CES quantity aggregator, as is evident from the definitions of $\tilde{\lambda}_{ij}(\varphi)$ and $\tilde{\varphi}_{ij}$ in the Online Appendix. The main difference is that the productivity measures derived from the CES quantity aggregator are net of the output lost through the variable costs of trade, whereas revenue-based productivity is measured using revenue at the factory gate.

IV. SYMMETRIC COUNTRIES

To provide intuition for the properties of the model, we begin by solving for general equilibrium for symmetric countries, allowing for general continuous distributions of firm ability and product attributes. Each country trades with itself and $n \geq 1$ foreign countries. All countries have identical labor endowments and the same firm ability and product attributes distributions, $g(\varphi)$ and $z(\lambda)$, respectively. Variable trade costs take the same value for all foreign countries and domestic trade is costless: $\tau_{ij} = \tau_{ji} = \tau > 1$ for all $i \neq j$ and $\tau_{ii} = 1$. Product and market fixed costs take the same value for all foreign countries, so we can distinguish solely between the domestic and export markets: $f_{ij} = f_x > 0$ for all $i \neq j$ and $f_{ii} = f_d > 0$; $F_{ij} = F_x > 0$ for all $i \neq j$ and $F_{ii} = F_d > 0$. We choose the wage in one country as the numeraire, which, as countries are symmetric, implies that the wage in all countries is equal to 1.

As shown in the Online Appendix, there exists a unique symmetric country equilibrium, which is referenced by six variables: the firm ability cutoff in the domestic market (φ_d^*); the firm ability cutoff in the export market (φ_x^*); the product cutoff for the lowest ability firm in the domestic market ($\lambda_d^*(\varphi_d^*)$); the product cutoff for the lowest ability firm in the export market ($\lambda_x^*(\varphi_x^*)$); the price index for each product (P) and aggregate revenue (R).

With symmetric countries, the free entry condition (14) can be written solely in terms of the domestic and exporting cutoffs for firm ability and product attributes $\{\varphi_d^*, \varphi_x^*, \lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$. Because the product cutoffs for the lowest ability firm in each market $\{\lambda_d^*(\varphi_d^*), \lambda_x^*(\varphi_x^*)\}$ are determined as a function of parameters alone in Equation 11, the free entry condition implies a downward-sloping relationship between φ_d^* and φ_x^* , where autarky corresponds to the limiting case of infinitely large trade costs for which $\varphi_x^* \rightarrow \infty$. As the closed economy is opened to trade, the exporting cutoff for firm ability (φ_x^*) falls to a finite value, which implies that the domestic cutoff for firm ability (φ_d^*) must necessarily rise for the expected value of entry to remain equal to the unchanged sunk entry cost.

IV.A. Multiproduct Firms and Productivity Growth

In contrast to the standard heterogeneous firm model with single-product firms, the opening of the closed economy to international trade raises firm productivity ($\theta(\varphi)$) through within-firm reallocations of resources across products and markets.

PROPOSITION 1. The opening of the closed economy to trade increases firm productivity $\theta(\varphi)$:

- (a) All firms drop low-attribute products from the domestic market, which reallocates resources toward higher-attribute products and hence raises firm productivity.
- (b) High-ability firms begin to export, and hence add products with high attributes in the export market, which further reallocates resources toward higher-attribute products and hence raises firm productivity.

Proof. See Online Appendix. ■

The key to understanding this result is recognizing that opening a closed economy to trade has uneven effects across firms depending on whether they begin to export and uneven effects across products within firms depending on whether these products begin to be exported. Since there is a positive ex ante probability of drawing a firm ability high enough to export, the opening of trade increases the expected value of entry. As a result, there is increased entry, which enhances product-market competition in the domestic market and raises the zero-profit cutoff for firm ability (φ_d^*) below which firms exit. This rise in φ_d^* reduces the average prices of varieties supplied by competing firms, which

induces surviving firms to drop products with lower values of product attributes and also reduces the revenue of all surviving products in the domestic market. At the same time, entry into exporting generates new revenue in export markets for products with high values of product attributes. Each of these responses shifts the composition of firm revenue toward products with higher values of product attributes, which increases firm productivity $\theta(\varphi)$. For brevity we concentrate on the opening of the closed economy to trade, but similar results hold for reductions of variable trade costs in the open economy equilibrium, as discussed further in the Online Appendix.

In our multiproduct-firm setting, each firm chooses optimally the range of products to supply to each market. These optimal choices of product range affect aggregate productivity, as can be shown by decomposing the change in aggregate productivity following trade liberalization into the contributions of within- and between-firm reallocations:

$$\begin{aligned} \theta_i^t - \theta_i^a = & \underbrace{\frac{1}{1 - G(\varphi_{ii}^{*t})} \int_{\varphi_{ii}^{*t}}^{\infty} \frac{r_i^a(\varphi) \theta_i^a(\varphi)}{L} g(\varphi) d\varphi}_{\text{Term A}} - \underbrace{\frac{1}{1 - G(\varphi_{ii}^{*a})} \int_{\varphi_{ii}^{*a}}^{\infty} \frac{r_i^a(\varphi) \theta_i^a(\varphi)}{L} g(\varphi) d\varphi}_{\text{Term B}} \\ & + \underbrace{\frac{1}{1 - G(\varphi_{ii}^{*t})} \int_{\varphi_{ii}^{*t}}^{\infty} \frac{r_i^t(\varphi) \theta_i^t(\varphi)}{L} g(\varphi) d\varphi}_{\text{Term C}} - \underbrace{\frac{1}{1 - G(\varphi_{ii}^{*t})} \int_{\varphi_{ii}^{*t}}^{\infty} \frac{r_i^a(\varphi) \theta_i^t(\varphi)}{L} g(\varphi) d\varphi}_{\text{Term D}}, \\ & \underbrace{\frac{1}{1 - G(\varphi_{ii}^{*t})} \int_{\varphi_{ii}^{*t}}^{\infty} \frac{r_i^t(\varphi) \theta_i^t(\varphi)}{L} g(\varphi) d\varphi}_{\text{Term E}} - \underbrace{\frac{1}{1 - G(\varphi_{ii}^{*t})} \int_{\varphi_{ii}^{*t}}^{\infty} \frac{r_i^t(\varphi) \theta_i^a(\varphi)}{L} g(\varphi) d\varphi}_{\text{Term F}}, \end{aligned}$$

where the superscript a indicates the value of a variable under autarky; the superscript t indicates the value of a variable under costly trade; and $\varphi_{ii}^{*t} > \varphi_{ii}^{*a}$.

The difference between terms A and B captures the change in aggregate productivity due to the change in the range of abilities where firms produce, holding constant firm revenue and productivity at their autarky values. The difference between terms C and D corresponds to the change in aggregate productivity due to the change in firm revenue shares, holding constant firm productivity and the range of abilities where firms produce. The difference between terms E and F captures the change in aggregate productivity due to the change in firm productivity, holding constant firm revenue shares and the range of abilities where firms produce. In our framework, these changes in firm productivity (terms E and F)

arise from reallocations across products and destinations within firms, where the relative magnitude of these terms depends, in general, on the distributions of firm ability and product attributes and the closed and open economy cutoffs (φ_{ii}^{*a} and φ_{ii}^{*t}). To show the equivalence of the left-hand and right-hand sides of the above expression, note that terms A and D are the same as one another, as are terms C and F , and use the definition of aggregate revenue-based productivity.

In Proposition 1, trade liberalization reduces the range of products supplied to the domestic market. In the common-product-attributes specification, this reduction necessarily translates into a decrease in the range of products produced, since no product is exported without also being supplied to the domestic market. In contrast, in the country-specific-product-attributes specification, trade liberalization can induce a firm to add products in export markets that are not supplied domestically, because product attributes vary across countries. As a result, the net effect of trade liberalization on the range of products produced depends on whether this addition of new products for export markets exceeds the reduction in the range of products supplied to the domestic market. Empirical studies tend to find that the number of products exported but not supplied domestically is relatively small (see, for example, [Iacovone and Javorcik 2010](#)), suggesting that the reduction in the range of products supplied to the domestic market is likely to dominate.

IV.B. Margins of Trade and Trade Costs

The impact of variable trade costs on aggregate trade flows is also mediated by the optimal choice of product range for each firm ability.

PROPOSITION 2. A reduction in variable trade costs (τ):

- (a) increases the share of products exported to a given country by existing exporters (within-firm product extensive margin),
- (b) increases the expected number of countries to which a given product is supplied by existing exporters (within-firm country extensive margin),
- (c) increases the share of firms that export (across-firm extensive margin),
- (d) increases exports of a given product to a given country by a given firm but has an ambiguous effect on average

exports per firm-product-country because of changes in export exposition (intensive margin).

Proof. See Online Appendix. ■

The intuition for these comparative statics is as follows. A reduction in variable trade costs reduces the price of products in each export market, which, with elastic demand, increases revenue and variable profits. As a result, some products with low attributes that were previously only supplied to the domestic market can now be profitably exported. Therefore reductions in variable trade costs raise aggregate exports through the share of products exported to a given country by incumbent exporters (*within-firm product extensive margin*) and increase the expected number of countries to which a given product is supplied by incumbent firms (*within-firm country extensive margin*). Reductions in variable trade costs also induce some lower-ability firms that previously only served the domestic market to enter export markets. Hence aggregate exports also increase because of a rise in the share of firms that export (the *across-firm extensive margin*).

In the common-product-attributes specification, country symmetry implies that each product is either only supplied domestically or supplied to all countries worldwide. In this case, reductions in variable trade costs increase the probability that each product is exported to all countries worldwide, and hence increase the expected number of countries to which the product is exported. In contrast, in the country-specific-product attributes specification, realized values of product attributes vary across export markets, which implies that a given product can be exported to some foreign markets but not others.¹⁶ In this case, reductions in variable trade costs increase the probability that each product is exported to each market, and again increase the expected number of countries to which the product is exported.

Selection within firms implies that the effect of the reduction in variable trade costs on the *intensive margin* depends on whether one controls for export composition. On one hand, the reduction in variable trade costs reduces the price of products that are already exported in each foreign country, which increases

16. More generally, when countries are asymmetric, as considered in the next section, there is a hierarchy of export markets in terms of their zero-profit cutoffs for product attributes and firm ability. As a result, even in the common-product-attributes specification, a given product can be exported to some foreign markets but not others.

exports of a given product to a given destination by a given firm. On the other hand, the reduction in variable trade costs causes products with lower attributes to be exported. As these lower-attribute products are exported in smaller amounts, this change in export composition reduces average exports per firm-product-country. The net change in average exports is ambiguous and depends on the functional form of the distribution for product attributes. For the special case of a Pareto distribution of product attributes, the two forces exactly offset one another, so average exports per firm-product-country are independent of variable trade costs, as shown in the next section and the Online Appendix.¹⁷

IV.C. Margins of Trade across Firms

Multiproduct firms' choices of optimal product range also influence the dispersion of sales across firms. Higher-ability firms have larger total exports than lower ability firms, not only because they export more of a given product to a given country, but also because they export to more countries and export more products to each country.

PROPOSITION 3. Higher firm ability φ :

- (a) increases the share of products exported by the firm to a given country (within-firm product extensive margin),
- (b) increases the expected number of countries to which a given product is exported by the firm (within-firm country extensive margin),
- (c) increases exports of a given product to a given country but has an ambiguous effect on average exports per firm-product-country through changes in export composition (intensive margin).

Proof. See Online Appendix. ■

The intuition for these comparative statics follows a similar logic as for changes in variable trade costs. Higher-ability firms

17. Although a reduction in the fixed costs of exporting also increases aggregate exports through the extensive margins, it has a different effect on the intensive margin from a reduction in variable trade costs. As the fixed costs of exporting fall, a lower value of product attributes is required to generate sufficient revenue to profitably export, which reduces average exports per firm-product-country through a change in export composition, as shown in the Online Appendix for a Pareto distribution of product attributes.

charge lower prices for their products, which increases revenue and variable profits for a given value of product attributes. As a result, higher-ability firms export a larger share of products to each country (the *within-firm product extensive margin*) and on average export products to more countries (the *within-firm country extensive margin*). With common product attributes, products are either supplied only domestically or to all markets worldwide. In contrast, with country-specific product attributes, a given product can be exported to some foreign markets but not others.¹⁸

Selection within firms implies that the effect of higher firm ability on the *intensive margin* depends on whether one controls for export composition. On the one hand, higher firm ability increases exports of a given product with given product attributes to a given market. On the other hand, higher firm ability causes products with lower attributes to be exported. The net change in average exports per firm-product-country is ambiguous as before. For the special case of a Pareto distribution of product attributes, average exports per firm-product-country are independent of firm-ability, as shown in the next section and the Online Appendix.¹⁹

V. ASYMMETRIC COUNTRIES

To examine further the properties of the model with asymmetric countries, we characterize equilibrium analytically by assuming specific functional forms for the distributions of firm ability and product attributes. Firm ability is assumed to be Pareto distributed with the probability density function in country i given by $g_i(\varphi) = a\varphi_{\min i}^a \varphi^{-(a+1)}$. Countries can differ in terms of their size (labor endowment) and their production technology (as captured by the lower limit of the support of the firm ability distribution, $\varphi_{\min i}$). Product attributes are assumed to be Pareto distributed with the same probability density function across countries: $z(\lambda) = z\lambda_{\min}^z \lambda^{-(z+1)}$. To ensure that firm revenue has a finite mean with

18. As noted, even in the common-product-attributes specification, a given product can be exported to some foreign countries but not others if countries are asymmetric, as considered in the next section.

19. The lack of correlation between the intensive margin and firm ability requires both a Pareto distribution of product attributes and a product fixed cost of exporting, f_x , that is independent of product attributes, λ . Even with a Pareto distribution of product attributes, the correlation between the intensive margin and firm ability is negative if f_x is increasing in λ , and is positive if f_x is decreasing in λ , as shown in the Online Appendix.

these Pareto distributions, we assume $a > z > \sigma - 1$. Variable trade costs are allowed to differ across country-partner pairs and need not be symmetric, with domestic trade costless: $\tau_{ij} > 1$ and $\tau_{ii} = 1$. Similarly, product and market fixed costs can differ across country-partner pairs and need not be symmetric: $F_{ij} > 0$ and $f_{ij} > 0$.

V.A. *Asymmetric Country Equilibrium*

To determine the asymmetric country equilibrium with Pareto distributions, we use the recursive structure of the model, as shown in the Online Appendix. General equilibrium can be referenced by the wage (w_i) and mass of entrants (M_{ei}) in each country. While the wage in each country is determined as the unique solution to a system of equations, the mass of entrants can be solved for as a function of parameters. All other endogenous variables of the model can be determined as a function of wages and parameters.

Using the product attributes cutoff condition, the firm ability cutoff condition, the free entry condition, the labor market clearing condition, and the Pareto distributions of product attributes and firm ability, the mass of entrants can be solved for as a function of parameters:

$$(20) \quad M_{ei} = \frac{\sigma - 1}{a\sigma} \frac{L_i}{f_{ei}}.$$

Using the mass of entrants, the probability of serving each destination market, average firm revenue in each source country from serving each destination market, the Pareto distributions of product attributes and firm ability, and the equality between each country’s total labor income and total expenditure on its products, wages are implicitly defined by:

$$(21) \quad w_i L_i = \sum_{j=1}^J \alpha_{ij} w_j L_j,$$

$$\alpha_{ij} = \frac{(L_i/f_{ei}) (\varphi_{\min i})^a \tau_{ij}^{-a} w_i^{-\left(\frac{a\sigma - (\sigma - 1)}{\sigma - 1}\right)} F_{ij}^{-\left(\frac{a-z}{z}\right)} f_{ij}^{-\frac{a}{z} \left(\frac{z - (\sigma - 1)}{\sigma - 1}\right)}}{\sum_{k=1}^J (L_k/f_{ek}) (\varphi_{\min k})^a \tau_{kj}^{-a} w_k^{-\left(\frac{a\sigma - (\sigma - 1)}{\sigma - 1}\right)} F_{kj}^{-\left(\frac{a-z}{z}\right)} f_{kj}^{-\frac{a}{z} \left(\frac{z - (\sigma - 1)}{\sigma - 1}\right)},}$$

which provides a system of J equations that determines the unique equilibrium value of the wage in each of the J countries, as shown in the Online Appendix.

Using the product attributes cutoff, the firm ability cutoff, the measures of firms supplying each market, and the Pareto distributions of product attributes and firm ability, the price index (18) can be expressed in terms of wages and parameters:

$$(22) \quad P_j^{-a} = \frac{\kappa_P}{w_j^{(1-\frac{a}{\sigma-1})} L_j^{(1-\frac{a}{\sigma-1})}} \sum_{i=1}^J \frac{L_i (\varphi_{\min i})^a \tau_{ij}^{-a} F_{ij}^{-\left(\frac{a-z}{z}\right)} f_{ij}^{-\frac{a}{z} \left(\frac{z-(\sigma-1)}{\sigma-1}\right)}}{f_{ei} w_i^{(a-1+\frac{a}{\sigma-1})}},$$

where the constant κ_P is defined as a function of parameters in the Online Appendix.

The expression for the trade share, α_{ij} , in Equation 21 takes a similar form as in the Melitz (2003) model with a Pareto productivity distribution (see Arkolakis et al. 2008 and Chaney 2008). However, a key difference is that the trade share in our framework depends on parameters that influence multiproduct firms' choice of the optimal range of products to export. As a result, the trade share depends on product fixed costs (f_{ij}) as well as market fixed costs (F_{ij}), and depends on the dispersion of product attributes within firms ($1/z$) as well as the dispersion of ability across firms ($1/a$).²⁰ In this special case of our model with Pareto distributions for firm ability and product attributes, a country's trade share with itself is a sufficient statistic for the welfare gains from trade, as in Arkolakis, Costinot, and Rodríguez-Clare (2010). Even within this special case, our model points to new determinants of the endogenous value of the trade share, as already discussed. More generally, if there are departures from Pareto distributions for firm ability and product attributes, a country's trade share with itself, α_{jj} , is no longer a sufficient statistic for the welfare gains from trade.

The core implications of the model in Propositions 1–3 carry over from symmetric to asymmetric countries, as shown in the Online Appendix. For example, the opening of the closed economy to trade again increases the ability cutoff below which firms exit (φ_{ij}^*), which reallocates resources within firms toward higher-attribute products and hence raises firm productivity. In addition, the introduction of country asymmetries gives rise to a hierarchy

20. Note that the exponent on wages in the trade share, α_{ij} , differs from the exponent on variable trade costs, because product and market fixed costs are denominated in terms of source-country labor. If the product and market fixed costs were instead denominated in terms of destination-market labor, the exponent on wages would be the same as the exponent on variable trade costs and equal to $-a$.

of export markets in terms of their zero-profit cutoffs for product attributes and firm ability. Across countries, markets with lower variable trade costs are, other things equal, served by more firms and supplied with more products by each firm. Across firms, higher-ability firms export to more markets than lower-ability firms. Within firms, the higher a firm's ability, the more markets to which it exports any given product.

V.B. Margins of Trade across Countries

In this section, we highlight the implications of the asymmetric country equilibrium with Pareto distributions for the margins of trade across countries. We show that the model yields log linear gravity equations for the extensive and intensive margins of trade that we estimate in our empirical analysis.

For each source country i , aggregate bilateral trade to destination markets j (X_{ij}) can be decomposed into the contributions of the number of firm-product observations with positive exports (the extensive margin, O_{ij}) and average firm-product exports conditional on positive trade (the intensive margin, X_{ij}/O_{ij}):

$$X_{ij} = O_{ij} \left(\frac{X_{ij}}{O_{ij}} \right).$$

The extensive margin is determined as follows:

$$O_{ij} = \frac{1 - G(\varphi_{ij}^*)}{1 - G(\varphi_{ii}^*)} M_{ii} \int_{\varphi_{ij}^*}^{\infty} [1 - Z(\lambda_{ij}^*(\varphi))] \frac{g_i(\varphi)}{1 - G_i(\varphi_{ij}^*)} d\varphi,$$

where the first term before the integral is the measure of exporting firms and the integral corresponds to the expected number of products per exporting firm.

Under the assumption of Pareto distributions of firm ability and product attributes, the extensive margin can be written as a log linear gravity equation comprising source country i characteristics related to country size (e.g., labor endowment and wage), destination market j characteristics related to market size (e.g., labor endowment and price index), variable and fixed trade costs that differ across bilateral trade pairs, and parameters:

(23)

$$O_{ij} = \kappa_O \left[w_i^{-\frac{a\sigma}{\sigma-1}} \varphi_{\min i}^a \frac{L_i}{f_{ei}} \right] \left[w_j^{\frac{a}{\sigma-1}} L_j^{\frac{a}{\sigma-1}} P_j^a \right] \left[\tau_{ij}^{-a} F_{ij}^{-\left(\frac{a}{\sigma}-1\right)} f_{ij}^{-\left(1+\frac{a}{\sigma-1}-\frac{a}{\sigma}\right)} \right],$$

where κ_O is a function of parameters, as shown in the Online Appendix.²¹

The intensive margin can be also expressed as a log linear gravity equation. Using the expression for aggregate trade and the extensive margin (23), average firm-product exports conditional on positive trade are:

$$(24) \quad \frac{X_{ij}}{O_{ij}} = \frac{z\sigma}{z - (\sigma - 1)} w_{ifij},$$

which is independent of variable trade costs and destination market size for asymmetric countries, as was the case for symmetric countries.

In contrast, a firm's exports of a product with given attributes are monotonically decreasing in variable trade costs, as follows immediately from Equation 4, which also takes the form of a log linear gravity equation in source country characteristics, destination market characteristics, and bilateral trade costs.

We examine these differing predictions for the extensive and intensive margins in our empirical work using the United States as our source country i . We estimate the equations for the extensive and intensive margin, Equations 23 and 24, respectively, using observations across destination countries j . We also estimate the intensive margin equation (4) using firm-product-destination observations on exports.

V.C. Margins of Trade across Firms

In the model, differences in exports across firms are driven by the interaction of firm ability and product attributes. Although both firm ability and product attributes are unobservable, we show here that they have observable implications for firms' extensive and intensive margins that we examine in our empirical work.

For a given destination market, the total exports of a firm of ability φ ($r_{ij}(\varphi)$) can be decomposed into the measure of exported products (the extensive margin, $[1 - Z(\lambda_{ij}^*(\varphi))]$) and average firm-product exports conditional on positive trade (the intensive margin, $\bar{r}_{ij}(\varphi, \lambda)$):

$$r_{ij}(\varphi) = [1 - Z(\lambda_{ij}^*(\varphi))] \bar{r}_{ij}(\varphi, \lambda).$$

21. The extensive margin of the measure of exporters exhibits a similar log linear gravity equation relationship, which again includes source country characteristics, destination market characteristics, and bilateral trade costs, as also shown in the Online Appendix.

Under the assumption of Pareto distributions of firm ability and product attributes, the measure of exported products is an increasing log linear function of firm ability:

$$(25) \quad [1 - Z(\lambda_{ij}^*(\varphi))] = \frac{z - (\sigma - 1)}{\sigma - 1} \frac{F_{ij}}{f_{ij}} \left(\frac{\varphi}{\varphi_{ij}^*} \right)^z.$$

In contrast, average firm-product exports conditional on positive trade are independent of firm ability, as already noted:

$$(26) \quad \bar{r}_{ij}(\varphi, \lambda) = \int_{\lambda_{ij}^*(\varphi)}^{\infty} r_{ij}(\varphi, \lambda) \frac{z(\lambda)}{1 - Z(\lambda_{ij}^*(\varphi))} d\lambda = \frac{z\sigma}{z - (\sigma - 1)} w_{ij} f_{ij}.$$

Nonetheless, exports of a product with given attributes (λ) to a given market are an increasing log linear function of firm ability, as evident from Equation 4.

Each of these predictions can be related to observables. Both the number of products (25) and average firm-product exports conditional on positive trade (26) are observed. While exports of a product with given attributes (4) are not observed, because product attributes are not observed, the model can be used to derive an observable counterpart. Each firm draws the same distribution of attributes across products within each destination market. Therefore there is a one-to-one relationship in the model between the rank of a product in a firm’s sales within a given market and the attractiveness of its attributes. Hence the observable counterpart of the exports of a product with given attributes is the exports of a product of a particular rank in the firm’s exports to a given market (e.g., exports of the firm’s largest product).

The model’s observable implications for firms’ extensive and intensive margins can therefore be summarized as follows. First, both the number of products exported and exports of a firm’s largest product are increasing log linear functions of firm ability. Therefore these variables should be positively related to one another in the data. Second, under the assumption of Pareto distributions for ability and product attributes, average firm-product exports are unrelated to firm ability. Hence the number of products exported should have a weaker relationship in the data with average firm-product exports than with exports of the firm’s largest product.

VI. EMPIRICAL EVIDENCE

Our empirical analysis makes use of two data sets: the U.S. Linked/Longitudinal Firm Trade Transaction Database (LFTTD) and the U.S. Census of Manufactures (CM). The LFTTD captures all U.S. international trade transactions from 1992 through 2004 and is derived from customs forms. For each export transaction, we observe the identity of the firm responsible for the export, the export product's 10-digit Harmonized System (HS) product classification, the value shipped, the date of the shipment, and the destination country. Although for much of our analysis we use the universe of export transactions, in some cases we restrict the sample to firms exporting up to 10 products to conform to census disclosure requirements. In these cases, we report additional information on the representativeness of our findings for the universe of export transactions.

The quinquennial CM collects information on manufacturing establishments' inputs and output in each census year. For several of our findings, we link manufacturing establishments in the CM to firms in the LFTTD using the bridge developed by [Bernard, Jensen, and Schott \(2009\)](#).

We interpret the approximately 8,000 10-digit HS and 1,500 5-digit SIC codes used to classify exports and production, respectively, as discrete partitions of the model's continuum of products, which become coarser as one increases the level of aggregation. With this interpretation, the model provides a natural explanation for single- and multiproduct firms. We think of firms producing a single product as those whose range of products falls within a single classification code. Multiproduct firms, on the other hand, are those whose product range is wide enough to span several classification codes.

VI.A. Trade Liberalization and Product Range

We use the Canada–U.S. Free Trade Agreement (CUSFTA) as a natural experiment to examine the relationship between trade costs and firm scope summarized in Proposition 1. CUSFTA, signed in 1988, came into effect on January 1, 1989, and involved substantial tariff reductions for a number of goods ([Trefler 2004](#)). In contrast to many trade liberalizations in developing countries, CUSFTA involved a clearly defined change in trade policy that did not come as part of larger package of reforms. Furthermore, there is a close match between the reciprocal liberalization considered

in the model and CUSFTA's more-or-less symmetric trade cost reductions in the United States and Canada, which enhanced export opportunities for firms in both countries. As our trade data do not start until after CUSFTA is implemented, we examine its impact on U.S. firms' production. We combine data from the CM on the number of five-digit SIC goods firms produced in the years 1987 and 1992, before and after the introduction of CUSFTA, with data on reductions in Canadian tariffs on U.S. manufacturing imports measured at the four-digit SIC level over this period.²²

In developing the model in Section III, we focused for simplicity on a single industry, which contains a continuum of products and has given values of trade barriers. In relating the model to the data, we recognize that there can be multiple industries, which each contain a continuum of products and have different values of trade barriers.²³ Following a trade liberalization that is symmetric across countries, industries experiencing greater reductions in trade barriers exhibit, other things equal, greater increases in the zero-profit cutoffs for firm ability and product attributes, because of enhanced export opportunities. As a result, these industries display greater reductions in the range of products supplied by firms to the domestic market. In the data, we allow for variation in the size of tariff reductions across industries by measuring a firm's exposure to CUSFTA as the domestic-shipment weighted average of tariff reductions in the four-digit SIC industries in which the firm was active in 1987:

$$\Delta\text{Tariff}_f = \frac{\sum_i v_{fi}^{87} (\Delta\text{Tariff}_i)}{\sum_i v_{fi}^{87}},$$

where f and i index firms and four-digit SIC industries, respectively; v_{fi}^{87} represents firm domestic shipments in industry i in 1987; and ΔTariff_i is the percentage point change in the Canadian tariff rate on U.S. manufacturing imports in industry i between 1989 and 1992.²⁴ As our measure of firm exposure to tariff

22. Five-digit SIC products and four-digit SIC industries are defined consistently across years as in Bernard, Redding, and Schott (2010). U.S. manufacturing consists of approximately 1,500 five-digit SIC products and 450 four-digit industries.

23. In the Online Appendix, we consider one such multi-industry extension of the model, which focuses on the separate issue of Heckscher-Ohlin-based comparative advantage.

24. Industry-level Canadian tariff data are from Trefler (2004) and are available from 1989 to 1992. We note that we obtain—as expected for a largely reciprocal liberalization—similar results when using U.S. tariff changes on Canadian four-digit SIC imports over the same period.

reductions is a weighted average across industries, there is no necessary relationship between $\Delta Tariff_f$ and the number of industries in which a firm is active. Across industries, the mean tariff reduction is 4.0 percentage points with a standard deviation of 2.9 percentage points. Across surviving firms in our data, the mean and standard deviation of $\Delta Tariff_f$ is 3.1 and 2.4 percentage points, respectively. We note that in the regressions we use $-\Delta Tariff_f$ as a covariate so that increases in this variable represent greater tariff reductions, that is, greater increases in export opportunities.

Our empirical strategy involves a “differences-in-differences” specification, in which we compare the number of products produced by firms before and after CUSFTA (first difference) for firms experiencing above- versus below-median Canadian tariff reductions (second difference). This “differences-in-differences” specification can be represented as the following regression:

$$(27) \quad Products_{ft} = \beta (Post_t \times Exposure_f) + \eta_f + d_t + u_{ft},$$

where f , again, denotes firms, $Products_{ft}$ is the number of five-digit SIC products produced by a firm in 1987 and 1992; $Post_t$ is a dummy variable that equals 0 in 1987 prior to CUSFTA and 1 in 1992 afterward; $Exposure_f$ is a dummy variable that equals 1 if a firm experienced above-median Canadian tariff reductions between 1989 and 1992 and 0 otherwise; η_f are firm fixed effects that control for unobserved heterogeneity in the determinants of a firm’s number of products; d_t are time dummies that control for common macro shocks; and u_{ft} is a stochastic error.

As we have two cross-sections of data in 1987 and 1992, the fixed effects specification in Equation 27 has an equivalent representation in first differences. Taking first differences in Equation 27, the left-hand-side variable becomes the change in the number of products between the two years, the right-hand-side variable becomes the $Exposure_f$ dummy variable for whether a firm experienced above- or below-median Canadian tariff reductions, the firm fixed effects η_f difference out, and the time dummies d_t become a constant.²⁵ We cluster the standard errors in this first-differences

25. We find similar results using alternative cutoffs, for example, comparing firms experiencing Canadian tariff reductions above the 75th percentile to those experiencing Canadian tariff reductions below the 25th percentile. Results are also robust to including the number of products firms produce in 1987, and to using the log difference in the number of products produced rather than levels.

specification by firms' main four-digit SIC industry to account for the fact that our firm-specific measure of exposure to CUSFTA is constructed using four-digit SIC data on Canadian tariff reductions.

Results are reported in the first row of Table I. In column 1, we find that firms experiencing above-median Canadian tariff reductions reduce the number of products they produce relative to firms experiencing below-median Canadian tariff reductions. In columns 2 and 3, we show that this result is robust to including additional controls for firms' major four-digit industry and log 1987 employment as a measure of initial firm size. These findings are in line with our theoretical predictions in the common-product-attributes specification. They also accord with our theoretical predictions in the country-specific-product-attributes specification, as long as the addition of new products for the export market that are not supplied domestically is small relative to the reduction in the range of products supplied to the domestic market.

As a robustness check, the second row of the table replaces the number of products on the left-hand side of Equation 27 with an alternative measure of firm diversification used by Baldwin and Gu (2009). This "entropy" measure is defined as $\sum_k s_{fkt} \ln(s_{fkt})$, where s_{fkt} represents the share of firm shipments accounted for by five-digit SIC product k . It captures the extent to which a firm's

TABLE I
U.S. MANUFACTURING FIRM SCOPE DURING THE CANADA-U.S. FREE TRADE
AGREEMENT

	[1]	[2]	[3]
Change in products	-0.059	-0.624	-0.572
	0.015	0.101	0.096
Change in entropy	0.011	0.156	0.153
	0.003	0.026	0.026
Firm observations	66,472	66,472	66,472
Major industry dummy variables	No	Yes	Yes
Log 1987 employment	No	No	Yes

Notes. Table reports mean difference in noted variable between surviving firms experiencing above- and below-median changes in Canadian export opportunities between 1987 and 1992. Each cell reports the mean difference and associated standard error from a separate OLS regression. Change in products refers to change in number of five-digit SIC categories produced in the United States. Change in entropy is defined in the text. Change in export opportunities refers to the output-weighted average change in Canadian tariffs across the four-digit SIC industries produced by the firm. Robust standard errors are clustered according to firms' main four-digit SIC industry. Additional covariates are included as noted.

output is skewed toward its largest rather than its smallest products. In our model, trade liberalization increases this measure of skewness by reducing the range of products produced. Estimating the regression specification again in first differences, column 1 shows that firms experiencing above-median Canadian tariff reductions exhibit a rise in entropy, that is, an increased concentration of production in their largest products, relative to firms experiencing below-median Canadian tariff reductions. Columns 2 and 3 show that this finding is robust to controlling for major four-digit industry and initial firm size. Overall, both sets of empirical results provide support for the idea that trade liberalization induces firms to concentrate production in their most successful products.²⁶

One remaining concern is that firms experiencing above- and below-median Canadian tariff reductions could exhibit different trends in the number of products even prior to CUSFTA (pre-trends). To address this concern, we undertake a placebo analysis, where we regress the change in the number of a firm's products between the 1982 and 1987 manufacturing censuses (prior to CUSFTA) on our *Exposure_f* dummy variable based on Canadian tariff reductions between 1989 and 1992. We find a negative but statistically insignificant relationship for dropped products (-0.10 with standard error 0.16) in the specification in column 3 of Table I and a negative and statistically significant relationship with respect to entropy (-0.11 with standard error 0.05) in the specification in column 3 of Table I. Therefore there is no evidence of pre-trends, which is consistent with the idea that CUSFTA did indeed induce firms to concentrate production in their most successful products.

VI.B. Margins of Trade across Countries

We examine the relationship between variable trade costs and firms' intensive and extensive margins (Proposition 2), using the gravity equations derived in Section V.B. for asymmetric countries and Pareto distributions.

26. These results are consistent with Baldwin and Gu (2009)'s findings for the impact of CUSFTA in Canada. Additional evidence in support of the model's predictions comes from Iacovone and Javorcik (2010), who find a decline in the number of goods produced and a rise in the number of goods exported at Mexican firms following the North American Free Trade Agreement (NAFTA). For further supportive evidence using import penetration data, see Bowen and Wiersema (2005) and Liu (2010).

We use data on U.S. exports across countries. We control for source-country characteristics with a constant, and employ distance and GDP as proxies for variable trade costs and market size, respectively.²⁷

$$\ln Z_c = \zeta_0 + \zeta_1 \ln \text{distance}_c + \zeta_2 \ln \text{GDP}_c + \varepsilon_c.$$

Z_c is either aggregate trade (x_c), the number of firm-product observations with positive trade (o_c), or average firm-product exports conditional on trade being positive (\bar{x}_c), where $\bar{x}_c = x_c/o_c$. To examine the sources of variation in the extensive margin, we also decompose o_c into the number of firms (f_c), the number of products (p_c), and a density term that captures the extent to which each firm supplies each product ($d_c = o_c/(f_c p_c)$), where $o_c = f_c p_c d_c$. For brevity, we report regression results using 2002 data, but note that results for other years are similar.²⁸

The first column of Table II echoes the well-known result that destination-country exports decline with distance and increase with market size. Columns 2 and 3 decompose this relationship according to intensive (\bar{x}_c) and extensive (o_c) margins. Consistent with the forces of selection emphasized in the model, the negative relationship between export value and distance in column 1 is due entirely to the extensive margin. In contrast, the coefficient for the intensive margin of average firm-product-country exports conditional on positive trade is positive but not statistically significant. Results in the next three columns consider the three components of o_c . They show that both the number of firms and the number of products decline with distance. The opposite is true of density, because each firm is active in a limited subset of products, which implies that the number of firm-product observations with positive trade (o_c) increases less than proportionately than the number of firms times the number of products ($f_c p_c$) as market size increases. Summing the coefficients for density and the number of products, or density and the number of firms, we find that the number of products with positive exports per firm (o_c/f_c) and the number of firms with positive exports per product (o_c/p_c)

27. Distance is a widely used proxy for trade costs in the large gravity equation literature, as reviewed by [Disdier and Head \(2008\)](#). For direct evidence on the relationship between trade costs and distance, see, for example, [Hummels \(2001\)](#) and [Limao and Venables \(2001\)](#).

28. Distance data are from CEPII (see [Mayer and Zignago 2006](#)) and GDP data are from the World Bank's World Development Indicators database.

TABLE II
GRAVITY AND THE MARGINS OF U.S. EXPORTS

	ln(Value _c)	ln(Avg Exports _c)	ln(Obs _c)	ln(Firms _c)	ln(Products _c)	ln(Density _c)	ln(Value _{fp,c})
ln(Distance _c)	-1.37	0.05	-1.43	-1.17	-1.10	0.84	-0.18
	0.17	0.10	0.17	0.15	0.15	0.13	0.080
ln(GDP _c)	1.01	0.23	0.78	0.71	0.55	0.48	0.25
	0.04	0.02	0.04	0.03	0.03	0.03	0.020
Constant	7.82	6.03	1.80	0.52	3.48	-2.20	4.79
	1.83	1.07	1.81	1.59	1.55	1.37	0.64
Observations	175	175	175	175	175	175	1,878,532
Fixed effects	No	No	No	No	No	No	Firm-Product
R ²	0.82	0.37	0.75	0.76	0.68	0.66	0.70

Notes: Table reports results of OLS regressions of U.S. export value or its components on trading-partners' GDP and great-circle distance (in kilometers) from the United States. The first six columns are country-level regressions and final column is a firm-product-country level regression. Robust standard errors are noted below each coefficient; they are adjusted for clustering by country in the final column. Data are for 2002.

decline with distance, which is line with the model's predictions that higher variable trade costs reduce both these extensive margins.

After controlling for export composition, the model predicts a negative relationship between exports of a given product by a given firm and variable trade costs. To assess this prediction, the final column of the table reestimates gravity at the firm-product-country level and includes firm-product fixed effects, so that the coefficients on distance and market size are identified from variation across countries within firm-product pairs. Consistent with the model, we find a negative and statistically significant coefficient on distance in this specification.

VI.C. Margins of Trade across Firms

We next examine the model's predictions for the relationship between the intensive and extensive margins across firms (Proposition 3), as characterized for asymmetric countries and Pareto distributions in Section V.C.

We consider two measures of the extensive margin, the number of exported products and the number of export destinations, which are both monotonically increasing in unobserved firm ability in the model. We also consider two other observables influenced by firm ability: firms' total exports and measured productivity. One caveat with the latter is that our model, and research by others (e.g., [De Loecker 2007](#)), suggests that measuring multiproduct firms' productivity can be problematic unless data on inputs, outputs, and prices are all available at the firm-product level. Though our CM data do not include information on firms' inputs at the product level, we nevertheless consider total factor productivity (TFP) and labor productivity (domestic shipments per worker) as alternative empirical measures of firm ability.²⁹

29. We measure firm TFP as the shipment-weighted average TFP of its plants using data from the 1997 CM, the latest year for which matched CM-LFTTD data are available. Plant TFP in a given census year is measured relative to other plants in its main industry in percentage terms using the multifactor superlative index number of [Caves, Christensen, and Diewert \(1982\)](#). This index accounts for plants' use of capital, production workers, nonproduction workers, and materials. Plant shipments, capital, and materials are deflated according to the four-digit SIC deflator of its major industry using deflators provided by [Bartelsman, Becker, and Gray \(2000\)](#). Wages are deflated by the U.S. consumer price index available at www.bls.gov. A plant's main industry is the four-digit SIC in which it has the largest value of shipments.

As a first measure of the intensive margin, we use exports of a product of given rank in the firm's exports (e.g., exports of the firm's largest product), which is also monotonically increasing in unobserved firm ability in the model. Because this relationship holds within each market, we examine exports of the firm's largest product across all markets. As a second measure of the intensive margin, we use average exports per product, which is influenced by export composition and is unrelated to unobserved firm ability in the model under the assumption of Pareto distributions.

Table III reports correlations between the extensive and intensive margins in our data. Results are for 1997, the last year for which the CM and LFTTD have been merged. As indicated in columns 1 and 6, both the number of products exported (left panel) and the number of destinations served (right panel) exhibit the expected positive and statistically significant relationship with the size of firms' largest products. The remaining columns show that the extensive margins of products and destinations are positively and significantly related to other observable counterparts of firm ability.³⁰

To provide evidence on the role of export composition in influencing the intensive margin, Figure I plots the mean size of firms' largest export product against the mean size of their average export product for firms exporting up to 10 products in 2002.³¹ Results are presented for all export products (left panel) and for the much narrower set of Machinery and Electrical products (HS 84-85) exported to Canada (right panel). Vertical axes in both panels use logarithmic scales. Consistent with the compositional changes emphasized in the model, exports of a product of a given

30. Because only the CM contains the information on inputs needed to estimate productivity, for consistency we restrict our analysis throughout the table to the subset of exporting firms from the LFTTD that also appear in the CM. Results very similar to those reported in columns 1-2 and 6-7 are found when the same specification is applied to the larger set of firms that appear in the LFTTD, both in 1997 and across years.

31. Firms exporting up to 10 products account for 85% of exporting firms and 10% of U.S. export value. Though census disclosure guidelines preclude publication of a version of Figure I showing results for individual numbers of products greater than 10, we note that we find a similar relationship among firms exporting more than 10 products. For example, a firm-level regression of the log difference between the exports of firms' largest and average products on the number of products exported by the firm yields a positive and statistically significant relationship, as does an analogous regression using a dummy for whether a firm exports more than 10 products, as reported in the Online Appendix. We find similar relationships for other years and for other combinations of countries and products.

TABLE III
CORRELATION OF U.S. FIRMS' EXTENSIVE AND INTENSIVE MARGINS

	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]
	ln(Products _f)					ln(Countries _f)				
ln(Size of Largest Product _f)	0.345 0.003					0.329 0.006				
ln (Size of 5 th -Largest Product _f)		0.405 0.004								0.345 0.005
ln(Exports _f)			0.384 0.004					0.347 0.006		
ln(TFP _f)				0.071 0.022					0.076 0.022	
ln(Output _f /Worker _f)					0.474 0.019		0.426 0.020			
Constant	-2.300 0.061	0.405 0.004	-3.022 0.053	1.894 0.006	0.436 0.096	-2.714 0.078	-0.797 0.101	-3.141 0.072	1.292 0.006	-1.733 0.051
Observations	27,987 0.56	16,215 0.50	27,987 0.69	27,987 0.13	27,987 0.18	27,987 0.55	27,987 0.24	27,987 0.60	27,987 0.21	16,215 0.53

Notes: Table reports results of firm-level OLS regressions of the log number of 10-digit HS products exported by the firm, or log the number of destination countries served by the firm, on noted covariates. All regressions include dummies for firms' main four-digit SIC industry, and robust standard errors are clustered on this dimension of the data. Results in columns 2 and 7 are restricted to firms exporting at least five products. Data are for 1997.

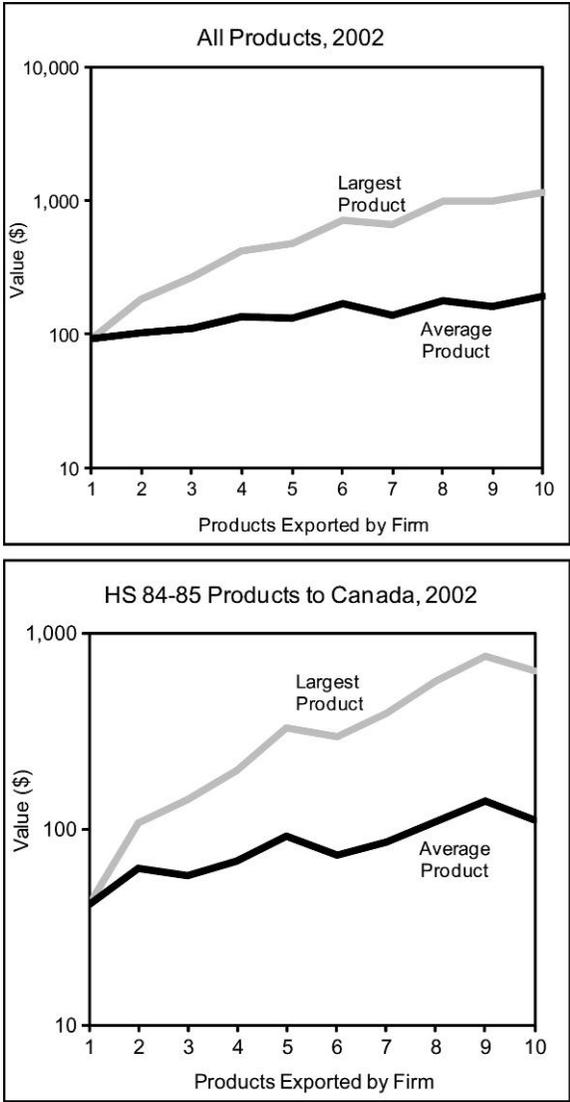


FIGURE I

“Exports of Firms’ Largest and Average Products by Number of Products Exported, 2002”

Figures display the mean value of firms’ largest and average export product across all firms exporting the noted number of products. The left panel displays trends for all exports. The right panel is restricted to exports of Machinery and Electrical products (HS 84-85) to Canada. Data are for 2002. Y-axes use log scales.

rank increase more sharply with the number of products exported than average exports of a product.³²

VI.D. Margins of Trade within Firms

In the model, the distribution of product attributes governs variation in exports across products within firms. Table IV documents the extent of this heterogeneity in exports within firms by reporting the average size distribution of products within firms across firms exporting 10 products in 2002.³³ As indicated in the table, the distribution of exports within firms is highly skewed, with the largest of a firm's products accounting for 49% of firm exports. The next two columns in the table demonstrate similar skewness when restricting observations to products exported to Canada and machinery products exported to Canada, respectively.

Under the assumption of a Pareto distribution of product attributes, the model implies a linear relationship between the log rank of products in firm exports and their log share of firm exports within destinations.³⁴ To assess this implication, we estimate an OLS regression of the log rank of products exported to Canada on their log share of firm exports to Canada using the data reported in the third column of Table IV. The fitted and actual values for log rank and log share are displayed in Figure II. Although the Pareto distribution provides a reasonable approximation to the data, actual values lie above the regression line in the middle of the distribution and below the regression line in the tails, implying thinner tails than the Pareto distribution.³⁵ These observed

32. As discussed in note 19, with Pareto-distributed product attributes and product fixed costs of exporting that are independent of product attributes, average exports per product are uncorrelated with the number of products exported. This implication is at odds with the moderate but statistically significant rise in average exports per product displayed in Figure I.

33. Similar heterogeneity is found among firms exporting fewer or more than 10 products. For example, the average size distribution across firms' top 10 products among firms exporting more than 10 products looks very similar to the distributions reported in Table IV, as reported in the Online Appendix. Again we find similar patterns across years.

34. If the distribution of firm exports across products within destinations is Pareto with minimum value k and shape parameter a , we have $\text{Prob}(x > x') = (k/x')^a$, where x denotes exports. Taking logarithms in this expression and rearranging terms yields the following relationships: $\log(\text{Rank}_p) = A - a \log(x_p) = B - a \log(\text{Share}_p)$, where Rank_p is the rank of x_p , $\text{Share}_p = x_p/X$, and A , B and $X = \sum_p x_p$ are constants.

35. Including a quadratic term in the log share of firm exports in the regression, we find that the null hypothesis of linearity implied by a Pareto distribution is rejected at conventional levels of statistical significance.

TABLE IV
DISTRIBUTION OF FIRM EXPORTS ACROSS PRODUCTS, 2002

Rank	All Exports	Products Exported to Canada	HS 84-85 Products Exported to Canada
1	49.0	47.4	47.9
2	18.6	19.4	19.3
3	10.5	11.1	11.0
4	6.7	7.0	7.0
5	4.6	4.8	4.7
6	3.4	3.4	3.3
7	2.5	2.5	2.4
8	1.9	1.9	1.8
9	1.5	1.5	1.4
10	1.1	1.1	1.1

Notes. Columns report the mean percent of firm exports represented by the product with the noted rank (from high to low) across firms exporting 10, 10-digit HS products in 2002. Second and third columns restrict observations to firms exporting 10 products to Canada, and firms exporting 10 Machinery and Electrical products (HS 84-85) to Canada, respectively. Sample sizes across the three columns are 1641, 983, and 322 firms, respectively.

departures from a Pareto distribution for exports within firms follow the same pattern as in the literature concerned with the distribution of sales across firms (see, for example, [Rossi-Hansberg and Wright 2007](#)).

VI.E. Hierarchies of Markets

When countries are asymmetric, the model predicts a hierarchy of markets in terms of their zero-profit cutoffs for product attributes. In the common-product-attributes specification, this hierarchy is strict. No product is exported to a less attractive market with a higher product cutoff that is not also exported to a more attractive market with a lower product cutoff. In contrast, in the country-specific-product attributes specification, this hierarchy is imperfect. A product can be exported to a market with a higher product cutoff and not exported to a market with a lower product cutoff, because realized values for product attributes vary across markets.

To provide evidence on the extent to which hierarchies of markets are observed in the data, we compare the markets to which a firm exports its largest product to the markets to which it exports all of its other, smaller products. If product attributes are common across countries, the destinations to which firms export

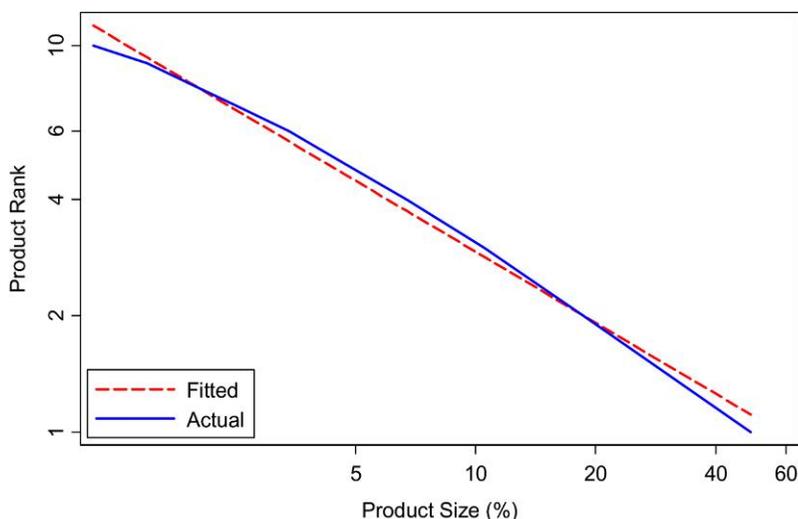


FIGURE II

Export-Product Rank versus Size (10-product exporters, 2002)

The solid line plots average within-firm product rank (low to high) against average within-firm product size across the 983 firms exporting 10 products to Canada. Both axes use log scales. The fitted line is the estimated relationship from an OLS regression of log product rank on log product size.

their smaller products should be subsets of the destinations to which they export their largest product. Empirically, we define a firm's largest product as the one it sends to the largest number of export destinations, with ties going to the product with the greatest export value. Let n_{kf} be the number of export destinations for smaller product k in firm f and \tilde{n}_{kf} be the number of destinations smaller product k has in common with the firm's largest product. Then $Nested_{kf} = \tilde{n}_{kf} / n_{kf}$ represents the share of "smaller" product k 's markets that are common to the firm's largest product. If firms' products exhibit a strict hierarchy, one would expect $Nested_{kf} = 1$. Instead, we find a mean value of 0.67 across the 151,204 firm-product observations in our sample, which bootstrap standard errors reveal to be significantly different from 1.³⁶

Although these findings are suggestive, one concern is that this specification does not control for variation in country-product

36. Averaging $Nested_{kf}$ first across products within firms and then across firms yields a value of 0.59, which bootstrapping also reveals is significantly different from unity. Results are similar if we restrict the sample to firms exporting 10 or more products.

attributes that are common to all firms but vary across destination markets, such as U.S. comparative advantage. To address this concern, we use the property of the model that a firm's exports of a given product to a given destination (4) are log linear in a firm's ability and product attributes, as well as in aggregate characteristics of the source and destination country (e.g., the price index that is influenced by comparative advantage). We examine the contribution of these components toward variation in U.S. exports by regressing log firm-product-country export value on a series of progressively more detailed fixed effects. Given a single source country (here, the United States), aggregate characteristics of the source and destination country can be captured by destination-product fixed effects. Because firm ability is common across destinations and products, it can be captured by a firm fixed effect. To the extent that product attributes are common across destinations for a given firm and product, they can be captured by a firm-product fixed effect.

Table V reports results using data for 2002, but results are similar across years. As noted in the first row of the table, country-product fixed effects explain 26% of the variation in log exports. Including firm fixed effects, in row 2, increases the regression R^2 to 41%, indicating that firm-level factors that are common across products and countries explain roughly 20% (15/74) of the remaining variation in log exports.

Augmenting the regression with firm-product rather than firm fixed effects increases the regression R^2 to 79%, highlighting the importance of product selection within firms. At the same time, this specification reveals that 29% (21/74) of the variation in U.S. exports that remains after including country-product fixed effects is due to idiosyncratic heterogeneity across countries within

TABLE V
COMMON AND IDIOSYNCRATIC COMPONENTS OF FIRM-PRODUCT SHIPMENTS

Fixed Effects	Cumulative R^2 -Squared
Country-Product	0.26
Country-Product + Firm	0.41
Country-Product + Firm-Product	0.79

Notes. Table reports R^2 s of a sequence of OLS regressions of the logarithm of U.S. firm-product-country exports in 2002 on the fixed effects noted in each row.

firm-products.³⁷ More generally, these results suggest that heterogeneity within firms makes a contribution of comparable magnitude to heterogeneity across firms in explaining cross-section variation in international trade.

VII. CONCLUSION

This article develops a general equilibrium model of multiple-product, multiple-destination firms and shows that the model provides a natural explanation for key features of the distribution of exports across firms, products, and countries. Through modeling firms' decisions about the optimal range of products to produce and export to each market, we both account for new features of disaggregated trade data and illuminate the mechanisms underlying more aggregate economic relationships.

Across countries, we find that the negative effect of distance on bilateral trade is largely explained by the extensive margins of the numbers of firms and products. Although distance reduces the intensive margin of exports of a given product by a given firm, average firm-product exports are largely uncorrelated with distance because of endogenous changes in export composition. Across firms, we find that trade liberalization induces firms to reduce the range of products they produce, and firms exporting many products also serve many destinations and export more of a given product to a given destination. Within firms, the distribution of exports across products is highly skewed, and product selection accounts for a substantial proportion of the overall variance of exports.

While a large theoretical and empirical literature has emphasized the importance of selection across firms in shaping the determinants and effects of trade, our analysis highlights the role of selection across products and countries within firms. These processes of selection within firms are worthy of further inquiry because they shape firm, industry, and aggregate productivity. A deeper understanding of the role of the extensive and intensive margins of trade can help, for example, shed light on the

37. These findings complement [Munch and Nguyen's \(2008\)](#) findings of imperfectly correlated firm rankings across export markets in Danish export data; [Eaton, Kortum, and Kramarz's \(2008\)](#) findings of idiosyncratic heterogeneity across countries within firms using French export data; and [Demidova, Kee, and Krishna's \(2009\)](#) findings of country-specific demand shocks for Bangladeshi apparel exporters.

mechanisms through which trade costs affect trade flows and the channels through which policy barriers more generally affect the allocation of resources to their most efficient use. A key implication of our findings is that reallocation may be even more important than hitherto thought in so far as it occurs within as well as across firms.

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