The Origins of Firm Heterogeneity: A Production Network Approach*

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Abstract

This paper quantifies the origins of firm size heterogeneity when firms are interconnected in a production network. We document new stylized facts about the universe of buyer-supplier relationships among all firms in Belgium during 2002-2014. These facts motivate a model in which firms buy inputs from upstream suppliers and sell to downstream buyers and final demand. Firms can be large not only because they have high production capability (i.e. productivity or product quality), but also because they interact with more, better and larger buyers and suppliers, and because they are better matched to their buyers and suppliers. The model delivers an exact decomposition of firm size into upstream and downstream margins with firm, buyer/supplier and match components. We establish three empirical results. First, downstream factors explain the vast majority of firm size heterogeneity, while upstream factors are only one fourth as important. Second, nearly all the variation on the downstream side is driven by network sales to other firms rather than final demand. By contrast, most of the variation on the upstream side reflects own production capability rather than network purchases from input suppliers. Third, most of the variance in network sales is determined by the number of buyers and the allocation of sales towards well-matched buyers of high quality, rather than by average buyer capability. By contrast, most of the variance in network purchases comes from average supplier capability and the allocation of purchases towards well-matched suppliers of high quality, rather than from the number of suppliers.

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1 Introduction

Why are firms large or small? Even within narrowly defined industries, there is evidence of massive dispersion in firm outcomes such as revenue, employment, labor productivity or measured total factor productivity (see Syverson (2011) for a recent overview). In Belgium, a firm at the 90th percentile of the size distribution has turnover more than 36 times greater than a firm at the 10th percentile in the same industry.[1] Understanding the origins of firm heterogeneity has important micro- and macro-economic implications. At the micro level, bigger firms perform systematically better along many dimensions, such as survival rate, innovation activity, and participation in international trade (e.g., Bernard et al. (2012a)). At the macro level, the skewness and granularity of the firm size distribution affect aggregate productivity, the welfare gains from trade, and the impact of idiosyncratic and systemic shocks (e.g., Melitz and Redding (2015), Pavcnik (2002), Gabaix (2011), di Giovanni et al. (2014), Gaubert and Itskhoki (2016)).

While the literature has made progress in identifying underlying firm-specific supply- and demand side factors driving firm size (e.g., Hottman et al., 2016), much less is known about the role of firm-to-firm linkages in production networks. In particular, the focus has been on one-sided heterogeneity in either firm productivity on the supply side (e.g., Jovanovic (1982), Hopenhayn (1992), Melitz (2003), Luttmer (2007)) or final-consumer preferences on the demand side (e.g., Foster et al. (2016), Fitzgerald et al. (2016)). To the extent that the literature has considered firm-to-firm trade, it has typically remained anchored in one-sided heterogeneity by assuming that firms source inputs from anonymous upstream suppliers or sell to anonymous downstream buyers, without accounting for the heterogeneity of all trade partners in the production network.

This paper examines how buyer-supplier connections in a complete production network shape the firm size distribution in the cross-section and its evolution over time.[2] The basic premise of the analysis is intuitive: firms can become large because they have inherently attractive capabilities such as productivity or product quality, because they interact with better and larger buyers and suppliers, and/or because they are particularly well matched to their buyers and suppliers. Alternatively, firms can improve their product quality or reduce their marginal costs if they enhance their own capabilities or if they buy more inputs from high-quality, efficient suppliers. Firms can expand sales if they appeal to more final consumers or if they match with more and with bigger downstream producers. There may be higher-order effects in a production network as well, because the customers of the customers

[1] Averaged across all NACE 4-digit industries in Belgium in 2014.

[2] Throughout the paper, firm size, sales, revenues and turnover are used interchangeably.

(and so on) of any one firm may ultimately also matter for that firm's economic performance.

The paper makes four main contributions. First, we document new stylized facts about a complete production network using 2002-2014 panel data on the universe of firm-to-firm domestic transactions in Belgium. Second, we provide a theoretical framework with minimal assumptions on production and demand that relates firm size to firm-specific characteristics, buyer and supplier characteristics, and buyer-supplier match characteristics. This allows the development of a new methodology for structurally estimating the primitives of the model from production network data. Third, we implement this methodology to decompose the Belgian firm size distribution into downstream and upstream components and to quantify the role of different firm-, buyer- and supplier characteristics. Finally, we use the model to simulate counterfactual shocks to firm capabilities and match quality, and assess the welfare impact of policy-relevant shocks to the production network.

We first document three stylized facts about the incidence, magnitude and two-sided heterogeneity of firm-to-firm transactions in a complete domestic production network, using comprehensive value-added tax (VAT) records for Belgium during 2002-2014. First, the distributions of firms' total sales, number of buyer- and supplier connections, and value of buyer-supplier bilateral sales exhibit high dispersion and skewness. Second, bigger firms have more upstream suppliers and downstream buyers. Third, the distribution of a firm's sales across its buyers does not vary with the number of its buyers, while the distribution of its purchases across its suppliers widens with the number of its suppliers. Together, these patterns suggest that the network of buyer-supplier links is key to understanding the firm size distribution.

Motivated by the stylized facts, we develop a theoretical framework that features two-sided firm heterogeneity in an input-output production network. This allows us to decompose firm sales into economically meaningful demand- and supply-side fundamentals. In the model, firms use a constant elasticity of substitution production technology that combines labor and inputs from upstream suppliers. Firms sell their output to final consumers, as well as to downstream domestic producers. Since we want to examine how the network contributes to size dispersion, we take the observed production network as given and do not model the firm-to-firm matching decision. However, key firm metrics such as marginal costs, employment, prices, and sales are nevertheless endogenous outcomes because they depend on the outcomes of all other firms in the economy.

In the framework, firms differ in production capability (a combination of efficiency and quality), as well as in sourcing capability (an input price aggregate that reflects the number and production capabilities of input suppliers). The value of a given firm-to-firm transaction depends on the production capability of the seller, the sourcing capability of the buyer, and the match quality of the specific seller-buyer pair. A new connection between two firms increases the total sales of both the seller and the buyer; for the seller this occurs mechanically because it gains a customer, while for the buyer this arises because a larger supplier base implies greater opportunities to source cheaper or higher-quality inputs.

At the firm level, total firm sales can thus be decomposed into two overall margins: upstream and downstream. The upstream margin can be further decomposed into own production capability and network supply (i.e. input costs), where the latter comprises the number of upstream suppliers, average production capability across suppliers, and the covariance of production capability and match quality across suppliers. Likewise, the downstream margin can be further decomposed into final demand and sales in the production network, where the latter comprises the number of downstream buyers, average sourcing capability across buyers, and the covariance of sourcing capability and match quality across buyers.[3]

We develop a three-step methodology to perform the exact model-based decomposition of firm size using the uniquely rich Belgian data. In the first step, we regress the value of bilateral firm-to-firm transactions on seller and buyer fixed effects, where the residual represents the bilateral match-specific component of the transactions. In the second step, we back out primitives of the model from these three terms. Intuitively, the seller and buyer fixed effects are related to firms' production and sourcing capability respectively, while the residual isolates firm-pair match quality. We also use balance sheet data to incorporate information on firms' activity outside the domestic production network (i.e. labor on the production side; sales to final consumers on the sales side). In the last step, we construct all components of the firm size decomposition, and regress each one on total firm sales. The coefficient estimates from this regression capture the contribution of each upstream- and downstream margin to the overall variation in firm size.[4]

This methodology has several appealing features. It demonstrates how to use production network data to break the reflection problem of distinguishing producers' own capability from that of their suppliers. It also provides an agnostic decomposition of firm size as it imposes no restrictions on the relative magnitude of different margins. This decomposition is moreover conceptually valid under alternative assumptions about the market structure (e.g. with or without monopolistic competition; with or without constant mark-ups). Finally, although we treat the production network as pre-determined, the approach produces unbiased estimates of its contribution to the overall firm size variation. Implicitly, this reflects the role of the

[3] While the firm size decomposition explicitly accounts only for direct linkages to firms' immediate buyers and suppliers, the complete network of firm-to-firm links is implicitly captured by the sourcing capability of all participants in the network.

[4] This variance decomposition is similar in spirit to Redding and Weinstein (2017).

intensive margin of firm connections even if firms endogenously match based on firm-specific attributes or firm-pair specific matching shocks, so long as these shocks are not correlated with a pairwise sales residual. Otherwise, the network component would reflect both the intensive and the extensive margins of firm connections. We explore this with exogenous mobility tests which speak against the latter.

We establish three main empirical results about the sources of firm size heterogeneity. We report results for 2014 but all of these results hold both in the cross-section of firms at a given point in time and in the evolution of firm size within firms over time. First, downstream factors explain the vast majority of firm size dispersion (82%), while upstream factors contribute significantly less (18%). Second, most of the variation on the downstream side is driven by network sales to other firms rather than final demand. On the upstream side by contrast, the variation is dominated by own production capability rather than network purchases from input suppliers. Overall, firm-to-firm linkages in the production network account for fully 83.7% of the firm size dispersion in the data. Together, these two results imply that trade in intermediate goods and firm-to-firm connections are essential to understanding firm-level performance and consequently aggregate outcomes. Models that feature only supply-side factors such as firm productivity or that ignore the input-output structure of the economy would thus fail to capture the vast majority of firm size heterogeneity.

Third, most of the variance in network sales is determined by the number of buyers (extensive margin) and the allocation of activity towards well-matched partners of high quality (covariance term), rather than by average partner capability (intensive margin). On the other hand, most of the variance in network purchases is determined by the covariance term and the intensive margin, rather than by the extensive margin. The main reason why the production network enables firms to sell more downstream is because they can sell to more buyers, and not because their buyers tend to purchase more intermediates. Firms also sell more when their products are especially well suited to the production needs of highly capable buyers. On the upstream side, the production network helps firms reduce marginal cost or improve quality because their suppliers are better on average, and not because they can match with many suppliers. Firms also benefit more when their production needs are especially well served by highly capable suppliers.

This paper contributes to several strands of literature. Most directly, the paper adds to the large literature on the extent, causes and consequences of firm size heterogeneity. The vast dispersion in firm size has long been documented, with a recent emphasis on the skewness and granularity of firms at the top end of the size distribution (e.g., Gibrat (1931), Syverson (2011)). This interest is motivated by the superior growth and profit performance of bigger firms at the micro level, as well as by the implications of firm heterogeneity and superstar firms for aggregate productivity, growth, international trade, and adjustment to various shocks (e.g., Bernard et al. (2012a), Gabaix (2011), Freund and Pierola (2015), Gaubert and Itskhoki (2016)).

Traditionally, this literature has looked to own-firm characteristics on the supply side as the driver of firm size heterogeneity. The evidence indicates an important role for firms' production efficiency, management ability, and capacity for quality products (e.g., Jovanovic (1982), Hopenhayn (1992), Melitz (2003), Sutton (2007), Bender et al. (2016)). Recent work has built on this by also considering the role of either upstream suppliers or downstream demand heterogeneity, but not both. Results suggest that access to inputs from domestic and foreign suppliers matters for firms' marginal costs and product quality, and thereby performance (e.g., Goldberg et al. (2010), Manova et al. (2015), Fieler et al. (ming), Bernard et al. (2015), Antràs et al. (2017)), while final-consumer preferences affect sales on the demand side (e.g., Foster et al. (2016), Fitzgerald et al. (2016)).

By contrast, we provide a comprehensive treatment of both own firm characteristics and production network features, on both the upstream and the downstream sides. The paper is thus related to Hottman et al. (2016) who also find that demand rather than supply is the primary factor driving firm size dispersion. However, as they do not observe the production network, they cannot distinguish between the impact of serving more customers, attracting better customers, and selling large amounts to (potentially few) customers. Since they have no information on the supplier margin, they also cannot compare own vs. network supply factors.

The paper also adds to a growing literature on buyer-supplier production networks (see Bernard et al. (2017) for a recent survey). On the empirical side, Bernard et al. (2015) study the impact of domestic supplier connections on firms' marginal costs and performance in Japan, whereas Bernard et al. (ming) and Eaton et al. (2016) explore the matching of exporters and importers using data on firm-to-firm trade transactions for Norway and US-Colombia, respectively. While we confirm some of the findings in these papers about the distributions of buyers and suppliers, we examine transaction-level data on a complete domestic production network and focus on the implications of two-sided heterogeneity and production networks for the firm size distribution. Using the Belgian production network data, Magerman et al. (2016) analyzes the contribution of the network structure of production to aggregate fluctuations.

Finally, the methodology in this paper is related to the econometrics of two-sided heterogeneity in other economic contexts (see Arellano and Bonhomme (2017) for a review). In particular, we estimate seller fixed effects, buyer fixed effects, and residual seller-buyer match effects from data on seller-buyer sales. This is similar in spirit to gravity models of international trade flows by exporting country - importing country pair, where exporter, importer and bilateral characteristics play a role (e.g., Helpman et al. (2008)). Another recent contribution is Kramarz et al. (2016), who estimate buyer and seller effects in a bipartite trade network. Our work also builds on employer-employee matching models in the labor literature (e.g., Abowd et al. (1999), Card et al. (2013)). However, each economic agent plays a unique role in the labor market - either a firm or a worker - such that both panel data and worker transitions across firms are necessary to identify the employer, employee and match effects. By contrast, each firm can in principle be both a buyer and a supplier in a production network, such that cross-sectional data is sufficient to identify the effects of interest.

The rest of the paper is organized as follows. Section 2 introduces the data and presents novel stylized facts. Section 3 outlines the theoretical framework. Section 4 operationalizes the first two steps of the estimation strategy to construct all necessary firm size components from the data. Section 5 presents the results of the firm size decomposition. Section 6 provides a general equilibrium formulation of the model which enables counterfactual welfare exercises in Section 7. The last section concludes.

2 The Belgian Production Network

2.1 Data

We exploit several comprehensive data sources on annual firm operations in Belgium over the 2002-2014 period: (i) the NBB B2B Transactions Dataset, containing the universe of domestic firm-to-firm sales relationships, (ii) annual accounts, with typical firm characteristics for firms above a minimum size threshold, (iii) VAT declarations, with more limited firm characteristics for small firms, and (iv) the Crossroads Bank of Enterprises dataset, containing firms' sector affiliation and geographic location. Unique firm identification numbers allow us to unambiguously match these datasets. We can thus examine an entire economy in unprecedented detail: we observe the complete domestic production network in Belgium, with information on seller firm characteristics, buyer firm characteristics, and seller-buyer transaction values.

The primary data source is the NBB B2B Transactions Dataset, administered by the National Bank of Belgium (NBB), which documents both the extensive and the intensive margins of domestic buyer-supplier relationships in Belgium.[5] The dataset reports the sales relationships between any two VAT-liable enterprises across all economic activities in Bel-

[5] See Dhyne et al. (2015) for details on the construction of this dataset.

gium.[6] In particular, an observation is the sum m_{ij} of sales invoices (in euro, excluding any value-added tax due) from enterprise i to enterprise j in a given calendar year.[7] Observations are directed, as $m_{ij} \neq m_{ji}$. Coverage is quasi universal, as all annual sales of at least 250 euros must be reported, and pecuniary sanctions on late and erroneous reporting ensure very high data quality.[8]

We use data on total sales (turnover), total input purchases, employment and labor costs from firm annual accounts maintained by the Central Balance Sheet Office (CBSO) at the NBB.[9] Annual accounts are collected by fiscal year and have been annualized to match the calendar year in the NBB B2B data. Since there is a firm-size threshold for reporting turnover and input purchases to CBSO, we access data on these two variables for small firms below the threshold from firms' VAT declarations. We keep only firms with at least one full-time equivalent employee. We observe the main economic activity of each enterprise at the NACE 4-digit level (harmonized over time to the NACE Rev. 2 (2008) version) and its geographic location at the zip-code level from the Crossroads Bank of Enterprises.

We combine these data sources to create the variables necessary for the firm size decomposition in Section 5. We construct firms' sales to final demand as the difference between their turnover and the sum of all their B2B sales to other enterprises in the domestic production network. Final demand thus contains sales to final consumers at home, potentially unobserved links in B2B with small transaction values, and exports. We likewise measure firms' purchases from outside the observed production network (including imports) as the difference between their total input costs and the sum of all their B2B purchases.[10] We compute the labor share in production at the NACE 4-digit level as the sum of total employment

[6] We use "enterprise" and "firm" interchangeably in this paper. The unit of observation is the unique firm identification number, which corresponds to the legal entity of the enterprise. Hence, we take the information on firms and their relationships as they are presented in the data, and do not consider individual plants, establishments, or conversely groups of firms that might be (in)directly owned through financial participations. Our assumption is that all production decisions are made at the level of the legal entity.

[7] While we do not observe the specific product content of each transaction, our analysis does not require such information. Our theoretical and empirical approach builds on the premise that firms assemble multiple inputs potentially sourced from multiple suppliers into a single product that they sell to other firms and to final consumers.

[8] While it is impossible to compare aggregated micro data to the national accounts of economic activity due to the different methodologies used by these data sources, the two aggregates are very close and have similar growth rates. See Dhyne et al. (2015) for further details.

[9] Total input purchases are the sum of material and service inputs, and include both new inputs and net changes in input stocks. Employment is reported as average full-time equivalent employees. Total labor costs include wages, social security, and pension contributions.

[10] Since our estimation procedure requires a connected network component of firm relationships, some isolated B2B links (less than 1%) drop out from the analysis (see Section 4.1). To ensure full internal consistency, we calculate final demand and unobserved inputs based on the B2B links that we keep in the analysis. The value of any dropped B2B relationships thus accrues to final demand and outside-network input purchases.

expenses across all firms in a sector, divided by total turnover in that sector.[11] Similarly, we calculate average wages by sector as the sum of total labor costs divided by total employment. In robustness exercises, we alternatively use NACE 2-digit sector averages, as well as information on firms' zip codes to calculate the bilateral distance between any two enterprises in Belgium.

Finally, for the counterfactual analysis in general equilibrium, we calculate firm-level markups as turnover over total input costs, and obtain aggregate final consumption by summing over all firms' final demand. We provide further details on data coverage and preparation in Appendix B.

2.2 Stylized Facts

We document three stylized facts about firm size and firm linkages in the Belgian domestic production network.[12] These facts provide evidence that buyer-supplier relationships are key to understanding the firm size dispersion in an economy, and motivate the subsequent theoretical and empirical analysis. We present cross-sectional evidence for the most recent year in our sample, 2014, but the patterns we establish are stable over the 2002-2014 period.

Fact 1. *The distributions of firms' total sales, buyer-supplier connections, and buyer-supplier bilateral sales exhibit high dispersion and skewness.*

Firm size varies dramatically in Belgium, as in other countries. Table 1 provides summary statistics for firm sales in 2014, both overall and within six broad industries (primary and extraction, manufacturing, utilities, construction, market services, and non-market services).[13] Across the 109,908 firms with sales data that are active in the production network, average turnover was 6.7 million euro, with a standard deviation of 145 million euro. Similar patterns hold within each broad industry category, although there is substantial heterogeneity across industries. The biggest number of firms is active in market services, while there are few firms in utilities. At the same time, firms in utilities are on average much larger than those in market services or other industries.

The cross-sectional distribution is, however, extremely skewed. Overall, firms at the 90th percentile generate turnover over 36 times higher than firms at the 10th percentile, while the top 10% of firms account for 84% of aggregate sales. Although there is some variation

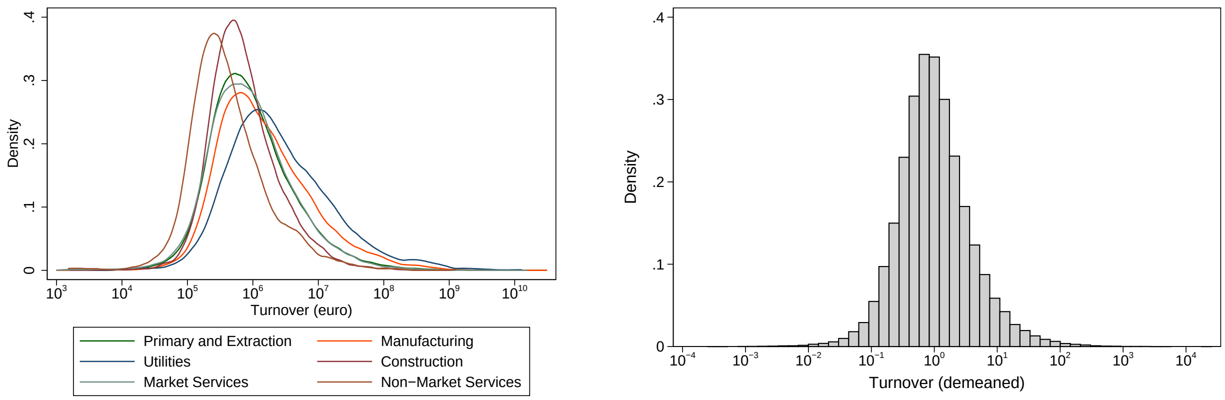
[11] Our assumption on the Cobb-Douglas upper tier of the production function implies that these shares are also the elasticity of output with respect to employment at the firm level.

[12] These stylized facts echo patterns established for the extensive margin of firm-to-firm linkages in the domestic production network in Japan (Bernard et al. (2015)) and for both the extensive and the intensive margins of firm-to-firm export transactions in Norway (**?**).

[13] See Table 13 in Appendix B for the classification of industry groups at the 2-digit NACE level.

Figure 1: Firm sales distribution (2014).

(a) Across industry groups. (b) Within NACE 4-digit industries.



in average firm size across industry groups, the dispersion is similar, with large firms being up to four orders of magnitude bigger than their industry mean, as shown in Figure 1a. The histogram in Figure 1b illustrates the full firm size distribution, after demeaning at the NACE 4-digit sector level..[14] Even within narrowly defined sectors, these patterns remain, with some firms generating turnover 10,000 times larger than their sector average.

Turning to firm-to-firm connections in the domestic production network, we find that the number of downstream customers per seller (out-degree) and the number of upstream suppliers per buyer (in-degree) are also very skewed. In 2014, we observe 17.3 million sales relationships among 859,733 firms within Belgium.[15] Of these, 590,271 enterprises sell to other firms in the network, while 840,607 buy from other firms in the network. Hence 31.5% of firms sell only to final demand, while a small minority of 2.2% do not purchase inputs from the domestic production network (or do so in an amount less than 250 euro). Conditional on trading with others in the network, 74% of producers have more than one supplier and 88% have more than one buyer.

Table 2 summarizes the overall distribution of buyer and supplier connections , as well as by broad industry. Across all sellers, the average number of customers is 29.3, with a standard deviation of 394. Across all buyers, the average number of suppliers is 20.6, with a standard deviation of 49.5. The average firm thus has more buyers than suppliers, and the distribution of buyers per seller is more dispersed than that of suppliers per buyer. Firm-to-firm links in the network are also highly concentrated among a few very connected

[14] All results reported in this section hold whether we demean by NACE 4-digit or NACE 2-digit industry.

[15] The number of firms in the B2B production network is much larger than the number of firms in the matched B2B-CBSO samplewith turnover data, because B2B contains many small firms that do not have to submit full annual accounts to CBSO.

Table 1: Firm sales (million euro, 2014).

Industry	N	Mean	St Dev	10th	25th	50th	75th	90th	95th	99th
Primary and Extraction (NACE 01-09)	3,063	11.9	433	0.2	0.4	0.7	1.9	4.8	9.5	52.0
Manufacturing (NACE 10-33)	18,086	14.4	251	0.2	0.5	1.1	3.8	13.8	34.6	202
Utilities (NACE 35-39)	897	39.2	443	0.3	0.7	1.9	6.9	25.7	68.6	496
Construction (NACE 41-43)	20,206	2.3	13.4	0.2	0.3	0.6	1.4	3.6	6.9	25.9
Market Services (NACE 45-82)	65,323	5.5	79.8	0.2	0.3	0.8	2.1	6.3	13.3	63.8
Non-Market Services (NACE 84-99)	2,333	2.2	26.2	0.1	0.2	0.3	0.8	2.6	5.5	24.9
All	109,908	6.7	145	0.2	0.3	0.8	2.1	6.6	14.3	78.3

Note: Summary statistics for the matched CBSO-B2B data. 10th, 25th, etc. refers to values at the 10th, 25th, etc. percentile of the distribution.

Table 2: Number of firm buyers and suppliers (2014).

(a) Number of downstream buyers.

Industry	N	Mean	St Dev	10th	25th	50th	75th	90th	95th	99th
Primary and Extraction (NACE 01-09)	50,706	12.1	60.1	1	2	4	8	18	40	154
Manufacturing (NACE 10-33)	57,976	47.5	284.9	1	2	7	26	98	192	603
Utilities (NACE 35-39)	2,734	192.7	3,305	1	2	6.5	36	154	336	1,514
Construction (NACE 41-43)	104,566	14.6	107.9	1	2	4	10	24	45	174
Market Services (NACE 45-82)	351,773	32.9	394.6	1	1	3	11	48	112	453
Non-Market Services (NACE 84-96)	22,516	14.1	183.1	1	1	2	6	19	38	153
All	590,271	29.3	394	1	1	4	11	42	98	400

(b) Number of upstream suppliers.

Industry	N	Mean	St Dev	10th	25th	50th	75th	90th	95th	99th
Primary and Extraction (NACE 01-09)	60,508	20.5	29.6	2	5	13	27	44	57	117
Manufacturing (NACE 10-33)	72,698	38	89.5	2	5	15	38	89	148	348
Utilities (NACE 35-39)	3,401	62.8	180.7	2	4	14	55	146	235	757
Construction (NACE 41-43)	130,358	24.5	48.3	2	5	13	29	52	77	178
Market Services (NACE 45-82)	506,145	18.3	41.5	1	3	8	19	42	64	150
Non-Market Services (NACE 84-96)	67,497	9.7	37.1	1	2	4	9	19	30	90
All	840,607	20.6	49.5	1	3	9	22	46	71	177

Note: Summary statistics for the B2B data. 10th, 25th, etc. refers to values at the 10th, 25th, etc. percentile of the distribution.

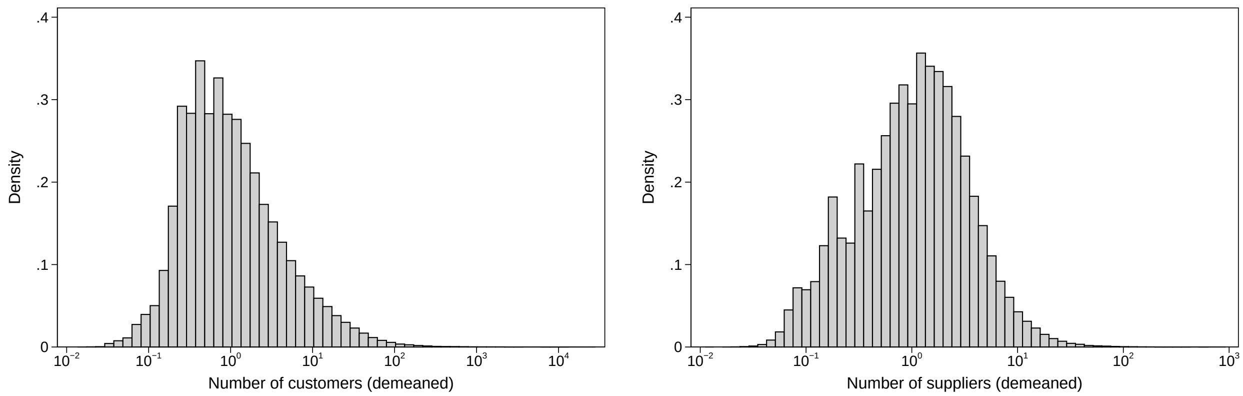
participants: The median number of customers and suppliers is only 4 and 9 respectively, while the top 1 percent of firms transact with more than 400 buyers and 177 sellers. This dispersion and skewness across firms within NACE 4-digit industries is also evident in the histograms in Figure 2. Again, firms with the most customers or suppliers are several orders of magnitude more connected than the average firm, in their industry.

Of note, the in-degree and out-degree distributions have similar features within different broad industries, but they also display some heterogeneity in line with priors. For example, the number of buyers and suppliers is highest for firms in utilities, which are followed closely by manufacturing firms. These numbers are intermediate for producers in primary materials and extraction, and lowest among service providers.

The intensive margin of firm-to-firm bilateral sales is also very dispersed and skewed, with the vast share of economic activity concentrated in a small number of buyer-supplier transactions, as demonstrated in Table 3. The mean transaction across the 17,304,408 buyer-supplier links in 2014 amounts to 28,893 euro. At the same time, the median purchase totals only 1,392 euro, while the standard deviation reaches nearly 3 million euro and the top 10% of relationships account for 92% of all domestic firm-to-firm sales by value. This dispersion in transaction values in a buyer-supplier production network was first documented in the

Figure 2: Distribution of firm buyer and supplier connections (2014).

(a) Number of downstream buyers. (b) Number of upstream suppliers.



Note: The number of customers and suppliers is demeaned at the NACE 4-digit level.

Table 3: Firm-to-firm transaction values (euro, 2014).

Industry	N	Mean	St Dev	10th	25th	50th	75th	90th	95th	99th
Primary and Extraction	613,868	39,898	5,409,863	419	840	2,490	9,150	33,789	81,626	387,573
Manufacturing	2,755,457	44,303	2,007,421	359	613	1,661	6,185	25,436	63,467	411,379
Utilities	526,932	59,953	7,410,682	366	615	1,388	3,744	11,560	28,382	281,181
Construction	1,529,078	24,500	386,201	375	676	1,926	7,000	27,186	64,585	339,523
Market Services	11,562,445	24,373	2,886,213	341	546	1,266	4,060	15,579	37,960	224,363
Non-Market Services	316,628	8,036	318,863	315	472	996	2,736	8,396	18,920	92,732
All	17,304,408	28,893	2,988,881	348	571	1,392	4,669	18,280	44,770	269,153

Note: Summary statistics for the B2B data. 10th, 25th, etc. refers to values at the 10th, 25th, etc. percentile of the distribution. Industry refers to the main industry of activity of the seller.

Belgian data by Dhyne et al. (2015). As with firm size and the extensive margin of firm connections, the intensive margin of firm linkages exhibits qualitatively similar properties within broad industries, with notable variation in magnitudes across industries.

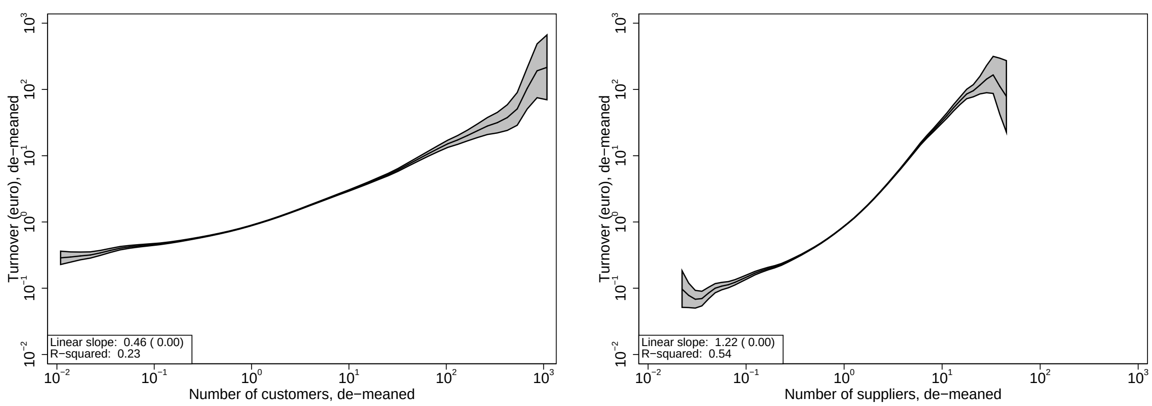
Fact 2. *Bigger firms have more buyers and suppliers.*

A sharp pattern in the data is that bigger firms interact with more buyers and suppliers in the production network. Figure 9a plots the fitted line and 95% confidence interval based on a local polynomial regression of firm turnover on the number of firm downstream customers, on a log-log scale. Both variables have been demeaned by their NACE 4-digit sector average, such that the latter corresponds to the point with coordinates (1,1) in the graph. Figure 9b repeats the exercise for the relationship between firm sales and number of upstream suppliers. Both figures display tightly estimated upward-sloping lines. Implied elasticities and R-squared from OLS regressions with NACE 4-digit industry fixed effects are also reported in the lower left corner of each graph.

The estimates indicate that relative to the industry mean, a firm with 10 times more customers has approximately 4.6 times higher sales, while a producer with 10 times more suppliers attains 12.2 times higher sales. Since the out-degree elasticity of turnover is less than 1, average sales per customer decrease as the number of customers increases. Conversely, with an in-degree elasticity exceeding 1, turnover rises more than proportionally with the number of suppliers. We report similar results using downstream sales withing the B2B domestic network instead of total turnover in Appendix C.

Figure 3: Firm size and number of buyers and suppliers (2014).

(a) Firm sales and number of buyers. (b) Firm sales and number of suppliers.



Note: Firm turnover and number of customers and suppliers are demeaned at the NACE 4-digit level. Graphs are trimmed at the 0.1st and 99.9th percentiles of the number of customers and suppliers respectively.

Fact 3. *The distribution of sales across buyers does not vary with the number of buyers. The distribution of purchases across suppliers widens with the number of suppliers.*

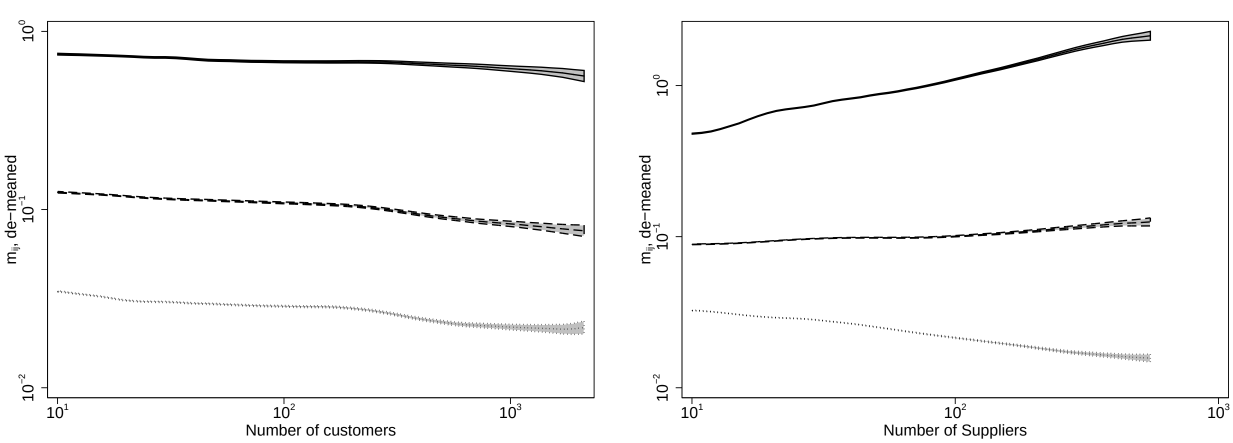
Facts 1 and 2 reveal broadly symmetric patterns in the extensive margin of firms' interactions with upstream suppliers and with downstream buyers in the production network. In contrast, Fact 3 uncovers asymmetry between the input and output sides along the intensive margin of firm-to-firm transactions: While the distribution of a firm's bilateral sales across customers does not vary with the number of customers, the distribution of its input purchases across suppliers widens monotonically with the number of suppliers.

Figure 4a illustrates the dispersion of downstream sales across buyers within a seller. For each firm with at least 10 customers, we take the 10th, 50th and 90th percentile values of its bilateral sales, and demean these by NACE 4-digit industry. We plot the fitted lines from local polynomial regressions of these percentile values against firms' out-degree, including 95% confidence interval bands. The three lines we obtain are almost parallel and slightly declining. In other words, sales to the bottom, median and top customer are essentially the same, or somewhat smaller, for firms with 100 customers and for firms with 10 customers. The slight decline is consistent with the out-degree elasticity of turnover in Figure 9a. Together with Fact 2, this suggests that larger sellers have higher sales primarily because they serve more customers, but they neither sell more to their buyers nor vary their sales more across buyers.

Figure 4b demonstrates the distribution of input purchases across upstream suppliers within a buyer. For each firm with at least 10 input providers, we obtain the 10th, 50th and 90th percentile values of its bilateral purchases, and demean by its NACE 4-digit industry. We graph the fitted lines from local polynomial regressions of these percentile values against firms' in-degree, with 95% confidence interval bands. While purchases from the median supplier are essentially unchanged across firms with broad and narrow supplier bases, however, firms that source inputs from more suppliers systematically buy more from their largest suppliers and less from their smallest. Together with Fact 2, this implies that larger buyers have higher purchases both because they transact with more suppliers and because they vary their purchases more across suppliers.

Figure 4: Sales distribution across buyers and suppliers within firms.

(a) Number of buyers and bilateral sales. (b) Number of suppliers and bilateral purchases.



Note: Local polynomial regressions for the the value of firm-to-firm transactions at the 10th, 50th and 90th percentile of the distribution. Firm-to-firm sales are demeaned by the NACE 4-digit industry of the seller and the customer in each figure respectively. The number of customers and suppliers respectively has been trimmed at the 0.1st and 99.9th percentiles.

Summary. We have documented three stylized facts which suggest that buyer-supplier linkages in a production network are key to understanding the origins of the firm size distribution. In particular, they signal an important role for (i) downstream input demand relative to final output demand, (ii) the number of buyers and suppliers of a firm, (iii) seller and buyer firm characteristics, and (iv) seller-buyer match characteristics. Motivated by these stylized facts, we next develop a unified theoretical framework that accommodates them by introducing two-sided firm heterogeneity in an input-output production network. Importantly, this model allows us to decompose the variation in the firm size distribution into economically meaningful components related to both own-firm characteristics and the production network. Of note, existing models of one-sided firm heterogeneity such as differentiated firms producing only for final consumers cannot account for (ii)-(iv), while existing models of two-sided firm heterogeneity have so far ignored either (i) or (iv).

3 Theoretical Framework

This section develops a theoretical framework that serves several purposes. First, the model allows for various sources of firm heterogeneity both on the demand size (e.g., being connected to many or large customers) and the supply side (e.g., having access to cheap intermediate inputs). Second, the framework gives a clear mapping between model parameters and firm-level estimated coefficients from production network data. Section 4 below describes the identification and estimation of those coefficients. Third, the framework allows a decomposition of firm sales into various downstream and upstream margins (a model-based decomposition). And finally, the model can be used for counterfactual analyses (Section 6).

Our starting point is a model where firms are heterogeneous in productivity or quality, as in Melitz (2003). Firms sell to other firms and to final demand, and how many and which buyers they meet will affect firm size. In addition, firms source inputs from one or more suppliers, and those input prices will determine output prices and consequently also firm sales. Since the main aim of the paper is to understand the role of the network in generating heterogeneity, we take the observed production network as given, i.e. we do not model the firm-to-firm matching decision itself.

3.1 Technology

To implement our approach, we start with the following production function of firm i:

$$y_i = \kappa z_i l_i^{\alpha} v_i^{1-\alpha},$$

where y_i is output, z_i is productivity, l_i is labor, α is the labor share, and $\kappa > 0$ is a normalization constant.[16] v_i is a constant elasticity of substitution (CES) input bundle:

$$v_i = \left(\sum_{k \in \mathcal{S}_i} (\phi_{ki} \nu_{ki})^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)},$$

where ν_{ki} is the quantity purchased from firm k, $\mathcal{S}_i$ is the set of suppliers to firm i, and $\sigma > 1$ is the elasticity of substitution across suppliers. ϕ_{ki} is a demand shifter that captures the idea that firms (and industries) may have very different production technologies, and that their purchases from a given supplier may vary greatly. We allow for heterogeneity in α and σ across industries, however for ease of notation we drop industry subscripts for now. The corresponding input price index is $P_i^{1-\sigma} = \sum_{k \in \mathcal{S}_i} (p_{ki}/\phi_{ki})^{1-\sigma}$, where p_{ki} is the price charged by supplier k to firm i. The marginal cost of the firm is then

$$c_i = \frac{w^\alpha P_i^{1-\alpha}}{z_i}. \tag{1}$$

3.2 Firm-to-Firm Sales, Total Sales and Purchases

Each firm faces demand from other firms as well as from final demand. Given the assumptions about technology, sales from firm i to j are

$$m_{ij} = \left(\frac{\phi_{ij}}{p_{ij}} \right)^{\sigma-1} P_j^{\sigma-1} M_j, \tag{2}$$

where M_j are the total intermediate purchases of firm j. In the baseline decomposition, final demand is directly observed as the difference between total sales S_i (including exports) and firm-to-firm sales, and as such it is unnecessary to model it explicitly, see Section 2.1. In this part of the paper, we therefore take final demand as given, while Section 6 extends the model with endogenous final demand. We define β_i^c as the ratio between total sales and sales to the network,

$$\beta_i^c \equiv \frac{S_i}{\sum_{j \in \mathcal{C}_i} m_{ij}} \geq 1 \tag{3}$$

where $\mathcal{C}_i$ is the set of network customers of firm i. In our data, we only observe firm-to-firm links in the domestic economy. Hence, demand from foreign firms (exports) will be part of S_i but not $\sum_{j \in \mathcal{C}_i} m_{ij}$. In a similar manner, we define β_i^s as the ratio between total purchases (including imports) and purchases from the network,

$$\beta_i^s \equiv \frac{M_i}{\sum_{k \in \mathcal{S}_i} m_{ki}} \geq 1, \tag{4}$$

[16]In particular, $\kappa = \alpha^{-\alpha} (1-\alpha)^{\alpha-1}$. This normalization simplifies the expression for the cost function without any bearing on our results.

where $\mathcal{S}_i$ is the set of network suppliers of firm i.

In the following, it will be useful to collapse parameters that are related to either the buyer, the seller, or the match. We assume that the match quality term ϕ_{ij} can be written as $\phi_{ij} = \phi_i \tilde{\phi}_{ij}$, where ϕ_i captures the average quality of firm i and $\tilde{\phi}_{ij}$ is an idiosyncratic match term. In a similar fashion, we assume that the price p_{ij} can be written as $p_{ij} = \tau_i \tilde{\tau}_{ij} c_i$, where c_i is marginal cost, τ_i captures the average mark-up and trade cost of i, and $\tilde{\tau}_{ij}$ is the match-specific trade cost/mark-up term.[17] $\tilde{\tau}_{ij}$ can reflect any type of price variation, e.g. heterogeneity in mark-ups across customers. Equation (2) can therefore be rewritten to

$$m_{ij} = \psi_i \theta_j \omega_{ij}, \tag{5}$$

where $\psi_i \equiv (\phi_i / (\tau_i c_i))^{\sigma-1}$ is a seller effect, $\theta_j \equiv P_j^{\sigma-1} M_j$ is a buyer effect and $\omega_{ij} \equiv \left(\tilde{\phi}_{ij} / \tilde{\tau}_{ij}\right)^{\sigma-1}$ is a match effect.

The exact decomposition in Section 3.4 only requires the assumptions described so far (the production function and the functional forms of p_{ij} and ϕ_{ij}). In particular, there is no need to assume anything about market structure, firms' pricing behavior or the elasticity of substitution. However, a few additional elements are required to solve the general equilibrium and to perform counterfactuals. We therefore introduce those assumptions when needed in Section 6.

3.3 Mapping Buyer and Seller Effects to Model Parameters

Section 4 describes how we can estimate the parameters ψ_i, θ_i and ω_{ij} from production network data. Given knowledge about those parameters and total input expenditure M_i, we can invert the expressions for ψ_i, θ_i and ω_{ij} and back out structural parameters. Using equation (1) and rearranging yields:

$$P_j^{\sigma-1} = \frac{\theta_j}{M_j} \tag{6}$$

$$\left(\frac{\phi_i z_i}{\tau_i w^{\alpha}}\right)^{\sigma-1} = \psi_i \left(\frac{\theta_i}{M_i}\right)^{1-\alpha} \tag{7}$$

$$\left(\frac{\tilde{\phi}_{ij}}{\tilde{\tau}_{ij}}\right)^{\sigma-1} = \omega_{ij}. \tag{8}$$

Loosely speaking, the buyer effect θ_j reflects the magnitude of average purchases controlling for the size of suppliers. We refer to θ_j as *sourcing capability.* Intuitively, it is an

[17] In the empirical application, $\tilde{\phi}_{ij}$ and $\tilde{\tau}_{ij}$ will be normalized such that $(1/n_i^c) \sum_{j \in \mathcal{C}_i} \tilde{\phi}_{ij} = 1$ and $(1/n_i^c) \sum_{j \in \mathcal{C}_i} \tilde{\tau}_{ij} = 1$, where n_i^c is the number of customers of firm i. Intuitively, this normalization separates the systematic variation across firms from the variation across buyers and suppliers within firms, such that the former is fully loaded on ϕ_i and τ_i.

input price aggregate that implicitly reflects the number and production capabilities of input suppliers. Hence, if the buyer effect is small and total purchases M_j are large, it must mean that purchases are spread out over many suppliers such that the input price index P_j in equation (6) is small.

Conversely, the seller effect ψ_i reflects the magnitude of average sales controlling for the size of customers. After adjusting for input costs $P_i^{1-\alpha}$, ψ_i thus identifies the productivity (z_i) or average attractiveness (ϕ_i/τ_i) of the seller in equation (7). In the following, it will become useful to define the left hand side of equation (7) as $\tilde{z}_i \equiv (\phi_i z_i/(\tau_i w^\alpha))^{\sigma-1}$. We refer to $\tilde{z}_i$ as *production capability.* Note that by exploiting the production network data, our methodology overcomes a standard reflection problem in the productivity literature: It permits the estimation of a firm's own productivity separately from the productivity of its input suppliers embedded in its marginal cost.

According to our model, the seller and buyer effects for firm i are negatively related. Rearranging equation (7), we get $\psi_i = \tilde{z}_i (M_i/\theta_i)^{1-\alpha}$. Hence, a marginal increase in the buyer effect is associated with a reduction in the seller effect of $1-\alpha$ (holding total input purchases and production capability constant). This occurs because a higher buyer effect, all else constant, implies higher input costs. This translates into higher output prices and therefore lower sales. We test this prediction in Section 4.1.[18]

Finally the match-specific component ω_{ij} jointly captures the demand (taste) shifter $\tilde{\phi}_{ij}$ and the supply (mark-up, trade cost) shifter $\tilde{\tau}_{ij}$ in equation (8).

3.4 An Exact Firm Sales Decomposition

In this section, we develop an exact decomposition of firm sales into different margins related to downstream and upstream factors. Combining equations (3) and (5) above, log total sales are

$$\ln S_i = \ln \psi_i + \ln \xi_i + \ln \beta_i^c, \tag{9}$$

where $\xi_i \equiv \sum_{j\in\mathcal{C}_i} \theta_j \omega_{ij}$.

The components ψ_i and ξ_i represent upstream and downstream fundamentals in explaining firm size, respectively, while β_i^c represents the importance of final demand. As we show below, we can identify $\ln \psi_i$, $\ln \theta_i$ and $\ln \omega_{ij}$ from the production network data. Furthermore, $\ln S_i$ and $\ln \beta_i^c$ are directly observed in our data. Hence, all components of equation (9) are known.

[18]While the model predicts a relationship between the buyer and seller effects, it has no prediction about the relationship between productivity/quality $\phi_i z_i$ and the input price index P_i. In frameworks with endogenous network formation, the correlation between these two will generally depend on the matching model and the market structure.

In order to assess the role of each margin, we follow the literature (Eaton et al. (2004), Hottman et al. (2016)) and regress each component ($\ln \psi_i$, $\ln \xi_i$, and $\ln \beta_i^c$) on log sales. By the properties of ordinary least squares, the sum of those three coefficients will sum to unity, and the coefficient magnitudes will represent the share of the overall variation in firm size explained by each margin.

We can further decompose the upstream and downstream margins into various sub-margins. Starting with the downstream side, the parameter $\ln \xi_i$ can be rewritten as

$$\ln \xi_i = \ln n_i^c + \ln \bar{\theta}_i + \ln \Omega_i^c, \tag{10}$$

where n_i^c is the number of customers and $\bar{\theta}_i \equiv \left(\prod_{j \in \mathcal{C}_i} \theta_j\right)^{1/n_i^c}$ is the average customer capability.[19] The covariance term Ω_i^c is defined as

$$\Omega_i^c \equiv \frac{1}{n_i^c} \sum_{j \in \mathcal{C}_i} \omega_{ij} \frac{\theta_j}{\bar{\theta}_i}.$$

Each of these components has an intuitive economic interpretation. First, firms face high demand if they are linked to many customers (high n_i^c). Second, they face high demand if the average customer has high sourcing capability (high $\bar{\theta}_i$). Third, they face high demand if the covariance term Ω_i^c is large. This would be the case if large customers (high θ_j) also happen to be a good match (high ω_{ij}). As with the overall decomposition, we will regress each component in equation (10) on $\ln \xi_i$.

Next, we turn to the upstream decomposition. A firm may be large because it has high production capability (high $\tilde{z}_i$), or because it benefits from cheap or high-quality inputs (low P_i). Just as above, the input price index can be decomposed into components for the number of suppliers, average supplier capability, and a covariance term. This can be shown in three steps. First, from the inversion in equation (7), the production capability of a firm, $\tilde{z}_i$, is a function of the estimated buyer and seller effects. Second, combining equations (4) and (5) above, log total purchases are

$$\ln M_j = \ln \theta_j + \ln \sum_{i \in \mathcal{S}_j} \psi_i \omega_{ij} + \ln \beta_j^s. \tag{11}$$

Third, solving equation (7) for $\ln \psi_i$ and substituting $\ln (M_i/\theta_i)$ using equation (11) yields

$$\ln \psi_i = \ln \tilde{z}_i + (1 - \alpha) \left[\ln n_i^s + \ln \bar{\psi}_i + \ln \Omega_i^s + \ln \beta_i^s\right], \tag{12}$$

[19] By the properties of ordinary least squares, the average term $(1/n_i^c) \sum_{j \in \mathcal{C}_i} \ln \omega_{ij} = (1/n_i^s) \sum_{k \in \mathcal{S}_i} \ln \omega_{ki} = 0$ and therefore omitted from the expression.

where n_i^s is the number of suppliers, $\bar{\psi}_i \equiv \left(\prod_{k \in \mathcal{S}_i} \psi_k\right)^{1/n_i^s}$ is average supplier capability, and the covariance term Ω_i^s is

$$\Omega_i^s \equiv \frac{1}{n_i^s} \sum_{k \in \mathcal{S}_i} \omega_{ki} \frac{\psi_k}{\bar{\psi}_i}.$$

Detailed derivations are found in Appendix A.1. Again, each component of this expression is either observed directly (α, β_i^s and n_i^s) or can be estimated from the production network data ($\tilde{z}_i$, ψ_i, $\bar{\psi}_i$ and Ω_i^s).

The interpretation of each element is as follows. A firm has a large market share among customers (high ψ_i) because it is inherently productive or high-quality (high $\tilde{z}_i$), because it has many suppliers (large n_i^s), because those suppliers are on average attractive suppliers (high $\bar{\psi}_i$), or because attractive suppliers also happen to be a good match (high Ω_i^s). As with the overall decomposition, we regress each component in equation (12) on $\ln \psi_i$. The coefficient estimates will mechanically sum to one because the left and right hand side of equation (12) are by construction identical.[20]

We summarize the overall decomposition of firm size in the equation below for ease of reference in the empirical analysis. Firm size is determined by an upstream factor and a downstream factor. The upstream factor comprises own production capability and network supply, where the latter constitutes the network input share, number of input suppliers, average production capability across suppliers, and a covariance term. The downstream factor includes final demand and network demand, where the latter contains the number of customers, average sourcing capability across customers, and a covariance term.

$$\overset{Size}{\ln S_i} = \overbrace{\underset{ProdCapability}{\ln \tilde{z}_i} + \overbrace{(1-\alpha)\left[\underset{\#Suppliers}{\ln n_i^s} + \underset{AvgSupProdCapab}{\ln \bar{\psi}_i} + \underset{Covariance}{\ln \Omega_i^s} + \underset{NetworkInputShare}{\ln \beta_i^s}\right]}^{NetworkSupply}}^{\overset{Upstream}{\ln \psi_i}}$$
$$+ \overbrace{\overbrace{\underset{\#Customers}{\ln n_i^c} + \underset{AvgCustSourceCapab}{\ln \bar{\theta}_i} + \underset{Covariance}{\ln \Omega_i^c}}^{NetworkDemand} + \overset{FinalDemand}{\ln \beta_i^c}}^{\overset{Downstream}{\ln \xi_i + \ln \beta_i^c}}. \tag{13}$$

The limited assumptions we have placed on the economic environment imply that this is an agnostic firm size decomposition that allows us to evaluate the contribution of different margins to the overall variation in firm size. Our approach imposes no restrictions on the

[20] This holds for any α. A change in α, e.g. due to measurement error, would lead to different coefficient estimates of each component, but the components would still sum to one.

absolute and relative contribution of these margins. In particular, we have not explicitly modeled the endogenous formation of the production network, and we do not aim to explain why some firms match with more or with more capable buyers and suppliers. Instead, our goal is to understand how these implicit firm decisions account for the observed firm size distribution.

4 Estimation

The exact firm size decomposition consists of three steps. In *Step One*, we estimate seller, buyer and match effects from the production network data ($\ln \psi_i$, $\ln \theta_j$ and $\ln \omega_{ij}$). In *Step Two*, we use the first-stage estimates and observed firm outcomes to calculate unobserved firm outcomes ($\ln \xi_i$, $\ln \tilde{z}_i$, $\ln \bar{\theta}_i$, $\ln \bar{\psi}_i$, $\ln \Omega_i^c$ and $\ln \Omega_i^s$). In *Step Three*, we perform the variance decomposition itself, regressing each component of firm size on total sales $\ln S_i$, the downstream (demand-side) factor $\ln \xi_i$ and the upstream (supply-side) factor $\ln \psi_i$, using equations (9), (10) and (12), respectively.

We discuss the first two steps of the econometric analysis in this section and present the firm size decomposition in Section 5. Our ultimate goal is to understand the cross-sectional variation in firm size at a given point in time, as well as the relative importance of different margins to changes in firm size over time. Since the production network continuously evolves, we therefore perform *Step One* and *Step Two* separately for each year in the 2002-2014 sample period. We report detailed results for these two steps for the most recent year in the data, 2014. The patterns for other years are extremely stable and available upon request.

4.1 Step One: Buyer, Seller and Match Effects

Our first step is to estimate the buyer, seller and buyer-seller match effects from the B2B data on the Belgian domestic production network. This step exploits the granularity of firm-to-firm transactions to inform the micro-foundations of firm size in a way that would be impossible without such rich data.

We estimate a two-way fixed effects specification for firm-to-firm sales based on equation (5):

$$\ln m_{ij} = \ln \psi_i + \ln \theta_j + \ln \omega_{ij}. \tag{14}$$

In this OLS regression, the seller effect $\ln \psi_i$ is identified from the variation in input purchases across the suppliers of an average buyer. Intuitively, attractive suppliers account for a large share of input expenditures for all their downstream customers and receive a high $\ln \psi_i$. Analogously, the buyer effect $\ln \theta_j$ is identified from the variation in sales across the customers of an average producer. Intuitively, attractive buyers purchase a disproportionate share of upstream suppliers' sales and receive a high $\ln \theta_j$. The estimated residual $\ln \omega_{ij}$ is by construction orthogonal to the fixed effects. It thus reflects match-specific characteristics that induce a given firm pair to trade more with each other, even if they are not fundamentally attractive trade partners. In the model, $\ln \omega_{ij}$ combines bilateral trade costs, demand shocks (e.g. how well the seller's product fits the production needs of the buyer), and heterogeneous mark-ups.[21]

In order to estimate the two-way fixed effects model, firms must have multiple connections. Specifically, identification of a seller fixed effect requires a firm to have at least two customers, and identification of a buyer fixed effect requires a firm to have at least two suppliers. Thus all one-to-one, one-to-many and many-to-one links are dropped in the estimation procedure. Furthermore, dropping customer A might result in supplier B having only one customer left. Supplier B would then also be removed from the sample. This iterative process continues until a connected network component remains (i.e. a within-projection matrix of full rank), in which each seller has at least two customers, and each customer has at least two suppliers. This component is known as a mobility group in the labor literature on firm-employee matches (e.g. Abowd et al. (1999)).

Note that our setting is more general than standard bipartite networks in the matching literature in which each economic agent plays a unique role, such as the labor market for firms and workers, the marriage market for men and women, or the organ market for donors and recipients. In the labor market for example, both panel data and worker transitions across firms are necessary to identify the employer, employee and match effects. By contrast, firms can be both buyers and suppliers in a production network, such that cross-sectional data is sufficient to identify the effects of interest. Importantly, this also attenuates the incidental parameter problem as the number of suppliers per customer and the number of customers per supplier is relatively large (see Section 2).

In practice, the estimation sample covers the vast majority of observations in the production network. This underlines the highly connected structure of the Belgian production network across all economic activities, even while it is relatively sparse. For the baseline year, 2014, we retain 17,054,274 firm-to-firm transactions which capture 99% of all links in the data and 95% of their sales value. We thus obtain seller fixed effects for 436,715 firms and buyer fixed effects for 743,326 firms. We report the characteristics of the initial and estimation samples in Table 4.

Figure 5 summarizes the estimation results for 2014. Three patterns stand out. First, the

[21] In Section 5.5, we add (the log of) geographic distance as a proxy for trade costs to equation (14) in order to shed light on the components of the match effect.

Table 4: Full sample vs. first-stage estimation sample (2014).

Full Sample			Estimation Sample			
# Links	# Sellers	# Buyers	Links	Value	Sellers	Buyers
17,304,408	590,271	840,607	99%	95%	74%	88%

Note: Summary statistics for firm-to-firm transactions in the raw B2B data and in the estimation sample in *Step One*.

variation in the seller effect $\ln \psi_i$ is large compared to that in the buyer effect $\ln \theta_j$ (standard deviations of 1.05 and 0.50 respectively). Second, the R^2 from the regression is 0.43, and the dispersion in the residual $\ln \omega_{ij}$ (standard deviation of 1.20) exceeds that in the buyer and seller effects. This signals the importance of buyer-supplier match quality to the value of firm-to-firm sales. Finally, while the estimation imposes no constraints on the relationship between the buyer and seller fixed effects for a given firm i, the theoretical framework implies that they should be negatively correlated (see Section 3.3). In the model, this occurs because firms that have higher input purchases and more suppliers can better allocate inputs towards more capable suppliers. By reducing their quality-adjusted production costs, this makes such firms more attractive suppliers to other firms in the network and thereby increases their sales. Our results confirm that this is borne out in practice: The correlation between $\ln \psi_i$ and $\ln (\theta_i / M_i)$ is -0.13 and significant at 1%.

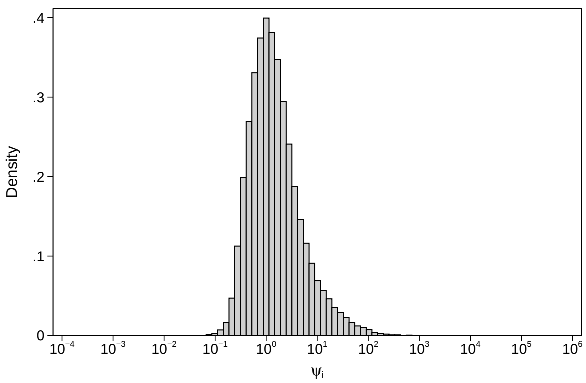
4.2 Conditional Exogenous Mobility

Our methodology will allow us to agnostically estimate the contribution of different margins to the overall firm size variation, without taking a stance on the mechanisms that generate the production network, which we take as given. It is important to note that conceptually, our estimates of the seller and buyer fixed effects are therefore per se unbiased. However, their interpretation depends on the mechanisms underlying the network formation process. $\ln \psi_i$ and $\ln \theta_j$ will identify the role of seller and buyer characteristics for the intensive margin of firm-to-firm sales if firms match either exogenously or endogenously based on firm attributes or firm-pair matching shocks, so long as these matching shocks are not correlated with pairwise sales shocks. Otherwise, the seller and buyer fixed effects will reflect elements from both the intensive and the extensive margins of firm connections. To explore this issue, in this subsection we provide evidence from exogenous mobility tests, and find support for the former interpretation.

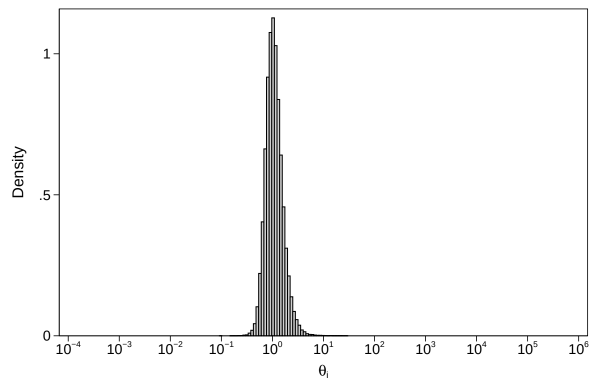
Equation (14) is a two-way fixed effects model similar to the models that are used in

Figure 5: Distribution of seller and buyer effects.

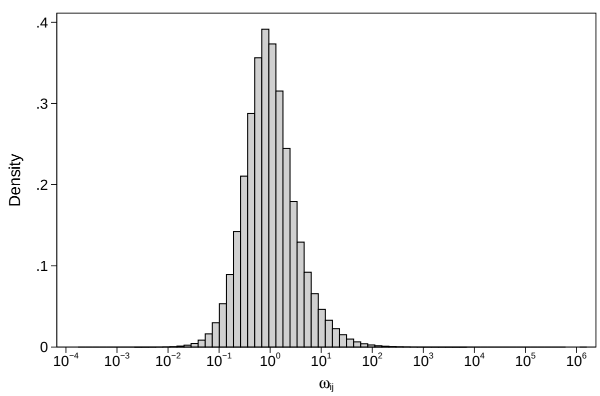
(a) Seller Fixed Effect, $\ln \psi_i$. N=436,715.



(b) Buyer Fixed Effect, $\ln \theta_j$. N=743,326.



(c) Match Effect (Residual), $\ln \omega_{ij}$. N=17,054,274.



the employer-employee literature (e.g., (Abowd et al., 1999; Card et al., 2013)).[22] As in that literature, our OLS estimates of $\ln\psi_i$ and $\ln\theta_j$will identify the effect of seller and buyer characteristics purely on the intensive margin of firm-to-firm sales only if the following moment conditions are satisfied:

$$\begin{cases} E[s_i'r]=0 & \forall i \\ E[b_j'r]=0 & \forall j \end{cases}$$

Here $S=[s_1,...,s_N]$ is the $N^*\times N_s$ seller fixed effects design matrix, $B=[b_1,...,b_N]$ is the $N^*\times N_b$ buyer fixed effects design matrix, and r is the $N^*\times 1$ vector of residual match effects. The first condition states that for each seller i, the average $\ln\omega_{ij}$ across buyers j is zero, while the second condition states that for each buyer j, the average $\ln\omega_{ij}$ across sellers i is zero. In other words, these moment conditions require that the assignment of suppliers to customers is exogenous with respect to ω_{ij}, so-called conditional exogenous mobility. This assumption is violated if a positive shock both increases the likelihood of matching and raises ω_{ij} and thereby bilateral sales conditional on matching. In our setting, this implies that a firm cannot charge a high markup or trade costs (high $\ln\omega_{ij}$) with a large buyer (high $\ln\theta_j$).

It is instructive to review some cases when these moment conditions hold. First, they hold if firms match based on supplier and customer capability, i.e. their buyer and seller effects. Second, they hold if firms match based on unobserved fixed costs that do not matter for sales, such as fixed search-and-match costs. The models of Bernard et al. (2017) and Lim (2017) are examples of the first and second cases. Third, the assumption holds if firms match based on idiosyncratic pair-wise shocks that are unrelated to ω_{ij}. Eaton et al (2015) develop a quasi-random matching model which would be consistent with this third case.

To explore the possibility that matching shocks are also correlated with sales shocks, we test for conditional exogenous mobility using a methodology inspired by Card et al. (2013). The key idea is to check whether a switch from a small to a large customer increases sales, while a switch from a large to a small buyer lowers sales, and that these changes are of equal magnitude in absolute value. Under the exogenous mobility assumption, the expected change in sales when moving from customer k to j is identical to the change when moving from j to k (in absolute value):

$$E\left[\ln m_{ij}-\ln m_{ik}\right]=-E\left[\ln m_{ik}-\ln m_{ij}\right]=\ln\theta_j-\ln\theta_k.$$

[22] We follow the empirical literature on matching markets and adopt a linear fixed-effects estimation approach. This procedure imposes no restrictions on the seller and buyer fixed effects, unlike random or mixed effects models would. With random effects, one also needs to model the network formation game to assess the plausibility of the required distributional assumptions for unobserved heterogeneity (see Bonhomme (2017)).

Intuitively, if exogenous mobility fails, then a switch from large to small may not result in a big sales decline because both matching and sales are driven by positive unobserved shocks.

We adapt the methodology in Card et al. (2013) to our setting, because firms have many connections both upstream and downstream, while in labor markets a worker is typically linked to one employer at a time. First, we estimate the fixed effects model from equation (14) for the 2005 cross-section ($t = 0$). We group firms into quartiles based on the magnitude of their estimated buyer effect. These quartiles are denoted by q_k, $k = 1, 2, 3, 4$. Second, we consider the set of firms that have at least one q_1 buyer at $t = 0$ and add at least one q_4 buyer at $t = 1$ (year 2006), i.e. *upgraders*. For each upgrading firm, we calculate the change in bilateral sales when moving from a q_1 to a q_4 customer, $\ln m_{ij(q_4),t=1} - \ln m_{ij(q_1),t=0}$, where $j(q_k)$ denotes a customer in quartile q_k. Since firms may add many q_4 buyers at $t = 1$ (and potentially have many q_1 buyers at $t = 0$), we form the average of all possible combinations and denote it $\bar{\Delta}_i^{Up}$. Third, we take the average of $\bar{\Delta}_i^{Up}$ across all upgraders. In a similar way, we calculate the outcomes among firms that have a q_4 buyer at $t = 0$ and add a q_1 buyer at $t = 1$, i.e. *downgraders*, and denote the firm-level change $\bar{\Delta}_i^{Down}$. We find that the mean of $\bar{\Delta}_i^{Up}$ is 0.49 and the mean of $\bar{\Delta}_i^{Down}$ is -1.46. Hence, the results suggest that there is asymmetry in upgrading and downgrading. However, this asymmetry goes in the opposite direction to what one would expect under endogenous mobility. Specifically, endogenous mobility would imply that downgrading leads to a smaller change in sales compared to upgrading (in absolute value).

4.3 Step Two: Firm Size Components

In *Step One*, we estimated equation (5). In *Step Two*, we now interpret the buyer, seller and match effects from the first step through the lens of the model. In particular, we combine the estimates from the first stage ($\ln \psi_i$, $\ln \theta_j$ and $\ln \omega_{ij}$) with observed measures of firm activity ($\ln S_i$, $\ln M_i$, $\ln \beta_i^c$, $\ln \beta_i^s$, $\ln n_i^c$, $\ln n_i^s$, $\mathcal{C}_i$ and $\mathcal{S}_i$) to back out model-consistent measures for unobserved firm attributes necessary for the firm size decomposition ($\ln \xi_i$, $\ln \tilde{z}_i$, $\ln \bar{\psi}_i$, $\ln \bar{\theta}_i$, $\ln \Omega_i^s$ and $\ln \Omega_i^c$).

To construct the observed firm metrics, we combine Business-to-Business (B2B) network data with data from the Central Balance Sheet Office (CBSO) in Belgium. We have information on turnover for 94,357 of the firms that enter the estimation in *Step One*, and we perform the size decomposition on this subsample. Importantly, all firms with identified fixed effects are part of the first step and thus contribute to the buyer and supplier margins of firms in the decomposition sample, even if the former lack sales data to be part of the latter.

We measure firm sales S_i with total reported turnover from CBSO. The network sales ratio, β_i^c, is calculated as total sales divided by the sum of all sales to other firms in the domestic network from B2B. Since we observe firm-to-firm links only within Belgium, sales to foreign firms (exports) are classified as part of final demand.[23] We measure firm purchases M_i with total input expenditures from CBSO.[24] The network input ratio, β_j^s, is then total input purchases divided by the cost of inputs from suppliers in the domestic network . We obtain directly from B2B the number n_i^c and the set $\mathcal{C}_i$ of firms' domestic customers, as well as the number n_i^s and the set $\mathcal{S}_i$ of firms' suppliers.

Using the first-stage estimates, the observed variables just described, and equations (1), (6), (7) and (8), we solve for firms' unobserved production capability $\ln \tilde{z}_i$, input price index $\ln P_i$, and marginal production costs $\ln c_i$. This requires three parameter values: the labor share α in the Cobb-Douglas production technology, the wage rate w, and the elasticity of substitution σ. To accommodate the variation in production technologies and factor costs across industries, we proxy α with the ratio of the total wage bill across all firms operating in a NACE 4-digit industry to the total production costs of all firms in that industry. We likewise measure w with the total wage bill in an industry, divided by total employment in that industry. We take a standard value for σ from the literature and set it equal to 4. This choice is however not consequential given the log-linear specification of the OLS regression in *Step One* and the fact that we demean all firm size components by industry after *Step Two*.[25]

Finally, we back out each firm's network demand $\ln \xi_i$, the average production capability of its suppliers $\ln \bar{\psi}_i$, the average sourcing capability of its buyers $\ln \bar{\theta}_i$, its supply and demand covariance terms $\ln \Omega_i^s$ and $\ln \Omega_i^c$. This completes the second step of the econometric analysis, as we now have measures for all firm size components. Table 5 provides summary statistics for these components, while Table 14 in Appendix B reports all two-way correlation coefficients among them.

[23] This assumption reduces the importance of firm-to-firm sales as almost all international trade is between firms. Using the Broad Economic Categories (BEC) classification of the UN, around 2/3 of Belgian exports are in intermediate goods.

[24] We consider all import transactions to be purchases of foreign inputs to production. For our purposes, even if a firm imports final rather than intermediate goods and services to sell alongside its own-manufactured products, such imports are part of its overall expenses in serving downstream buyers and final consumers. Their contribution to the firm size decomposition will therefore be the same as that of other production inputs. See (Bernard et al., 2012b) on the role of carry-along trade.

[25] Additional results with sector demeaning at 2-digit levels are almost identical and available upon request.

Table 5: Firm size components (demeaned by NACE 4-digit sector, 2014).

Firm Size Component	Estimated?	N	Mean	Median	St Dev
Total Sales, $\ln S_i$		94,357	0.000	-0.112	1.318
Overall Decomposition of $\ln S_i$					
Upstream Supply, $\ln \psi_i$	Y	94,357	0.000	-0.130	1.000
Downstream Network Demand, $\ln \xi_i$	Y	94,357	0.000	0.002	1.627
Final Demand, $\ln \beta_i^c$		94,357	0.000	-0.261	1.199
Upstream Decomposition of $\ln \psi_i$					
Production Capability, $\ln \tilde{z}_i$	Y	94,357	0.000	-0.108	1.294
# Suppliers, $\ln n_i^s$		94,357	0.000	-0.000	0.773
Avg Supplier Capability, $\ln \bar{\psi}_i$	Y	94,357	0.000	-0.016	0.215
Supplier Covariance, $\ln \Omega_i^s$	Y	94,357	0.000	-0.069	0.635
Outside-Network Supply, $\ln \beta_i^s$		94,357	0.000	-0.118	0.537
Downstream Decomposition of $\ln \xi_i$					
# Customers, $\ln n_i^c$		94,357	0.000	-0.006	1.366
Avg Customer Capability, $\ln \bar{\theta}_i$	Y	94,357	0.000	-0.033	0.318
Customer Covariance, $\ln \Omega_i^c$	Y	94,357	0.000	-0.127	0.739

5 Results

In the last step of the econometric analysis, we perform the firm size decomposition according to equation (13). In Section 5.1, we begin by analyzing the contribution of different upstream and downstream margins to the cross-sectional variation in firm size in the baseline year, 2014. In Section 5.2, we repeat this cross-sectional decomposition separately for each year to examine trends in the 2002-2014 panel. In Section 5.3, we turn to the evolution of firm size within firms over time, and evaluate how changes along different upstream and downstream margins shape firm growth. In Section 5.4, we explore the variation in patterns across sectors. Finally, in Section 5.5, we briefly discuss several robustness checks.

A potential concern is that industries are inherently different, and those differences may be systematically related to upstream or downstream characteristics. We therefore demean all observed and constructed variables by their NACE 4-digit industry average after the second step. For example, the overall decomposition from (9) becomes

$$\Delta_S \ln S_i = \Delta_S \ln \psi_i + \Delta_S \ln \xi_i + \Delta_S \ln \beta_i^c,$$

where Δ_S denotes the difference between the outcome of firm i and the average outcome in that sector. We then regress each component, e.g. $\Delta_S \ln \psi_i$, on $\Delta_S \ln S_i$. The baseline variance decomposition therefore estimates the importance of each margin in explaining

Table 6: Overall Decomposition (2014).

	N	Upstream $\ln\psi_i$	Downstream $\ln\xi_i$	Final Demand $\ln\beta_i^c$
$\ln S_i$	94,357	$.18^{***}$	$.81^{***}$	$.01^{**}$
		(.00)	(.00)	(.00)

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on total firm sales. All variables in logs. Standard errors in parentheses. Significance: * < 5%, ** < 1%, *** <0.1%.

within-industry size heterogeneity.[26]

5.1 Baseline Results

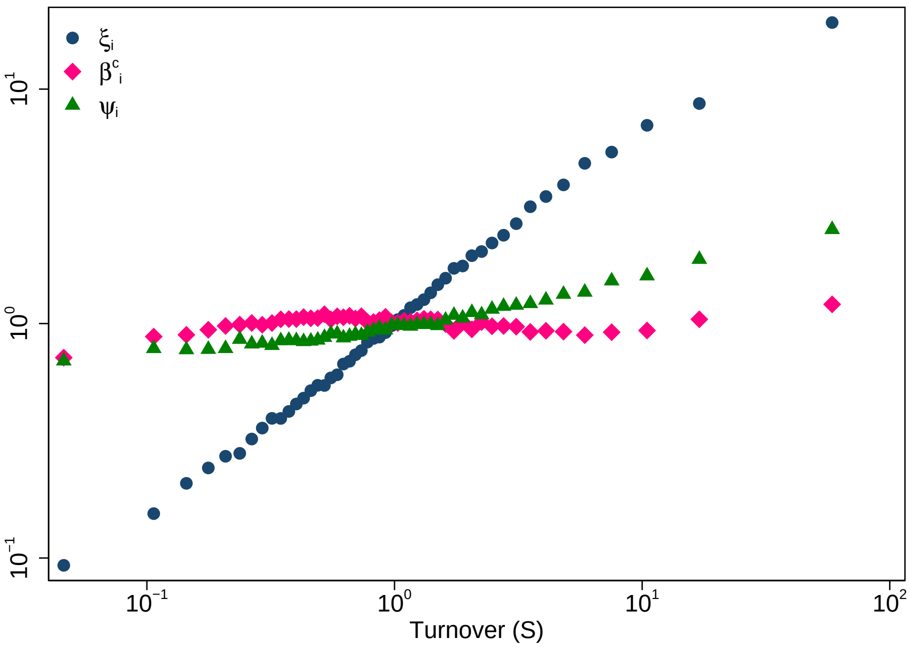
5.1.1 Top-tier Decomposition

We first examine the origins of firm size heterogeneity in the cross-section for 2014, the most recent year in the data. We start with the top-tier decomposition of firm sales $\ln S_i$ into final demand $\ln\beta_i^c$, the upstream factor $\ln\psi_i$ and the downstream factor $\ln\xi_i$, from equation (9), by regressing each factor on $\ln S_i$. Recall that by the properties of OLS, the coefficient estimates from these two regressions sum to 1 by construction, and indicate what fraction of the total variation in firm sales can be attributed to each factor. We report the results in Table 6. The downstream side accounts for 81% of the size dispersion across firms, the upstream fundamentals explain 18%, while final demand explains only 1%.

What is the interpretation of these results? The upstream factor $\ln\psi_i$ represents, loosely speaking, the average market share of i among its customers. Hence, the relatively small role for upstream fundamentals means that average market share is not strongly correlated with total firm sales. In other words, being an important supplier to your customers is only weakly related to overall firm success. This does not mean, however, that upstream factors in general are unimportant in explaining firm size. Rather, the results suggest that supply-side factors that are orthogonal to average market share might be important. Examples of such factors are efficiency in marketing or skills in finding and attracting a customer base. Differences in final demand across firms, as captured by the ratio of total firm sales to sales to final consumers $\ln\beta_i^c$, account for an economically negligible 1% of the overall variation in firm size. Hence, practically the entire downstream factor is governed by demand from other firms in the production network, rather than from final demand. In other words, large firms

[26] We perform sensitivity analysis by demeaning at the NACE 2-digit level and alternatively without demeaning in Section 5.5.

Figure 6: Overall Decomposition (2014).



Note: This binned scatterplot groups firms into 20 equal-sized bins by log sales, computes the mean of log sales and the components $\ln \psi_i$, $\ln \xi_i$ and $\ln \beta_i^c$ within each bin, and graphs these data points. The result is a non-parametric visualization of the conditional expectation function.

are not systematically selling relatively more (or less) to final demand than small firms.

We can also visualize the importance of each component using a binned scatterplot. In Figure 6, we group log sales into 20 equal-sized bins, compute the mean of log sales and the components $\ln \psi_i$, $\ln \xi_i$ and $\ln \beta_i^c$ within each bin, and then create a scatterplot of these data points. The result is a non-parametric visualization of the conditional expectation function, where the sum of the three components on the vertical axis equals log sales on the horizontal axis. Again, we observe the dominance of the downstream component, and furthermore that the relationship is close to linear across the entire distribution of firm sales.

These findings suggest that the key to understanding the vast firm size heterogeneity observed in modern economies is in how firms manage their sales activities, and specifically how they match and transact with buyers in the production network. This does not imply that the production side is irrelevant: models of the production process within firms inform various important aspects of firm operations beyond firm sales, such as value added and profits. In addition, results below for the upstream decomposition lend support to the large class of models that focus on a single firm attribute on the production side (e.g. productivity),

Table 7: Downstream Decomposition (2014).

	# Customers $\ln n_i^c$	Avg Customer Capability $\ln \bar{\theta}_i$	Customer Covariance $\ln \Omega_i^c$
$\ln \xi_i$	$.71^{***}$	$.03^{***}$	$.26^{***}$
	(.00)	(.00)	(.00)

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on the downstream factor, $\ln \xi_i$. All variables in logs. Standard errors in parentheses. Significance: * $<$ 5%, ** $<$ 1%, *** $<$0.1%.

such as Melitz (2003).

Finally, this top-tier firm size decomposition speaks to the stylized facts we presented in Section 2. At a basic level, the evidence here suggests that there is an intimate relationship between the skewed distributions of firms' total sales and various aspects of their production network activity, as summarized in *Fact 1.* In turn, the important role we uncover for the upstream and downstream factors of firm size indicates that the other *Facts 2* and *3* also implicitly reflect how the production network shapes the firm size distribution.

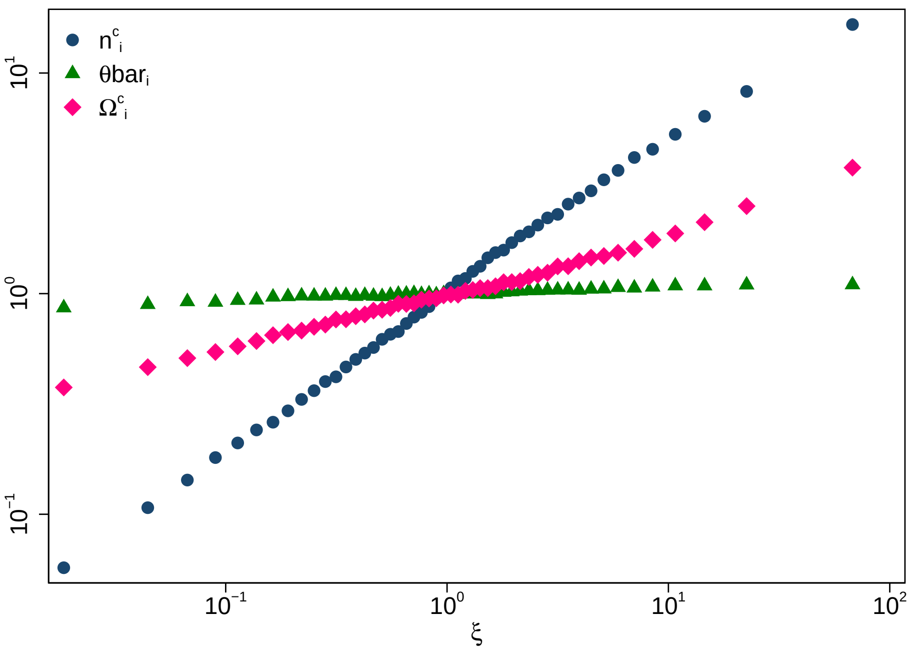
5.1.2 Downstream Decomposition

We next decompose the downstream component into its constituent parts, from equation (10), to assess the specific channels through which the production network shapes firm sales. Table 7 reports the results from regressing each downstream sub-component on $\ln \xi_i$, such that the coefficient estimates quantify their relative importance. An overwhelming 71% of the variation in the downstream component across firms can be attributed to the extensive margin, i.e. the number of (domestic) buyers $\ln n_i^c$ that producers sell to. On the other hand, the average sourcing capability across a firm's customers $\ln \bar{\theta}_i$ and the customer covariance term $\ln \Omega_i^c$ contribute a much more modest 3% and 26%, respectively. As above, we also report the results using a binned scatterplot in Figure 7.

We conclude that on the sales side, the single most important advantage of large firms is that they successfully match with many buyers. The covariance term is also substantial, suggesting that relative to smaller firms, bigger firms also concentrate sales among large buyers with high sourcing capability that are also very good bilateral matches. On the other hand, large firms are not matching with more capable buyers on average.

The downstream decomposition also sheds light on several stylized facts in Section 2. It powerfully illustrates *Fact 2* that bigger firms have more downstream buyers. The limited role of $\ln \bar{\theta}_i$ reinforces *Fact 3* that the distribution of a firm's sales across customers is

Figure 7: Downstream decomposition (2014).



Note: This binned scatterplot groups firmsinto 20 equal-sized bins by downstream sales component $\ln \xi_i$, computes the mean of $\ln \xi_i$, and its sub-components $\ln n_i^c$, $\ln \bar{\theta}_i$ and $\ln \Omega_i^c$ within each bin, and graphs these data points. The result is a non-parametric visualization of the conditional expectation function.

Table 8: Upstream Decomposition (2014).

	Own Prod Capability $\ln \tilde{z}_i$	# Suppliers $\ln n_i^s$	Avg Suppl. Capability $\ln \bar{\psi}_i$	Suppl. Cov. $\ln \Omega_i^s$	Outside-Network Supply $\ln \beta_i^s$
$\ln \psi_i$	.85***	-.01***	.04***	.08***	.04***
	(.00)	(.00)	(.00)	(.00)	(.00)

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on the upstream factor, $\ln \psi_i$. All variables in logs. Standard errors in parentheses. Significance: * $<$ 5%, ** $<$ 1%, *** $<$0.1%.

generally invariant with the number of its customers.

5.1.3 Upstream Decomposition

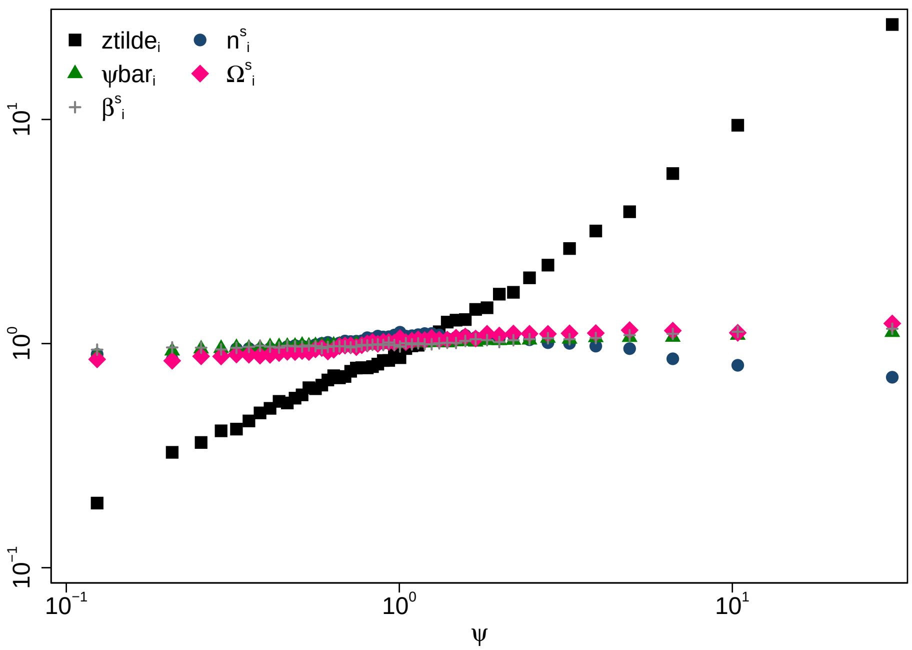
We complete the firm size decomposition by unbundling the upstream margin of firm sales, $\ln \psi_i$, from equation (12). Table 8 reports the results from regressing each sub-component on the upstream factor $\ln \psi_i$. As above, we also report the results using a binned scatterplot in Figure 8.

The seller-specific production capability $\ln \tilde{z}_i$ drives the vast majority of the upstream factor (85%). The remaining factors are loaded on average supplier capability (4%), the covariance term (8%), and the non-network input share (4%). Differently from the downstream side, the number of suppliers does not explain variation in the firm size.

These results reveal how successful firms are able to increase their market shares among customers. First and foremost, inherent firm characteristics, such as productivity or quality (the $\ln \tilde{z}_i$ term), explain differences in market shares. According to our results, firms that have good suppliers (the $\ln \bar{\psi}_i$ term), or that source relatively more from good suppliers that they are well-suited to (the $\ln \Omega_i^s$ term), are also more successful in terms of sales, although the economic magnitude of these effects is less pronounced.

These patterns would be consistent with the combination of search frictions and asymmetric information in the production network. In particular, producers may have to pay fixed search costs in order to meet input suppliers, while also facing ex-ante uncertainty about the primitive production capability of these suppliers and their pairwise match quality (e.g. how well suited the widget produced by a given supplier is to my own production process). In such an environment, exogenously more capable firms may be able to invest in meeting more suppliers on the extensive margin than less capable firms, but with a similar average supplier capability. On the intensive margin, more capable firms may also more effectively allocate their input purchases towards suppliers with both higher production capability and match quality.

Figure 8: Upstream Decomposition (2014).



Note: This binned scatterplot groups firms into 20 equal-sized bins by upstream sales component $\ln \psi_i$, computes the mean of $\ln \psi_i$ and its sub-components $\ln \tilde{z}_i$, $\ln n_i^s$, $\ln \bar{\psi}_i$, $\ln \Omega_i^s$ and $\ln \beta_i^s$, and graphs these data points. The result is a non-parametric visualization of the conditional expectation function.

Table 9: Firm Size Decomposition by Year (2002-2014).

Year	N	Upstream $\ln\psi_i$	Downstream $\ln\xi_i$	Final Demand $\ln\beta_i^c$
2002	81,410	.17***	.78***	.05***
2003	83,817	.17***	.78***	.05***
2004	85,174	.18***	.78***	.04***
2005	86,617	.17***	.78***	.04***
2006	88,714	.17***	.79***	.04***
2007	91,172	.18***	.79***	.03***
2008	92,465	.18***	.79***	.03***
2009	92,528	.17***	.79***	.04***
2010	92,903	.17***	.79***	.04***
2011	94,282	.18***	.80***	.03***
2012	95,558	.18***	.79***	.03***
2013	94,324	.18***	.80***	.02***
2014	94,357	.18***	.81***	.01**

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on total firm sales. All variables in logs. Significance: * $<$ 5%, ** $<$ 1%, *** $<$0.1%.

5.2 Results by Year

We next explore the evolution of the firm size distribution in Belgium over the 2002-2014 period in the sample. We find that despite the increase in the number of firms and in their sales dispersion over time, the sources of firm size heterogeneity have remained remarkably stable.

We perform the three-step firm size analysis separately for each year in the data, and list the results for the top-tier decomposition in Table 9. The importance of the upstream side has firmly stood at 17-18%. The downstream side has gradually risen from 78% to 81%, closely following a decline in final demand from 5% to 1%.

We observe similarly stable patterns when we consider the lower-tier decomposition of the downstream and upstream components (available upon request). These findings suggest that there may be inherent drivers of the firm size distribution whose relative importance persists despite the rise in production fragmentation across firm and country boundaries over the last 15 years.

5.3 Firm Growth

The baseline decomposition relates the variance of sales across firms to the variance of different sales margins. A related question is what explains the variance of firm growth. We proceed as follows. First, we estimate equation (14) on two cross-sections, the baseline year 2014 ($t = 1$) and year 2002 ($t = 0$). We then calculate the change in every demeaned variable in the decomposition. For example, the overall decomposition from (9) becomes

$$\Delta_T \ln S_i = \Delta_T \ln \psi_i + \Delta_T \ln \xi_i + \Delta_T \ln \beta_i^c,$$

where Δ_T denotes the change from $t = 0$ to $t = 1$, e.g. $\Delta_T \ln S_i = \Delta_S \ln S_{i1} - \Delta_S \ln S_{i0}$. We then demean all variables at the NACE 4-digit level. Finally, we regress each component, e.g. $\Delta_T \ln \psi_i$, on $\Delta_T \ln S_i$. This decomposition allows us to assess the importance of the network in explaining firm growth. Note that long differencing is only feasible for firms that are observed with non-missing sales as well as buyer and seller effects in both years, such that we cannot perform the decomposition on firms that enter or exit during the sample period. However, the decomposition accounts for the adding and dropping of customers and suppliers, i.e. the terms $\Delta_T \ln \psi_i$ and $\Delta_T \ln \xi_i$ may change because of extensive margin adjustments.

The results are summarized in Table 10. At a broad level, the contribution of each component is quite close to what we found in the cross-sectional analysis in Section 5.1, yet there are some notable differences in magnitudes. For example, the downstream component dominates in the overall decomposition, with the same contribution as in the cross-section of 81% (column 3). However, the upstream and final demand components are now equally important at 9-10%, whereas final demand played a trivial role of 1% before. On the upstream side, practically the entire growth over time comes from improvements in own production capability (98%), while this factor accounts for less of the cross-sectional dispersion (82%) (column 5). On the downstream side, the number of customers is the primary driver of variation in both firm growth and firm sales in the cross-section, but it generates 61% of the former compared to 71% of the latter (column 4). This is counterbalanced by a greater role for the customer covariance term in the growth decomposition (38%) relative to the levels decomposition (26%). Alternative long differences from 2002-2008 and 2008-2014 respectively give very similar results (results available upon request).

We draw three conclusions about the sources of firm growth from these patterns. First, the vast heterogeneity in growth rates across firms in the panel stems from some firms successfully expanding their sales to downstream buyers in the production network. This entails adding more customers over time, but also effectively redirecting sales towards buyers that are both big and well matched. The greater importance of the latter margin for firm

Table 10: Firm Growth Decomposition (2002-2014).

	Firm Size Component		Sales S_i	Downstream ξ_i	Upstream ψ_i
S_i	Upstream	ψ_i	.09***		
	Downstream	ξ_i	.81***		
	Final Demand	β_i^c	.10***		
ξ_i	# Customers	n_i^c		.61***	
	Avg Customer Capability	$\bar{\theta}_i$		.01***	
	Customer Covariance	Ω_i^c		.38***	
ψ_i	Own Production Capability	$\tilde{z}_i$			.98***
	# Suppliers	n_i^s			-.03***
	Avg Supplier Capability	$\bar{\psi}_i$			.01***
	Supplier Covariance	Ω_i^c			.03***
	Outside-Network Supply	β_i^s			.01***
	N		41,185	41,185	41,185

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the row heading) on total firm sales (column 3), lnξ_i (column 4) or lnψ_i (column 5). All variables in logs. Significance: * < 5%, ** < 1%, *** <0.1%.

growth relative to the cross-section would be consistent with the presence of matching costs and ex-ante imperfect information about buyers. Bigger firms may be able to match with more buyers at a given point, as well as to more effectively reallocate sales amongst them as match qualities are revealed over time, with both forces contributing to faster sales growth.

Second, while the variation in final demand is not important for firms' relative performance in the cross-section, tapping final consumers helps surviving firms expand revenues to a greater degree. Note that in our data, this corresponds to a rise in sales to final domestic consumers as well as to foreign markets.

Finally, faster growing firms enhance their efficiency and/or product quality mainly by increasing their own production capability. While big firms do benefit from more effective input sourcing in the cross-section, firms' sales growth does not come from further optimizing their sourcing behavior. To the limited extent that fast-growing firms do adjust along this dimension, they reduce the number of input suppliers and shift purchases towards well-matched suppliers in equal measure, without changing average supplier capability. The contrasting results for the upstream and downstream network components of firm growth indicate that firms may face different matching frictions and information assymetries in their interactions with buyers and suppliers which translate into different firm dynamics on the

Table 11: Firm Size Decomposition by Industry (2014).

NACE	Industry	N	Upstream ψ_i	Downstream ξ_i	Final Demand β_i^c
01-09	Primary and Extraction	2,838	.24***	.79***	-.03**
10-33	Manufacturing	16,905	.26***	.75***	-.01**
35-39	Utilities	852	.15***	.81***	.04**
41-43	Construction	19,008	.12***	.99***	-.10**
45-82	Market Services	53,604	.18***	.77***	.04**
84-96	Non-Market Services	1,150	.12***	.84***	.04

Note: The table reports coefficient estimates from OLS regressions of a firm size margin (as indicated in the column heading) on total firm sales. All variables in logs. Significance: * $<$ 5%, ** $<$ 1%, *** $<$0.1%.

production and sales side.

5.4 Results by Industry

Finally, we explore the stability of our results across different industries. Table 11 provides the size decomposition separately for six broad industry groups. Across all of these groups, the estimated coefficients are relatively close to the baseline findings in Section 5.1, underscoring their robustness. One exception is construction (NACE 41 to 43), where the final demand term β_i^c enters with a coefficient of -0.10. However, this is expected, as large construction firms typically sell relatively less to final demand compared to small construction firms.

We also perform the analysis separately for every NACE 2-digit industry. For the large majority of industries, we find that the overall decomposition looks strikingly similar to the baseline. For brevity, in Table 12 we report the mean, standard deviation, and coefficient of variation (CV) for the contribution of each size component to firm size heterogeneity across all 2-digit industries . Most components have a CV smaller than 1, indicating little variation across sectors even at this disaggregated level. Two sectors deserve special attention: wholesale and retail. Intuitively, downstream factors and final demand may be relatively more important in these sectors. For wholesalers, we observe the following top-tier decomposition: $\psi_i = .29$, $\xi_i = .78$, and $\beta_i^c = -.07$, consistent with priors that larger wholesalers sell less to final demand. For retailers, we find $\psi_i = .04$, $\xi_i = .67$, and $\beta_i^c = .29$, such that final demand constitutes the dominant component as anticipated.

Table 12: Firm Size Decomposition by Industry (2014).

	Firm Size Component		Mean	St Dev	CV
S_i	Upstream	ψ_i	.17	.14	.80
	Downstream	ξ_i	.78	.30	.39
	Final Demand	β_i^c	.05	.29	5.89
ξ_i	# Customers	n_i^c	.70	.11	.15
	Avg Customer Capability	$\bar{\theta}_i$	.02	.04	1.57
	Customer Covariance	Ω_i^c	.27	.10	.37
ψ_i	Own Production Capability	$\tilde{z}_i$	.80	.23	.29
	# Suppliers	n_i^s	.01	.14	15.78
	Avg Supplier Capability	$\bar{\psi}_i$	.04	.04	.94
	Supplier Covariance	Ω_i^c	.11	.13	1.10
	Outside-Network Supply	β_i^s	.04	.07	1.77

5.5 Sensitivity Analysis

We perform several robustness tests that leave our results unchanged. We briefly discuss these findings here without tabulating them in the interest of space (avaialable upon request).

First, we execute all decomposition exercises demeaning at the NACE 2-digit level instead of the 4-digit level. Alternatively, we also directly analyze the size components we observe and construct without demeaning them by sector. In both cases, the results are very similar to our baseline setting.This suggests that firm size heterogeneity and its origins are not sector specific and present at very disaggregated sectors.

Second, we calculate wages and labor cost shares at the firm level rather than using sector averages, to accommodate the possibility that different firms within the same industry can adopt different production techniques, hire workers with different skills, and/or pay different wages for identical workers. Again, this bears no qualitative impact on our results. We prefer to use the sector averages in our baseline setting, as the available information from the annual accounts generates fairly noisy measures for these variables at the firm level.

Finally, we construct firm size components in *Step Two* and conduct the size decomposition in *Step Three* after adding firm-to-firm geographic distance in the two-way fixed effects regression in *Step One*. Recall that in the model, the residual match quality $\ln \omega_{ij}$ combines pair-specific trade costs, mark-ups and taste preferences. To the extent that distance proxies the former, controlling for it removes that component from $\ln \omega_{ij}$. In practice, this has no significant impact on our results for the firm size decomposition.

6 General Equilibrium

The estimation and decomposition presented in Sections 3 and 4 provide parameter values for firm-level fundamentals. What remains is to close the model and solve for the general equilibrium. This will become useful in the counterfactual experiments presented below.

Final Demand. To close the model, two additional assumptions are required. First, we need an assumption about final demand. We choose the simplest possible case and assume CES utility with the same elasticity of substitution σ across firms:

$$U = \left(\sum_k (\phi_k \nu_k)^{(\sigma-1)/\sigma} \right)^{\sigma/(\sigma-1)}.$$

Using the same functional form for final demand and firm demand enables us to utilize the estimates for production also for final demand. We consider the final consumer as an average input consumer, so that the terms $\tilde{\phi}_{ki}$ and $\tilde{\tau}_{ij}$ do not appear in final demand.[27] The value of final demand is then $\mathcal{F}_i = \left(\frac{\phi_i}{\tau_i p_i}\right)^{\sigma-1} \mathcal{P}^{\sigma-1} X$, where X is overall income, $\tilde{P}_i \equiv P_i^{1-\sigma}$ is producer i's input price index, and $\mathcal{P}$ is the CES consumer price index:

$$\mathcal{P}^{1-\sigma} = \sum_i (\tau_i p_i / \phi_i)^{1-\sigma} = \sum_i \tilde{P}_i^{1-\alpha} \tilde{z}_i. \tag{15}$$

Mark-ups. Second, we need an assumption about mark-ups. So far, we have been completely agnostic about market structure and price determination. To allow for maximum flexibility, we assume that the mark-up potentially varies across firms, but that it is constant across equilibria.[28] As a consequence, a firm's purchases relative to total sales, $\mu_i \equiv M_i/S_i$ is constant.[29] We therefore use data on μ_i when simulating the model. The set of firms is fixed and there is no free entry. We assume that the final consumer is the shareholder of the firms, so that aggregate profits Π become part of consumer income. Income X is therefore the sum of labor income and aggregate profits, $X = wL + \Pi$, where w is the wage and L is inelastically supplied labor. It can be shown that in equilibrium, $X = \rho w L$, where ρ is a constant term.

Backward fixed point. We need to determine how the input costs of firm j depend on the input costs of the suppliers of j. This can be solved by iterating on a backward fixed point

[27] Since final demand is modeled as a representative consumer, there is by construction no match-specific component ϕ_{ki}. Since $\phi_{ki} = \phi_k \tilde{\phi}_{ki}$ and $(1/n_i^c) \sum_i \tilde{\phi}_{ki} = 1$, this implies that the perceived quality of a firm k is identical for the final consumer and the average firm i.

[28] An alterantive, and in our opinion less flexible, approach would be to add more assumptions about market structure and pricing behavior.

[29] $\frac{M_i}{S_i} = \frac{1-\alpha}{S_i/TotalCosts_i} = \frac{1-\alpha}{Markup_i}$, which is constant given that $Markup_i$ is constant.

problem. The backward fixed point relates the price index of firm j to the price indices of its suppliers, using equations (1) and (8):

$$\begin{aligned}\tilde{P}_j &= \sum_{i \in \mathcal{S}_j} \left(\frac{p_{ij}}{\phi_{ij}}\right)^{1-\sigma} \\ &= \sum_{i \in \mathcal{S}_j} \tilde{P}_i^{1-\alpha} \tilde{z}_i \omega_{ij}.\end{aligned} \tag{16}$$

Firm j's input costs depend on the production capability of its suppliers, $\tilde{z}_i$, the suppliers' input costs, $\tilde{P}_i$, and the match terms ω_{ij}. We solve for the backward fixed point using data on α and our estimates of $\tilde{z}_i$ and ω_{ij} from above.

Forward fixed point. We also need to characterize how the sales of firm i relate to the sales of the customers of i. Total firm sales are $S_i = \mathcal{F}_i + \sum_{j \in \mathcal{C}_i} m_{ij}$. Using equations (1), (8) and (2) and defining $\tilde{\mathcal{P}} \equiv \mathcal{P}^{1-\sigma}$, the forward fixed point is then:

$$S_i = \tilde{z}_i \tilde{P}_i^{1-\alpha} \left(\frac{X}{\tilde{\mathcal{P}}} + \sum_{j \in \mathcal{C}_i} \frac{\mu_j S_j}{\tilde{P}_j} \omega_{ij} \right). \tag{17}$$

A detailed derivation is found in Appendix A.2. Firm i's sales depend on final demand, $X/\tilde{\mathcal{P}}$, the production and sourcing capability of the firm itself, $\tilde{z}_i$ and $\tilde{P}_i$, as well as the sales, sourcing capabilities and match effects of its customers, S_j, $\tilde{P}_j$ and ω_{ij}. We solve for the forward fixed point using (i) data on α, final demand X and μ_j, (ii) our estimates of $\tilde{z}_i$ and ω_{ij}, and (iii) $\tilde{P}_i$ and $\tilde{\mathcal{P}}$ using the solution to the backward fixed point in equation (16). Note that we can solve for the equilibrium distribution of sales without imposing any assumption on the elasticity of substitution σ.

Welfare. Indirect utility equals the inverse of the final demand price index $\mathcal{P}$. Hence, welfare can be evaluated with equation (15), using estimates of production capability $\tilde{z}_i$ and the solution for $\tilde{P}_i$ from the backward fixed point.

7 Conclusions

This paper quantifies the origins of firm size heterogeneity when firms are interconnected in a production network. We first document new stylized facts about a complete production network using data on the universe of buyer-supplier relationships among all firms in Belgium during 2002-2014. These stylized facts suggest that the network of buyer-supplier links is key to understanding the firm size distribution. Specifically, they signal the important roles played by downstream input demand as distinct from final demand, by both seller- and buyer-specific firm characteristics, and by seller-buyer match characteristics.

Motivated by these facts, we outline a model in which firms buy inputs from upstream suppliers and sell to downstream buyers and final demand. In the model, firms can be large for the standard reason that they have high production capability (i.e. productivity or product quality). However, firms can also be large because they interact with more, better and larger buyers and suppliers and because they are better matched to their buyers and suppliers. This framework delivers an exact decomposition of firm size into supply and demand margins with firm, buyer/supplier and match components. We design a three-stage estimation methodology that makes it possible to back out these firm size components from data on firm-level balance sheets and firm-to-firm transactions in a production network. We implement the methodology using detailed data for Belgium, and quantify the contribution of each component to the overall dispersion in firm size in the economy.

We establish three empirical results for the origins of firm size heterogeneity. These patterns hold in the cross-section of firms in each year of the panel, as well as in the evolution of firm size within firms over time. First, downstream (demand) factors explain 82% of firm size heterogeneity, while upstream (supply) factors only 18%. Second, nearly all the variation on the demand side is driven by network sales to other firms rather than by final demand. By contrast, most of the variation on the supply side reflects heterogeneity in own production capability rather than network purchases from input suppliers. Third, most of the variance in network sales is determined by the number of buyers and the allocation of sales towards well-matched buyers of high quality, rather than by average buyer capability. By contrast, most of the variance in network purchases comes from average supplier capability and the allocation of purchases towards well-matched suppliers of high quality, rather than from the number of suppliers.

These theoretical, methodological and empirical contributions open interesting avenues for future research. We have taken the production network as given in order to assess its role in shaping the firm size distribution. Our results nevertheless shed light on the various challenges and opportunities that firms face in the presence of input-output linkages in the economy. Future work can examine how firm-specific characteristics determine the matching of buyers and suppliers in the production network in light of our findings. Separately, we have dissected the origins of firm size heterogeneity, but not explored its implications for the aggregate economy. Future studies can analyze whether different sources of the dispersion in firm size have different implications for aggregate outcomes such as growth or income inequality. Finally, we have focused on the relationship between the production network and firm size heterogeneity in steady state. Future research can explore how this relationship affects the propagation and aggregate welfare impact of firm-specific and macroeconomic shocks.

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Appendix

A The Model

A.1 The supply side decomposition

From equation (7), we get

$$\ln \psi_i = \ln \tilde{z}_i + (1-\alpha)\left(\ln M_i - \ln \theta_i\right),$$

where $\tilde{z}_i \equiv \left(\phi_i z_i / \left(\tau_i w^\alpha\right)\right)^{\sigma-1}$. Substituting for $\ln M_i$ from equation (11) yields

$$\ln \psi_i = \ln \tilde{z}_i + (1-\alpha)\left(\ln \sum_{k\in\mathcal{S}_i} \psi_k \omega_{ki} + \ln \beta_i^s\right).$$

The term $\sum_{k\in\mathcal{S}_i} \psi_k \omega_{ki}$ can be further decomposed into

$$\ln \sum_{k\in\mathcal{S}_i} \psi_k \omega_{ki} = \ln n_i^s + \ln \bar{\psi}_i + \ln\left(\frac{1}{n_i^s}\sum_{k\in\mathcal{S}_i} \omega_{ki}\frac{\psi_k}{\bar{\psi}_i}\right),$$

where $\bar{\psi}_i = \left(\prod_{k\in\mathcal{S}_i} \psi_k\right)^{1/n_i^s}$. Combining the last two equations yields equation (12) in the main text.

A.2 Forward fixed point

Total firm sales are $S_i = \mathcal{F}_i + \sum_{j\in\mathcal{C}_i} m_{ij}$. We first derive expressions for final demand and then demand from other firms.

Final demand. Using equation (1) and defining $\tilde{P}_i \equiv P_i^{1-\sigma}$ and $\tilde{\mathcal{P}} \equiv \mathcal{P}^{1-\sigma}$, the final demand price index is

$$\tilde{\mathcal{P}} = \sum_i \left(\frac{\tau_i p_i}{\phi_i}\right)^{1-\sigma} = \sum_i \tilde{P}_i^{1-\alpha}\tilde{z}_i.$$

Using equation (1), final demand is

$$\begin{aligned}\mathcal{F}_i &= \left(\frac{\phi_i}{\tau_i p_i}\right)^{\sigma-1}\mathcal{P}^{\sigma-1} wL \\ &= \tilde{z}_i \tilde{P}_i^{1-\alpha}\frac{wL}{\tilde{\mathcal{P}}}.\end{aligned}$$

Firm Demand. Using (1), (8) and (2), firm demand is

$$\begin{aligned}\sum_{j\in\mathcal{C}_i} m_{ij} &= \sum_{j\in\mathcal{C}_i}\left(\frac{\phi_{ij}}{p_{ij}}\right)^{\sigma-1} P_j^{\sigma-1} M_j \\ &= \tilde{z}_i \tilde{P}_i^{1-\alpha}\sum_{j\in\mathcal{C}_i}\frac{\mu_j S_j}{\tilde{P}_j}\omega_{ij}.\end{aligned}$$

Combining the two sources of demand, we get total sales:

$$S_i = \tilde{z}_i \tilde{P}_i^{1-\alpha} \left(\frac{wL}{\tilde{\mathcal{P}}} + \sum_{j \in \mathcal{C}_i} \frac{\mu_j S_j}{\tilde{P}_j} \omega_{ij} \right).$$

A.3 Variance decompositions

This section derives statistical properties of the baseline variance decomposition. Consider the following identity:

$$s \equiv \sum_k a_k.$$

The variance of s is

$$var\left(s\right) = \sum_k \sigma_{kk} + \sum_k \sum_{i \neq k} \sigma_{ki}, \tag{18}$$

where $\sigma_{ki} = cov\left(a_k, a_i\right)$. In the baseline decomposition, we regress each element a_k on s. By the properties of OLS, the estimate is

$$\beta_k = \frac{cov\left(a_k, s\right)}{var\left(s\right)} = \frac{1}{var\left(s\right)} \left(\sigma_{kk} + \sum_{i \neq k} \sigma_{ki} \right). \tag{19}$$

Note that the sum of all β_k's equals one,

$$\sum_k \beta_k = \frac{1}{var\left(s\right)} \left(\sum_k \sigma_{kk} + \sum_k \sum_{i \neq k} \sigma_{ki} \right) = 1.$$

Also note that in the case with only two components, the covariance term in equation (19) is split equally among components:

$$\beta_1 = \left(\sigma_{11} + \sigma_{12}\right) / var\left(s\right)$$
$$\beta_2 = \left(\sigma_{22} + \sigma_{12}\right) / var\left(s\right).$$

B Data Sources and Data Construction

B.1 The Belgian VAT system

The Belgian value-added tax (VAT) system requires that the vast majority of enterprises located in Belgium and across all economic activities charge VAT on top of the delivery of their goods and services. This also includes foreign companies with a branch in Belgium and firms whose securities are officially listed in Belgium. Enterprises that only perform financial transactions, medical or socio-cultural activities such as education are exempt. The tax is levied in successive stages of the production and distribution process: at each purchase transaction, firms pay their input suppliers VAT on top of the value of the inputs sourced. At each sales transaction, enterprises charge their buyers VAT on top of the sales value, and in effect transfer to the tax authorities only taxes due on the value added at that stage. The tax is neutral to the enterprise (other than potentially through its effect on enterprises' pre-tax pricing strategy), and the full burden of the tax ultimately lies with the final consumer. The standard VAT rate in Belgium is 21%, but for some goods a reduced rate of 12% or 6% applies.[30]

B.2 Data sources

The empirical analysis draws on three main data sources administered by the National Bank of Belgium (NBB): (i) the NBB B2B Transactions Dataset, (ii) annual accounts from the Central Balance Sheet Office at the NBB supplemented by VAT declarations, and (iii) the Crossroads Bank at the NBB. Firms are identified by a unique number, which is common across these databases and allows for their straightforward merging.

Firm-to-firm relationships The confidential NBB B2B Transactions Dataset contains the values of yearly sales relationships among all VAT-liable Belgian enterprises for the years 2002 to 2014, and is based on the VAT listings collected by the tax authorities. At the end of every calendar year, all VAT-liable enterprises have to file a complete listing of their Belgian VAT-liable customers over that year.[31] An observation in this dataset refers to the sales value in euro of enterprise i selling to enterprise j within Belgium, excluding the VAT amount due on these sales. The reported value is the sum of invoices from i to j in a given calendar year. Whenever this aggregated value is larger than or equal to 250 euro, the relationship has to be reported. Sanctions for incomplete and erroneous reporting guarantee the high quality of the data. Note that the relationship is directed, as the observation from i to j is different from the observation from j to i. This dataset thus covers both the extensive and the intensive margins of the Belgian production network. A detailed description of the collection and cleaning of this dataset is given in Dhyne et al. (2015).

Firm-level characteristics We extract information on enterprises' annual accounts from the Central Balance Sheet Office at the NBB for the years 2002 to 2014. Enterprises above a certain size threshold have to file annual accounts at the end of their fiscal year.[32] We retain

[30] See ec.europa.eu/taxation_customs for a complete list of rates. These rates did not change over our sample period.

[31] Sample VAT listings forms can be found at here (French) and here (Dutch).

[32] See here for filing requirements and exceptions. See here for the size criteria and filing requirements for

information on the enterprise identifier (VAT ID), turnover (total sales in euro, code 70 in the annual accounts), input purchases (total material and services inputs in euro and net changes in input stocks, codes 60+61), labor cost (total cost of wages, social securities and pensions in euro, code 62), and employment (average number of full-time equivalent (FTE) employees, code 9087). We annualize all flow variables in the annual accounts from fiscal years to calendar years by pro-rating the variables on a monthly base. This transformation ensures that all firm-level information in our database is consistent with observations in the VAT listings data.[33]

Enterprises that report abbreviated annual accounts are not required to report turnover or input purchases. For these small enterprises, we supplement info on turnover and inputs from their VAT declarations. All VAT-liable enterprises have to file periodic VAT declarations with the tax administration.[34] The VAT declaration contains the total sales value (including domestic sales and exports), the VAT amount charged on those sales (both to other enterprises and to final consumers), the total amount paid for inputs sourced (including both domestic and imported inputs), and the VAT paid on those input purchases. This declaration is due monthly or quarterly depending on firm size, and it is the basis for the VAT due to the tax authorities every period. We aggregate the VAT declarations to yearly values to correspond to the annual accounts.

We obtain information on the main economic activity of each enterprise at the NACE 4-digit level from the Crossroads Bank of Belgium for the years 2002 to 2014. We concord NACE codes over time to the NACE Rev. 2 version to deal with changes in the NACE classification over our panel from Rev. 1.1 to Rev. 2. Table 13 lists industry groups at the NACE 2-digit level.

B.3 Data construction and cleaning

We calculate the final demand for enterprise i in year t as i's turnover minus the value of all of its B2B sales. This implies that final demand contains final domestic consumption, exports, and business transactions below 250 euro that are not observed in the B2B dataset. Similarly, we calculate observed i's input purchases as i's total input purchases minus the value of all of its B2B purchases. The residual, unobserved part of input expenditures then contains imports and unobserved B2B input acquisitions.

Wages are calculated as labor cost over FTE employment. Labor shares are calculated

either full-format or abridged annual accounts.

[33] 78% of firms have annual accounts that coincide with calendar years, while 98% of firms have fiscal years of 12 months.

[34] Sample VAT declaration forms can be found at here (French) and here (Dutch).

Table 13: NACE classification of industry groups.

NACE Section	NACE Division	Description	Industry
A	NACE 01-03	Agriculture, forestry and fishing	Primary and Extraction
B	NACE 05-09	Mining and quarrying	Primary and Extraction
C	NACE 10-33	Manufacturing	Manufacturing
D	NACE 35	Electricity, gas, steam and air conditioning supply	Utilities
E	NACE 36-39	Water supply; sewage, waste management and remediation activities	Utilities
F	NACE 41-43	Construction	Construction
G	NACE 45-47	Wholesale and retail trade; repair of motor vehicles and motorcycles	Market Services
H	NACE 49-53	Transportation and storage	Market Services
I	NACE 55-56	Accommodation and food service activities	Market Services
J	NACE 58-63	Information and communication	Market Services
K	NACE 64-66	Financial and insurance activities	Market Services
L	NACE 68	Real estate activities	Market Services
M	NACE 69-75	Professional, scientific and technical activities	Market Services
N	NACE 77-82	Administrative and support service activities	Market Services
O	NACE 84	Public administration and defence; compulsory social security	Non-Market Services
P	NACE 85	Education	Non-Market Services
Q	NACE 86-88	Human health and social work activities	Non-Market Services
R	NACE 90-93	Arts, entertainment and recreation	Non-Market Services
S	NACE 94-96	Other service activities	Non-Market Services
T	NACE 97-98	Activities of households as employers; undifferentiated goods- and services-producing activities of households for own use	–
U	NACE 99	Activities of extraterritorial organizations and bodies	–

as labor cost over turnover. We set the labor share equal to one if it is larger than one. For most of the analysis, we use wages and labor shares at the NACE 4-digit industry, by first summing over all firms' labor costs in that industry and then dividing by total FTE employment or total turnover in that industry. We drop firms that have missing employment information or less than one FTE employee.

Throughout the paper, we report statistics on both the full sample in the raw data and the estimation sample used in the firm size decomposition. For the full sample, we keep all B2B relationships in the NBB B2B dataset, even if there is missing firm-level information, as these contribute to the decomposition exercise. We thus keep all enterprises that show up in the network as either a buyer or a seller. For the estimation sample, in *Step One* we first estimate the two-way fixed effects regression on the full sample. Note that if a buyer or seller has only one business relationship, the fixed effect is not identified. This enterprise, together with its connections, is then dropped from the sample. This is done iteratively, until only enterprises that have at least two sellers or two buyers remain. Finally, for the decomposition exercise to contain the same number of observations across all (sub-)components, in *Step Two* and *Step Three* we keep only enterprises that show up as both buyers and sellers in the network. We index firm pairs by the Cantor pairing function to keep the pairing identity consistent over the panel.[35]

Finally, for the counterfactual exercise, we obtain firm-level markups as turnover over inputs and calculate aggregate final demand wL by summing over final demand for all enterprises that are part of the fixed point algorithm. Note that this obtained value is very close to observed GDP in the National Accounts (420 billion euro in 2014).

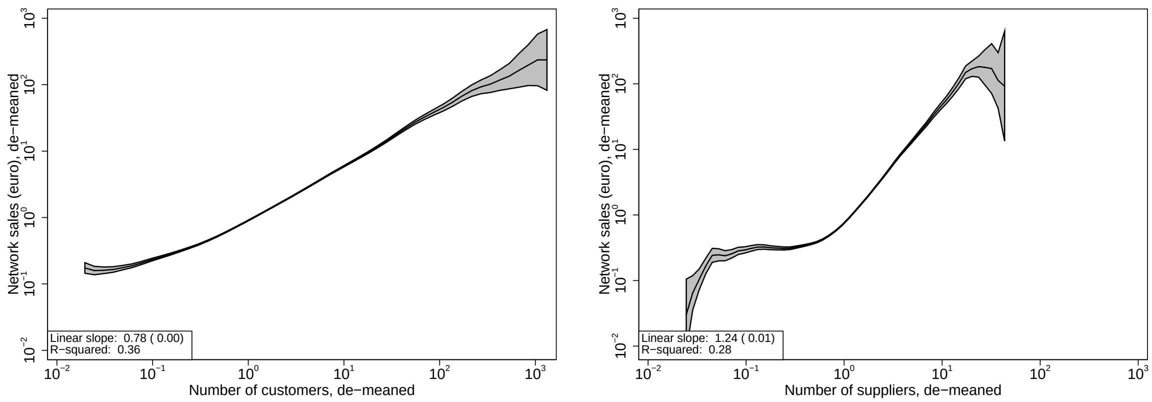
C Additional Descriptive Statistics

Figure 9 replicates Figure 9 in the main text using firms' total B2B sales in the domestic network instead of total turnover. Recall that in addition to downstream domestic network sales, total turnover also includes domestic final demand and exports. Figure 9 plots the fitted line and 95% confidence interval from a local polynomial regression of domestic network sales on the number of downstream customers or upstream suppliers, on a log-log scale. The pattern is very similar to the baseline in the main text: a strong monotonic relationship with implied elasticities of 0.78 and 1.24, respectively.

[35]In particular: $p_{ij} = \frac{1}{2}(a+b) \times (a+b+1) + b$, where p_{ij} is the pair ID and a and b are the seller and buyer ID respectively.

Figure 9: Firm size and number of buyers and suppliers (2014).

(a) Firm sales and number of buyers. (b) Firm sales and number of suppliers.



Note: The number of customers and suppliers is demeaned at the NACE 4-digit level. Graphs are trimmed at the 0.1st and 99.9th percentiles of the number of customers and suppliers respectively.

Table 14 reports the two-way correlation coefficients among all firm size components that we obtain in *Step One* and *Step Two* of the estimation procedure and that we use in the decomposition analysis in *Step Three*.

Table 14: Correlation among firm size components.

	Firm Size Component	$\ln S_i$	$\ln \psi_i$	$\ln \xi_i$	$\ln \beta_i^c$	$\ln \tilde{z}_i$	$\ln n_i^s$	$\ln \bar{\psi}_i$	$\ln \Omega_i^s$	$\ln \beta_i^s$	$\ln n_i^c$	$\ln \bar{\theta}_i$	$\ln \Omega_i^c$
	Total Sales, $\ln S_i$	1											
	Upstream Supply, $\ln \psi_i$	0.25	1										
$\ln S_i$	Downstream Network Demand, $\ln \xi_i$	0.65	-0.14	1									
	Final Demand, $\ln \beta_i^c$	0.01	-0.36	-0.52	1								
	Production Capability, $\ln \tilde{z}_i$	-0.52	0.64	-0.58	-0.30	1							
	# Suppliers, $\ln n_i^s$	0.76	-0.02	0.62	0.00	-0.61	1						
$\ln \psi_i$	Avg. Supplier Capability, $\ln \bar{\psi}_i$	0.19	0.28	0.01	-0.04	0.02	0.02	1					
	Supplier Covariance, $\ln \Omega_i^s$	0.65	0.18	0.39	0.04	-0.40	0.36	0.17	1				
	Outside-Network Supply, $\ln \beta_i^s$	0.10	0.08	0.00	0.04	-0.08	-0.14	-0.13	-0.27	1			
	# Customers, $\ln n_i^c$	0.46	-0.31	0.85	-0.37	-0.61	0.55	-0.07	0.26	-0.02	1		
$\ln \xi_i$	Avg. Customer Capability, $\ln \bar{\theta}_i$	0.21	0.21	0.15	-0.15	0.05	0.07	0.06	0.11	0.09	-0.18	1	
	Customer Covariance, $\ln \Omega_i^c$	0.46	0.18	0.57	-0.41	-0.18	0.33	0.13	0.32	-0.01	0.09	0.24	1

Note: All size components are demeaned at the NACE 4-digit level. All correlations are significant at 5%