MULTI-PRODUCT FIRMS AND TRADE LIBERALIZATION

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Abstract

This paper develops a general equilibrium model of multi-product firms and analyzes their behavior during trade liberalization. Firm productivity in a given product is modeled as a combination of firm-level “managerial ability” and firm-product-level “product expertise”, both of which are stochastic and unknown prior to the firm’s payment of a sunk cost of entry. Firm productivity is positively correlated across products, which in equilibrium induces a positive correlation between a firm’s intensive (output per product) and extensive (number of products) margins. Trade liberalization fosters productivity growth both within-firms and in aggregate by inducing firms to shed marginally productive products and forcing the lowest-productivity firms to exit. Though exporters produce a smaller range of products after liberalization, they increase the share of products sold abroad as well as exports per product. All of these adjustments are shown to be relatively more pronounced in countries’ comparative advantage industries.

Keywords: heterogeneous firms, endogenous product scope, love of variety, core competency

JEL classification: F12, F13, L11

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1. Introduction

Firms producing multiple products dominate domestic production and international trade. In the U.S., firms manufacturing more than one product account for over 90% of total manufacturing shipments, while firms that export multiple products represent more than 95% of total exports. Despite the overwhelming empirical dominance of multi-product firms, comparatively little theoretical research examines how firms’ product scope (i.e., the number of goods produced or exported) is determined or how it is influenced by globalization.

This paper develops a general equilibrium model of international trade in which firms that are heterogeneous in managerial ability and product expertise choose endogenous ranges of products to manufacture and sell abroad. The model is motivated by a series of stylized facts about multi-product firms that have emerged in the recent literature, most notably a positive correlation between the number of products a firm produces and its output per product. The framework captures these stylized facts and yields a number of new theoretical implications regarding firm behavior and aggregate productivity growth in both closed and open economies.

A key prediction of the model is that trade liberalization induces adjustments along firms’ extensive (i.e., number of products) and intensive (i.e., sales per product) margins as well as entry and exit. As trade costs fall, reallocation occurs both within firms, as surviving firms drop their marginally productive products, and across firms, as firms with the lowest overall productivity exit. Though exporters are among the firms dropping their least-productive products, the share of products they export as well as their sales per exported product rise. Due to these adjustments, productivity increases both within firms and in aggregate, and we show that all of these adjustments are relatively more pronounced in countries’ comparative advantage industries.

Our analysis extends existing models of industry dynamics (e.g., Jovanovic 1982, Hopenhayn 1992, Ericson and Pakes 1995, Melitz 2003 and Bernard, Redding and Schott 2006a) to multi-product firms with endogenous product scope. As in those models, the firms considered here are heterogeneous in productivity. In this paper, however, the “productivity” of a firm in a given product is a combination of two capabilities: firm-level “managerial ability” and firm-product-level “product expertise”. We assume these two components of overall firm productivity to be stochastic, independent, and unknown prior to firms paying a sunk cost of entry. These assumptions are meant to capture the idea that even though some firms are successful in a broad range of activities (e.g., General Electric), even the most successful firm may not be the most efficient producer of any given product. Though we assume that managerial ability and product expertise are uncorrelated, and that product expertise across products within firms is uncorrelated, a firm’s productivity across products is nevertheless positively correlated because higher managerial ability raises productivity in all products.

The model works as follows. After a firm incurs the sunk entry cost, and after managerial ability as well as product expertise in all products are revealed, the firm decides which (if any) of a continuum of products to manufacture. Firms that decide to enter must pay a fixed headquarters cost which is independent of the range of products manufactured, as well as fixed production costs for each product market they enter. Marginal costs of production vary across products, declining with managerial ability and product expertise. Managerial ability

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1These figures are taken from the Bernard, Redding and Schott (2006b) and Bernard, Jensen and Schott (2005). They define products as five-digit SIC and ten-digit Harmonized System categories, respectively.

2The importance of general managerial practice and its variation across firms has received renewed emphasis in recent work, including Garicano (2000), Bertrand (2003), Garicano and Rossi-Hansberg (2006) and Bloom and Van Reenen (2006). See also Lucas (1978) and Rossi-Hansberg and Wright (2006).
varies across firms and product expertise varies across products for a firm with a particular managerial ability. Firms with higher managerial abilities have a lower zero-profit cutoff for product expertise, i.e., the level of product expertise at which variable profit exceeds the fixed production costs. As a result, firms with greater managerial ability can profitably manufacture a larger range of products, extending their product scope. Though firms with high managerial ability enjoy higher average productivity than lower-managerial-ability firms, the percentage difference in average productivity is smaller than the percentage difference in managerial ability as a result of their endogenous expansion into lower-expertise products.

The equilibrium of the model exhibits conventional self selection of the most productive firms into production. Here, self selection across firms is driven by the interaction of stochastic managerial ability and fixed headquarters costs. Firms drawing low managerial ability are unable to generate enough variable profit to cover fixed headquarters costs from the small number of products in which they have high expertise. As a result, firms drawing the lowest managerial ability have the lowest average productivity and exit immediately.

With endogenous product scope, the model also features self selection within firms. Firms’ product selection is explained by the interaction of stochastic product expertise and fixed production costs for each product. Firms that become active in a product have systematically higher productivity in that product than in the products they choose not to produce. They also have systematically higher productivity than firms that decide not to produce the product.

Stochastic variation in managerial ability yields a positive correlation between firms’ intensive and extensive margins: more productive firms are on average larger not only because they sell more of a given product but because they manufacture more products. This positive correlation between intensive and extensive margins magnifies inequality in the firm-size distribution and has general equilibrium implications for firms’ entry and exit decisions. As higher-managerial-ability firms expand along the extensive margin into lower-expertise products, demand for labor rises, wages increase and the profitability of lower-managerial-ability firms declines. Relative to a setting with exogenous product scope, this mechanism drives up the zero-profit threshold for managerial ability above which firm entry is profitable.

One manifestation of the influence of endogenous product scope is the role it plays in dampening entry and exit responses to exogenous shocks to market structure. In conventional heterogeneous-firm models, an increase in sunk costs of entry, for example, reduces the zero-profit productivity cutoff necessary for survival, allowing firms with previously unprofitable levels of productivity to enter the market. This shift in the cutoff is driven by a reduction in the ex ante expected profitability of entry: when sunk entry costs increase, ex ante expected profitability decreases, entry from the fringe declines, competition is diminished and the ex post profitability of production increases to the point that it covers the increase in sunk entry costs. When product scope is endogenous, the reduction in competition induced by the rising sunk entry cost allows firms to expand along their extensive margins into lower-expertise products. As a result, some of the increase in the ex post profitability required to equal the new sunk cost of entry is achieved through an expansion of product scope, implying a dampened decline in the zero-profit productivity cutoff. While there is less reallocation through firm entry and exit than in a model with exogenous product scope, the extent of reallocation is greater than implied by firm entry and exit alone due to reallocation across products within firms.3

The open economy equilibrium permits examination of a wide range of firm responses to globalization. When international trade with fixed and variable trade costs is possible,  

3For empirical evidence on firm entry and exit as a source of reallocation and productivity growth, see Dunne et al. (1989), Baily et al. (1992) and Foster et al. (2001) among many others. For evidence on the role of reallocation across products within firms, see Bernard, Redding and Schott (2006b).
the model reproduces the conventional result that only the highest-productivity firms are profitable enough to overcome trade costs and become exporters. With endogenous product scope, however, exporters ship only a relatively small fraction of their overall output abroad. While exporters’ high-expertise products are profitable enough to overcome trade costs and are sent abroad, their lower-expertise goods are confined to the domestic market. In equilibrium the same product might be both exported and sold domestically by some firms and only sold locally by others.

A key result of the model is that trade liberalization pressures firms to focus on their “core competencies”. When product scope is endogenous, symmetric reductions in countries’ fixed or variable trade costs cause all firms to drop their lowest-expertise products. This result is driven by the general equilibrium influence of growing export opportunities on labor markets. As trade costs fall, exporters’ foreign markets expand, driving up their demand for labor and therefore wages. As wages rise, the profitability of firms’ lowest-expertise products falls below the point at which they can cover fixed costs. However, even though product scope contracts for all firms, declining trade costs result in an increase in both the share of products exporters sell abroad as well as their level of exports in each product. Because these reactions are driven by greater export opportunities, they are all relatively more pronounced in countries’ comparative advantage industries.

Firms’ intensive- and extensive-margin responses to trade liberalization drive productivity growth both within and across firms. As firms drop their lowest-expertise products, they raise the average productivity of products that survive and therefore raise overall firm productivity. Therefore, although firm managerial ability and product expertise are themselves parameters, the model generates firm-level productivity growth following trade liberalization due to reallocations of resources across products within firms. This contrasts with standard models of industry dynamics, where firms manufacture a single product whose productivity follows an exogenous stochastic process that is unaffected by trade liberalization. Endogenous product scope therefore provides a novel new micro-foundation for the much-discussed idea that international trade spurs firms to rationalize production, but one that does not imply money being left on the table prior to liberalization. Along with the conventional result that symmetric reductions in trade costs cause the failure of low-productivity firms, endogenous product scope contributes to aggregate productivity growth that is strongest in countries’ comparative advantage industries. Trade liberalization gives rise to endogenous Ricardian productivity differences at both the firm and industry level that magnify Heckscher-Ohlin-based comparative advantage and provide a new source of welfare gains from trade.

The positive correlation between firms’ intensive and extensive margins influences adjustment in the domestic and export markets. As trade costs fall, there is a contraction in production for the domestic market, through both a reduction in the amount of each product supplied and the range of products offered. At the same time, there is an expansion in production for the export market, through both the amount of each product exported and the range of products shipped abroad. Therefore, trade liberalization has a magnified impact on the firm-size distribution relative to a model with exogenous product scope. High managerial ability exporting firms are larger than low managerial ability non-exporting firms, not only

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4 The concept of core competencies appears in both the theory of the firm and the business strategy literature (see for example Aghion and Tirole 1995, Porter 1998 and Sutton 2005). As formalized here, core competencies refer to the products where firms have the greatest product expertise.

5 The hypothesis that trade liberalization raises firm productivity by reducing X-inefficiency has a long tradition, but theories of this form face the challenge of explaining X-inefficiency as an equilibrium phenomenon (see for example the discussion in Lawrence 2000).
because they sell more of a given product in the domestic market, but also because they manu-
ufacture a larger range of products and export a range of products. Again these implications of
trade liberalization vary with comparative advantage, so that firms in comparative advantage
industries export a greater share of their output of a given product and a greater fraction of
their products than those in comparative disadvantage industries.

The research in this paper complements several existing models of product scope in in-
dustrial organization and international trade. Early research on multi-product firms in the
industrial organization literature by Baumol (1977), Panzar and Willig (1977) and others (see
the survey in Bailey and Friedlaender 1982) emphasized supply-side economies of scope. More
recent efforts by Brander and Eaton (1984), Shaked and Sutton (1990) and Eaton and Schmidt
(1994), introduce demand-side considerations and focus on the strategic interaction of firms
in product markets. Our contribution relative to this literature is to examine multi-product
firms within a general equilibrium model of industry dynamics. We are therefore able to ex-
amine the implications of trade liberalization for the economy as a whole within a framework
where both firm entry and exit and the distribution of productivity within and across firms
are endogenous.

In the international trade literature, Baldwin and Ottaviano (2001) use a model of multi-
product multinationals to explain two-way Foreign Direct Investment (FDI) within industries.
In the presence of trade costs, multi-product firms are able to reduce inter-variety competition
by locating production of some varieties abroad through FDI. Eckel and Neary (2006) develop
a theory of multi-product firms based upon flexible manufacturing, in which each firm has an
ideal product but can produce additional products by incurring an adaptation cost that rises
with the number of products. Firms compete under conditions of Cournot oligopoly subject to
quasi-linear preferences. This framework captures strategic interaction between firms but, in
contrast to the analysis above, assumes that the number of multi-product firms is exogenous
and that multi-product firms are homogeneous within industries.6 Nocke and Yeaple (2006)
develop a theory of multi-product firms in which firms vary in terms of a single dimension,
"organizational capability", and in which a firm’s productivity for all of its products declines
with the number of products manufactured. In equilibrium, firms with higher organizational
capability produce a greater number of products but have lower overall productivity than
firms with low organizational capability. In contrast with this paper, and the stylized facts
discussed below, their model predicts an equal distribution of output across products within
multi-product firms, and a negative correlation between firms’ intensive and extensive margins.

The assumptions upon which our model is based are motivated by the large body of em-
pirical evidence that firm entry and exit and within-industry reallocation across heterogeneous
firms are key features of trade liberalizations (see for example Pavcnik 2002, Trefler 2004 and
the survey by Tybout 2003). Our model highlights the way in which trade liberalization raises
the expected value of industry entry and induces endogenous changes in firm entry and exit
that cause multi-product firms to focus on their core competencies. The correlation between
firms’ intensive and extensive margins, which is a key prediction of our framework and which
receives empirical support, is driven by the within-industry heterogeneity in managerial ability
across multi-product firms.

The remainder of the paper is structured as follows. Section 2 summarizes the empirical
motivation for the model. Section 3 introduces the structure of the model and Section 4 solves
for the closed economy equilibrium. In Section 5 we characterize a number of properties of

6A large empirical literature documents substantial heterogeneity across firms within industries. See, for
example, the survey by Bartelsman and Doms (2000). In a partial equilibrium version of their analysis, Eckel
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the closed economy equilibrium. Section 6 opens the economy to international trade. Section 7 derives a number of results about the open economy equilibrium and develops a number of predictions for the effects of trade liberalization. Section 7 generalizes the model to two industries and factors of production to consider the influence of comparative advantage. Section 8 concludes. Appendices A and B contain data and theory appendices respectively.

2. Empirical Motivation

Our model and its assumptions are motivated by a series of stylized facts about multi-product firms reported by Bernard, Redding and Schott (2006b) and Bernard, Jensen and Schott (2005), henceforth BRS and BJS, respectively. These facts document the importance of multi-product firms for aggregate production and trade as well as the substantial heterogeneity of products within firms. In this section we discuss these stylized facts and offer others that are inspired by our main theoretical results.

Multi-product firms dominate U.S. manufacturing output as well as overall U.S. trade. Analyzing quinquennial manufacturing censuses from 1972 to 1997, BRS find that an average 41 percent of U.S. manufacturing firms produce more than a single product, but that these firms account for an average of 91 percent of output. Moreover, net product adding and dropping by incumbent firms accounts for roughly one third of aggregate U.S. manufacturing growth between 1972 and 1997, a contribution that dwarfs that of new firm entry and exit. Similarly, BJS (2005), analyzing U.S. trade data for 2000, show that firms that export more than ten products make up 17 percent of exporting firms but account for 94 percent of all exports. Comparable numbers are reported with respect to importing firms.

BRS demonstrate that goods produced by multi-product firms exhibit substantial heterogeneity. The same firm, for example, often produces products in more than a single four- or even two-digit SIC industry. Firms’ product-size distribution, moreover, is highly skewed, with firms’ largest products accounting for an average of forty percent of overall firm output. BRS also show that product characteristics such as relative size and tenure with the firm are associated with firms’ decisions to drop products from their portfolios. A firm is more likely to exit a product where its output and tenure are small relative to those of its rivals after controlling for firm and product fixed effects.

Our model highlights the contribution of firms’ extensive margins to the firm-size distribution. Here, we augment the existing stylized facts on multi-product firms by examining this contribution empirically across U.S. firms. Total firm output \( Y_f \) is the sum of output of individual products \( y_{pf} \), which equals the number of products \( n_f \) times average output per product \( \bar{y}_f \),

\[
Y_f = n_f \bar{y}_f, \quad \bar{y}_f = \frac{1}{n_f} \sum_p y_{pf}
\]  

where \( f \) and \( p \) index firms and products respectively. This relationship provides the basis for a regression decomposition of variation in firm size into the contributions of the extensive and intensive margins. The log number of products and log average output per product are each separately regressed on log total firm output or exports,

\[
\begin{align*}
\ln n_f &= \alpha + \beta_1 \ln Y_f + u_f \\
\ln \bar{y}_f &= \alpha + \beta_2 \ln Y_f + \varepsilon_f
\end{align*}
\]

where \( u_f \) and \( \varepsilon_f \) are stochastic errors and, by the properties of OLS, \( \beta_1 + \beta_2 = 1 \). The coefficients \( \beta_1 \) and \( \beta_2 \) capture the partial correlations between total firm output and the extensive
and intensive margins respectively. When total firm output increases, $\beta_1$ captures the percentage contribution from the number of products (the extensive margin), holding constant average output per product (the intensive margin). Similarly, $\beta_2$ captures the percentage contribution of the intensive margin, holding constant the extensive margin.

Regression decompositions for both production and exporting are contained in Table 1. For brevity we report results for $\beta_1$ given that $\beta_2 = 1 - \beta_1$. The first two columns of the table summarize the results of regressing the log number of five-digit SIC manufacturing products produced by U.S. multi-product firms in 1997 on their log total output in that year. The dataset is identical to that examined by BRS and are drawn from the Longitudinal Research Database of the United States Bureau of the Census. Column one displays the decomposition from an OLS regression while column two reports results of a weighted OLS regression where firms’ total outputs are used as weights. As indicated in the table, firms’ extensive margins account for 13 and 19 percent of the variation in total firm output, respectively.

Table 1: Regression Decompositions – Shipments

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<tr>
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<tbody>
<tr>
<td></td>
<td>OLS</td>
<td>Weighted OLS</td>
</tr>
<tr>
<td>In Firm Shipments</td>
<td>0.13205</td>
<td>0.19002</td>
</tr>
<tr>
<td></td>
<td>0.00017</td>
<td>0.00017</td>
</tr>
<tr>
<td>In Firm Exports</td>
<td>0.00017</td>
<td>0.00017</td>
</tr>
<tr>
<td>Observations</td>
<td>109,267</td>
<td>109,267</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.85</td>
<td>0.92</td>
</tr>
</tbody>
</table>

Notes: Table reports firm-level OLS regressions of the number of products produced and exported by U.S.-based firms on the total shipments and exports of the firm, respectively, in the noted years. Columns two and four report results for weighted regressions, where firm shipments and exports, respectively, are used for weights. Sample includes all multi-product firms in the noted year. All estimates are significant at the 1 percent level.

Columns three and four of Table 1 report an analogous decomposition of U.S. firm-product exports in 2000 using trade data from BJS. These data differ from the manufacturing data in two important ways. First, the product classification is much more disaggregate. Second, the products span all sectors, i.e., they are not confined to manufacturing. Here, too, we find a substantial role for firms’ extensive margins, with coefficients indicating that variation in the number of products firms export is responsible for 23 and 29 percent of the variation in their total exports, respectively. The relatively high coefficients in the exporting data are likely due to that data’s more finely disaggregate product categories. Five-digit SIC codes are relatively coarse compared with ten-digit HS categories. As a result, product-mix changes occurring within five-digit SIC categories appear as intensive- rather than extensive-margin adjustments.

7In the U.S. production data, BRS define a product as a five-digit Standard Industrial Classification (SIC) category. This level of aggregation breaks manufacturing into roughly 1500 products. In the U.S. trade data, BRS define a product as a ten-digit Harmonized System (HS) category. This level of aggregation breaks all U.S. exports into roughly 8,500 products, with manufacturing products making up approximately two thirds of this total. We return to the implications of using more disaggregate trade than production data below.

8The firm-level regressions in equation (2) capture partial correlations, e.g., the contribution toward an increase in $Y_f$ made by $n_f$ holding $y_f$ constant. An alternate approach for investigating the importance of firms’
One of the distinguishing features of the model that we develop below is a positive correlation between firms' extensive and intensive margins. To examine this correlation in the data, we regress firms' average output or exports per product on the number of products the firm produces or exports,

$$\ln \bar{y}_f = \eta + \gamma \ln n_f + \chi_f,$$

where $\chi_f$ is a stochastic error. Results are reported in Table 2. As indicated in the table, firms' intensive and extensive margins are correlated positively and significantly in both production and exporting. These results demonstrate that larger firms produce and export a larger number of products and also exhibit larger output or exports per product than smaller firms.\(^9\)

Table 2: Correlation of Extensive and Intensive Margins

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>OLS</td>
<td>0.4910</td>
<td>0.6555</td>
</tr>
<tr>
<td>ln Number of Products</td>
<td>0.0105</td>
<td>0.0013</td>
</tr>
<tr>
<td>Observations</td>
<td>109,267</td>
<td>662,397</td>
</tr>
<tr>
<td>R-Squared</td>
<td>0.02</td>
<td>0.38</td>
</tr>
</tbody>
</table>

Notes: Table reports firm-level OLS regressions of the average log shipments or exports per product of U.S. firms on the log number of products produced and exported, respectively, in the noted years. Columns two and four report results for weighted regressions, where total firm shipments and exports, respectively, are used for weights. Sample includes all multi-product firms in the noted year. All estimates are significant at the 1 percent level.

The model developed in the remainder of the paper both captures the stylized facts discussed in this section and remains consistent with widely known attributes of firms from the literature, such as the existence of substantial heterogeneity of firms within industries and relatively high rates of overall firm exit and entry. We use the model to examine the implications of the new extensive margin of firm activity for the economy’s adjustment to trade liberalization.

3. The Model

3.1. Preferences

The representative consumer derives utility from the consumption of a continuum of products which we normalize to the interval [0, 1]. There is a constant elasticity of substitution extensive margins is to decompose the variance in firm output into parts attributable to the variance in the number of products, the variance in the average output per product, and the covariance of $n_f$ and $\bar{y}_f$,.

$$\text{var} (\ln Y_f) = \underbrace{\text{var} (\ln n_f)}_{\text{Term A}} + \underbrace{\text{var} (\ln \bar{y}_f)}_{\text{Term B}} + \underbrace{2\text{cov} (\ln n_f, \ln \bar{y}_f)}_{\text{Term C}}.$$

Since all terms on the right-hand side are positive, this equation provides the basis for a percentage decomposition. In the production and trade data described in the text, Terms A and C account for 11 and 70 percent of the total variance, respectively.

\(^9\)The results hold across time and are robust to changes in the sample.
across products so that the utility function takes the following form:

\[ U = \left[ \int_{0}^{1} C_i^\nu di \right]^{\frac{1}{\nu}}, \quad 0 < \nu < 1. \quad (4) \]

where \( i \) indexes products. Within each product market, a continuum of firms produce differentiated varieties of the product, so that \( C_i \) is a consumption index which also takes the constant elasticity of substitution form:

\[ C_i = \left[ \int_{\omega \in \Omega} c_i(\omega)^\rho d\omega \right]^{\frac{1}{\rho}}, \quad 0 < \rho < 1. \quad (5) \]

where \( \omega \) indexes varieties. We make the natural assumption that the elasticity of substitution across varieties within products is greater than the elasticity of substitution across products, \( \sigma \equiv \frac{1}{1-\rho} > \kappa \equiv \frac{1}{1-\nu} > 1 \), and the price index dual to (5) is:

\[ P_i = \left[ \int_{\omega \in \Omega} p_i(\omega)^{1-\sigma} d\omega \right]^{\frac{1}{1-\sigma}}. \quad (6) \]

3.2. Production Technology

The specification of entry and production follows Melitz (2003). However, we augment that model to allow firms to manufacture multiple products and to allow for uncertainty about productivity that is specific to individual product markets. There is a competitive fringe of potential firms who are identical prior to entry. In order to enter, firms must incur a sunk entry cost of \( f_e > 0 \) units of labor. After the sunk cost is paid, firms draw a “managerial ability” \( \varphi \in (0, \infty) \) that is common to all products from a distribution \( g(\varphi) \) and a “product expertise” \( \lambda_i \in (0, \infty) \) for each product \( i \) from a distribution \( z(\lambda) \). Therefore, \( \varphi \) varies across firms whereas \( \lambda_i \) is specific to individual pairings of firms and products. This conceptualization of firm productivity captures the idea that some components of firm productivity enhance efficiency for the firm as a whole while other components (e.g. specialized technical knowledge) only apply within particular product markets. The two distributions \( g(\varphi) \) and \( z(\lambda) \) are independent of one another and common to all firms, and the product expertise distribution is the same for all products. The continuous cumulative distribution functions for managerial ability and product expertise are denoted by \( G(\varphi) \) and \( Z(\lambda) \). We focus on an interior equilibrium where no firm manufactures either all or no products. Therefore, we assume \( \lim_{\varphi \to 0} g(\varphi) > 0 \), \( \lim_{\varphi \to \infty} G(\varphi) \to 1 \), \( \lim_{\lambda \to 0} z(\lambda) > 0 \), and \( \lim_{\lambda \to \infty} Z(\lambda) \to 1 \).

Incurring the sunk entry cost creates the blueprint for a horizontally differentiated variety of each product. Once the sunk cost has been paid, and managerial ability and product expertise are observed, firms decide whether to enter and in which product markets to participate. For now, labor is the sole factor of production and is in inelastic supply \( L \) (which also indexes the size of the economy).\(^{10}\) There is a fixed corporate headquarters cost of \( f_h > 0 \) units of labor which the firm must incur irrespective of the number of product markets in which it is active and a fixed production cost of \( f_p > 0 \) units of labor for each product. There is a constant marginal cost of production for each product which depends on the interaction of managerial ability and product expertise. Total labor employed by a firm is therefore:

\[ l(\varphi) = f_h + \int_{0}^{1} I_i \left[ f_p + \frac{q(\varphi, \lambda_i)}{\varphi \lambda_i} \right] di, \quad (7) \]

\(^{10}\)We analyze a setting with two factors of production, skilled and unskilled labor, in Section 9.
where \( I_i \) is an indicator variable which equals one if a firm produces product \( i \) and zero otherwise, and \( q(\varphi, \lambda_i) \) denotes output of product \( i \) by a firm with managerial ability \( \varphi \) and product expertise \( \lambda_i \).

Even though the distributions \( g(\varphi) \) and \( z(\lambda) \) have been assumed to be independent of one another, a firm’s productivity in a product market \( i \), \( \theta_i \equiv \varphi \lambda_i \), depends on both managerial ability and product expertise. In particular, a higher level of managerial ability \( \varphi \) raises productivity \( \theta_i \) across all products \( i \). Therefore, productivity (the combination of managerial ability and product expertise) is positively correlated across products, but the correlation is imperfect due to the stochastic variation in product expertise.\(^{11}\)

3.3. Firm-Product Profitability

Demand for a product variety depends upon the own variety price, the price index for the product and the price indices for all other products. If a firm is active in a product market, it manufactures one of a continuum of varieties and so is unable to influence the price index for the product. At the same time, the price of firm’s variety in one product market only influences the demand for its varieties in other product markets through the price indices. Therefore, the firm’s inability to influence the price indices implies that its profit maximization problem reduces to choosing the price of each product variety separately to maximize the profits derived from that product variety.\(^{12}\) This optimization problem yields the standard result that the equilibrium price of a product variety is a constant mark-up over marginal cost:

\[
p_i(\varphi, \lambda_i) = \frac{1}{\rho} \frac{w}{\varphi \lambda_i},
\]

where we choose the wage for the numeraire and so \( w = 1 \).

Substituting for the pricing rule, and using the choice of numeraire, equilibrium revenue and profits from a product variety are:

\[
r_i(\varphi, \lambda_i) = R_i(\rho P_i \varphi \lambda_i)^{\sigma - 1}, \quad \pi_i(\varphi, \lambda_i) = \frac{r_i(\varphi, \lambda_i)}{\sigma} - f_p,
\]

where \( R_i \) denotes aggregate expenditure on product \( i \). From equation (9), the ratio of revenue for two varieties of the same product depends solely on their relative productivities:

\[
r_i(\varphi'', \lambda_i') = (\varphi''/\varphi')^{\sigma - 1} (\lambda''/\lambda')^{\sigma - 1} r_i(\varphi', \lambda_i').
\]

A firm with a particular managerial ability draw \( \varphi \) and product expertise draw \( \lambda_i \) decides whether or not to produce a product based on a comparison of revenue and fixed production costs for the product. For each managerial ability \( \varphi \), there is a zero-profit cutoff for product expertise \( \lambda^*_i(\varphi) \), such that the firm will enter the product if it draws a value of \( \lambda_i \) equal to or greater than \( \lambda^*_i(\varphi) \). For each managerial ability \( \varphi \), the value of \( \lambda^*_i(\varphi) \) is defined by the following zero-profit condition:

\[
r_i(\varphi, \lambda^*_i(\varphi)) = \sigma f_p.
\]

\(^{11}\)Note this property does not depend on managerial ability and product expertise combining multiplicatively (\( \theta_i \equiv \varphi \lambda_i \)). All we require is that, for two firms with the same expertise (\( \lambda_i \)), the firm with a higher managerial ability (\( \varphi \)) has the higher productivity (\( \theta_i \)) in the product. This would be satisfied by for example a linear specification (\( \theta_i \equiv \varphi + \lambda_i \)).

\(^{12}\)The structure of our model eliminates strategic interaction within or between firms. This choice of model structure enables us to focus purely on the implications of introducing endogenous product scope into a standard model of industry dynamics that captures the ongoing entry and exit and firm heterogeneity that are central features of micro data.
We refer to the lowest level of managerial ability where a firm finds it profitable to incur the fixed headquarters cost and enter as the **zero-profit cutoff for managerial ability** \( \varphi^* \). The zero-profit cutoff for product expertise for a firm with managerial ability \( \varphi^* \) is defined by:

\[
    r_i (\varphi^*, \lambda_i^* (\varphi^*)) = \sigma f_p.
\]

Since there is a continuum of identical products, the zero-profit cutoff for product expertise is the same across all products: \( \lambda_i^* (\varphi) = \lambda^* (\varphi) \) for all \( i \). Furthermore, since product expertise is independently and identically distributed, the law of large numbers implies that the fraction of products manufactured by a firm with a particular level of managerial ability \( \varphi \) equals the probability of that firm drawing a product expertise above \( \lambda^* (\varphi) \), that is \([1 - Z (\lambda^* (\varphi))]\).  

### 3.4. Firm Profitability

A firm with a particular managerial ability draw \( \varphi \) decides whether or not to enter based on a comparison of the fixed headquarters cost and total profits across those products where its product expertise draw \( \lambda_i \) is greater than the zero-profit cutoff \( \lambda^* (\varphi) \).

With a continuum of identical products and independent draws for product expertise, the law of large numbers implies that a firm’s average revenue across the continuum of products equals its average revenue for an individual product. Since the continuum of products is of interval one, total and average revenue across products are the same and are both equal to average revenue for an individual product. Average revenue for an individual product is a function of managerial ability \( \varphi \) and equals the probability of drawing a product expertise above the zero-profit cutoff \( \lambda^* (\varphi) \) times average revenue conditional on profitable production. Therefore firm revenue across products is:

\[
    r (\varphi) = \int_0^1 \left[ \int_0^\infty r (\varphi, \lambda_i) \mu_z (\lambda_i) d\lambda_i \right] d\varphi = \int_0^\infty \left[ 1 - Z (\lambda^* (\varphi)) \right] \int_0^\lambda r (\varphi, \lambda) \left( \frac{z (\lambda)}{1 - Z (\lambda^* (\varphi))} \right) d\lambda \\lambda^* (\varphi) = \int_\lambda^\lambda^* (\varphi) r (\varphi, \lambda) z (\lambda) d\lambda,
\]

where the *ex post* distribution for product expertise \( \mu_z (\lambda_i) \) is a truncation of the *ex ante* distribution \( z (\lambda) \) above the zero-profit cutoff for product expertise \( \lambda^* (\varphi) \).

Total profits across products equal average profits for an individual product minus fixed headquarters costs. Firm profits are therefore equal to the probability of drawing a product expertise above the zero-profit cutoff \( \lambda^* (\varphi) \) times average profits conditional on profitable production minus fixed headquarters costs:

\[
    \pi (\varphi) = \left[ 1 - Z (\lambda^* (\varphi)) \right] \int_0^\infty \left( \frac{r (\varphi, \lambda)}{\sigma} - f_p \right) \left( \frac{z (\lambda)}{1 - Z (\lambda^* (\varphi))} \right) d\lambda - f_h = \int_\lambda^\lambda^* (\varphi) \left( \frac{r (\varphi, \lambda)}{\sigma} - f_p \right) z (\lambda) d\lambda - f_h
\]

The lower the managerial ability of a firm \( \varphi \), the higher the zero-profit cutoff for product expertise \( \lambda^* (\varphi) \), and so the lower the probability of drawing a product expertise sufficiently high to profitably manufacture a product. Firms with lower managerial abilities have lower
expected profits in an individual product market and manufacture a smaller fraction of products \([1 - Z (\lambda^* (\varphi))]\). For sufficiently low draws of managerial ability, the excess of revenue over fixed production costs in the small range of products that the firm finds it profitable to manufacture falls short of the fixed headquarters cost. Therefore, the requirement that firm profits net of fixed headquarters costs are greater than or equal to zero defines a zero-profit cutoff for managerial ability \(\varphi^*\), such that only firms who draw a managerial ability equal to or greater than \(\varphi^*\) enter:

\[
\pi (\varphi^*) = 0. \tag{15}
\]

### 3.5. Free Entry

Firms from the competitive fringe decide whether or not to enter based on a comparison of the expected value of entry and the sunk entry cost. We assume that there is a constant exogenous probability of firm death \((\delta)\), as a result of force majeure events beyond the managers control, which generates ongoing firm entry and exit. Therefore, the expected value of entry \((v_e)\) is the \(ex \ ante\) probability of successful entry, multiplied by average firm profits conditional on entry \((\bar{\pi})\), and discounted by the probability of firm death. Free entry requires:

\[
v_e = \frac{[1 - G (\varphi^*)]}{\delta} \bar{\pi} = f_e, \tag{16}
\]

where the \(ex \ ante\) probability of successful entry is \([1 - G (\varphi^*)]\).

Firm revenue for a particular value of managerial ability \(\varphi\) was determined above. Taking expectations over values for managerial ability yields average firm revenue conditional on entry:

\[
\bar{r} = \int_0^\infty r (\varphi) \mu_g (\varphi) d\varphi, \tag{17}
\]

\[
= \int_{\varphi^*}^\infty r (\varphi) \left( \frac{g (\varphi)}{1 - G (\varphi^*)} \right) d\varphi,
\]

where the \(ex \ post\) distribution for managerial ability \(\mu_g (\varphi)\) is a truncation of the \(ex \ ante\) distribution \(g (\varphi)\) above the zero-profit cutoff for managerial ability \(\varphi^*\) where entry occurs. The expression for average firm profits conditional on entry \(\bar{\pi}\) is analogous and takes expectations over values for managerial ability in equation (14).

### 3.6. Goods and Labor Markets

The steady-state equilibrium is characterized by a constant mass of firms entering each period, \(M_e\), and a constant mass of firms producing, \(M\). Each of the firms producing is active in a subset of product markets, and the steady-state mass of firms manufacturing an individual product, \(M_p\), is a constant fraction of the mass of firms producing that is the same across products. The steady-state equilibrium is also characterized by stationary distributions of managerial ability and product expertise that are determined by the zero-profit cutoffs \(\varphi^*\) and \(\lambda^* (\varphi)\) and that are again the same across products.

From the equilibrium pricing rule (8), the aggregate price index for each product can be written as a function of the mass of firms manufacturing the product and the price of a variety with a weighted average of managerial ability and product expertise:

\[
P = M_p^{\frac{1}{1-\sigma}} \frac{1}{\bar{\rho} \tilde{\varphi}}, \tag{18}
\]
where the weighted average $\tilde{\varphi}$ depends on the \textit{ex post} distributions of managerial ability and product expertise and is defined in the appendix.

Of the mass of firms with a particular managerial ability $\varphi$, a fraction $[1 - Z(\lambda^*(\varphi))]$ are active in an individual product market. Therefore, the mass of firms manufacturing an individual product is the following fraction of the mass of firms:

$$M_p = \left[ \int_{\varphi^*}^{\infty} [1 - Z(\lambda^*(\varphi))] \left( \frac{g(\varphi)}{1 - G(\varphi^*)} \right) d\varphi \right] M. \quad (19)$$

With a constant mass of firms producing in steady-state, the mass of firms that draw a managerial ability sufficiently high to enter must equal the mass of firms that die, implying the following steady-state stability condition:

$$[1 - G(\varphi^*)] M_e = \delta M. \quad (20)$$

Finally, labor market clearing implies that the demand for labor in production and entry equals the economy’s aggregate supply of labor:

$$L_q + L_e = \bar{L}, \quad (21)$$

where the subscripts $q$ and $e$ respectively denote labor used in production and entry.

4. Closed Economy Equilibrium

The closed economy equilibrium is completely referenced by the quadruple $\{\varphi^*, \lambda^*(\varphi^*), P, R\}$. All other endogenous variables can be written as functions of these four elements of the equilibrium vector.

4.1. Multi-Product Firms

We begin by determining the zero-profit cutoff for expertise for every firm as a function of its managerial ability and the zero-profit cutoff for managerial ability $\varphi^*$. Combining the zero-profit cutoff for expertise for a firm with managerial ability $\varphi$ in equation (11), the zero-profit cutoff for expertise for a firm with managerial ability $\varphi^*$ in equation (12), and the relationship between the revenues of varieties from equation (10), we obtain:

$$\lambda^*(\varphi) = \left( \frac{\varphi^*}{\varphi} \right) \lambda^*(\varphi^*). \quad (22)$$

Intuitively, an increase in managerial ability $\varphi$ reduces a firm’s zero-profit cutoff for expertise $\lambda^*(\varphi)$ because higher managerial ability raises productivity in each product, and so sufficient revenue to cover fixed production costs is generated at a lower value of expertise. In contrast, an increase in the zero-profit cutoff for managerial ability $\varphi^*$ reduces a firm’s zero-profit cutoff for expertise $\lambda^*(\varphi)$ because it raises the average productivity of rival firms’ products, and so intensifies product market competition, and hence increases the value for expertise at which sufficient revenue is generated to cover fixed production costs.

To determine $\lambda^*(\varphi^*)$, we combine the requirement that zero profits are made by a firm with managerial ability $\varphi^*$ in equation (15) with the expression for firm profits in equation (14). We substitute for product revenue using the relationship between relative variety revenues in equation (10) and the zero-profit condition for product expertise in equation (12). Together
these equations yield the following expression that implicitly defines a unique value of $\lambda^*_r (\varphi^*)$ as a function of model parameters alone:

$$\int_{\lambda^* (\varphi^*)}^{\infty} \left[ \left( \frac{\lambda}{\lambda^* (\varphi^*)} \right)^{\frac{\sigma - 1}{\sigma}} - 1 \right] f_p z (\lambda) d\lambda = f_h, \quad \Rightarrow \quad \lambda^* (\varphi^*) = \bar{\lambda}. \quad (23)$$

Equations (22) and (23) jointly determine the zero-profit cutoffs for expertise for a firm as a function of its managerial ability and the zero-profit cutoff managerial ability. Three parameters influence a firm’s zero-profit cutoff for expertise for given values of its managerial ability ($\varphi$) and the zero-profit cutoff managerial ability ($\varphi^*$): the fixed production cost ($f_p$), the elasticity of substitution ($\sigma$) and the fixed headquarters cost ($f_h$). An increase in the fixed production cost or the elasticity of substitution raises $\lambda^* (\varphi^*)$, which in turn raises the zero-profit cutoff for expertise for firms of all managerial abilities $\lambda^* (\varphi)$. A higher fixed production cost raises the value for product revenue at which zero profits are made and so increases the zero-profit cutoff for expertise. Similarly, a higher elasticity of substitution reduces the revenue derived from lower expertise products and so increases the zero-profit cutoff for expertise. A higher elasticity of substitution corresponds to a reduced love of variety and so diminishes a firm’s incentive to expand along the extensive margin into lower expertise products. An increase in the fixed headquarters cost has the opposite effect, reducing $\lambda^* (\varphi^*)$, and thereby reducing the zero-profit cutoff for expertise for firms of all managerial abilities $\lambda^* (\varphi)$. This third and somewhat surprising comparative static is a property of general equilibrium. As the fixed headquarters cost rises, the ex post profits of firms across all products decline, thereby depressing entry and diminishing product market competition. As product market competition declines, firms with lower product expertise draws can generate sufficient profits to cover fixed production costs, and so the zero-profit cutoff for product expertise fails.

The zero-profit cutoff for expertise determines the fraction of product markets where a firm produces, $[1 - Z (\lambda^* (\varphi))]$. Therefore, equations (22) and (23) together determine endogenous product scope as a function of a firm’s managerial ability $\varphi$, the zero-profit cutoff for managerial ability $\varphi^*$ and model parameters. The relationship between a firm’s product scope and its managerial ability is illustrated graphically in Figure 1. For firms that draw managerial abilities less than $\varphi^*$, average profits across products are insufficient to cover fixed headquarters costs, and the firm exits immediately. A firm that draws a managerial ability $\varphi^*$ manufactures varieties in a range of products $[1 - Z (\bar{\lambda})]$. The range of products manufactured by a firm is monotonically increasing in managerial ability and converges asymptotically towards one. Our analysis is consistent with a wide range of distributions for product expertise, and the exact shape of the relationship shown in Figure 1 will vary with different functional forms for the product expertise distribution.

4.2. Free Entry

Average firm revenue and average firm profits can be re-written so that they are functions of only the zero-profit cutoff managerial ability ($\varphi^*$) and the zero-profit cutoff for expertise for a firm with this managerial ability ($\lambda^* (\varphi^*) = \bar{\lambda}$), as shown in the appendix. Once rewritten in this way, it is clear that average firm revenue, $\bar{r} = \bar{r} (\varphi^*, \bar{\lambda})$, and average firm profits, $\bar{\pi} = \bar{\pi} (\varphi^*, \bar{\lambda})$, are monotonically decreasing in both $\varphi^*$ and $\bar{\lambda}$. Intuitively, a higher value of $\varphi^*$ implies a higher average managerial ability of a firm’s competitors and so lower average prices of competing varieties. Similarly, a higher value of $\bar{\lambda}$ implies a higher average product expertise of the varieties manufactured by competitors and hence lower average prices of competing varieties.
Substituting for average profits in the free entry condition, the requirement that the expected value of entry equals the sunk entry cost becomes:

\[ v_e = \frac{1}{\delta} \int_0^{\infty} \int_0^{\infty} \left[ \lambda^*(\varphi) \left( \frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_P d\lambda - f_h \right] g(\varphi) d\varphi = f_e, \quad (24) \]

which is monotonically decreasing in \( \varphi^* \), since \( \lambda^*(\varphi) = (\varphi^*/\varphi) \bar{\lambda} \), and implicitly defines a unique zero-profit cutoff for managerial ability as a function of model parameters alone.

The comparative statics of the free entry condition are intuitive. As the product fixed production cost \( (f_p) \) rises, so does the zero-profit cutoff for managerial ability \( (\varphi^*) \), since a higher productivity is required in each product to generate sufficient revenue to cover the fixed production cost, and greater managerial ability raises productivity in all products. Increases in the fixed headquarters cost \( (f_h) \) and the sunk entry cost \( (f_e) \) both reduce the zero-profit cutoff for managerial ability. As the fixed headquarters cost rises, the \textit{ex post} profits of firms across all products decline, thereby depressing entry and diminishing product market competition. As product market competition declines, firms with lower managerial ability can generate sufficient profits across all products to cover fixed headquarters cost and so the zero-profit cutoff for managerial ability falls. A rise in the sunk cost of entry also reduces entry, and so has a similar effect on the zero-profit cutoff for managerial ability.

4.3. Goods and Labor Markets

So far we have solved for the zero-profit cutoff for managerial ability \( \varphi^* \) and the zero-profit cutoff for expertise for a firm with this level of managerial ability \( \lambda^*(\varphi^*) = \bar{\lambda} \), which implies from equation (22) that we have pinned down the zero-profit cutoff for expertise \( \lambda^*(\varphi) \) and hence equilibrium product scope for firms of all managerial abilities \( \varphi \). Together \( \varphi^* \) and \( \bar{\lambda} \) determine the \textit{ex post} distributions of managerial ability and product expertise and hence pin down average variety prices in equation (18) as well as average firm revenue \( (\bar{r} = \bar{r}(\varphi^*, \bar{\lambda})) \) and average firm profits \( (\bar{\pi} = \bar{\pi}(\varphi^*, \bar{\lambda})) \).

Total payments to labor used in production equal aggregate revenue minus aggregate profits, \( L_q = R - \Pi \). Combining the steady-state stability and free entry conditions, equations (20) and (16), it can be shown that total payments to labor used in entry equal aggregate profits, \( L_e = \Pi \). Therefore, aggregate revenue equals the economy’s aggregate supply of labor, \( R = L \), and the labor market clears.

We are now in a position to determine the price index for each product, which depends on average variety prices and the mass of firms producing each product in equation (18). Average variety prices have already been determined above. The mass of firms producing each product, \( M_p \), is a constant fraction of the mass of firms producing any product, \( M \), in equation (19). The mass of firms producing any product equals aggregate revenue divided by average firm revenue, \( M = R/\bar{r} \), where we have solved for both aggregate and average firm revenue. The constant fraction of firms that produce each product depends solely on the zero-profit cutoff for managerial ability \( \varphi^* \) and the zero-profit cutoffs for expertise \( \lambda^*(\varphi) \) for which we have already solved. Therefore, we have determined the price index and completed the characterization of the equilibrium quadruple \( \{ \varphi^*, \lambda^*(\varphi^*), P, R \} \).

5. Properties of the Closed Economy Equilibrium

The equilibrium range of products manufactured by a firm is determined by a tension between two forces. On the one hand, the firm charges a lower price for products where it has
higher productivity and as a result enjoys higher sales. This force for specialization causes the firm to expand along the intensive margin in those products where it has higher productivity. On the other hand, consumer preferences display a love of variety and there is diminishing marginal utility to consuming more of any one variety. Diminishing marginal utility implies that the firm faces a downward-sloping demand curve for each product, which constrains sales and limits expansion along the intensive margin. This force for diversification induces the firm to expand along the extensive margin into products where it has a lower productivity. Nonetheless, the firm must derive sufficient revenue from each product to cover fixed production costs, which constrains the firm’s incentive to expand along the extensive margin.

**Proposition 1** There is a positive correlation between firms’ intensive margins (how much of a product is produced) and their extensive margins (how many products are produced).

**Proof.** See appendix. ■

Proposition 1 summarizes a key prediction of our framework, namely the positive correlation between firms’ intensive and extensive margins which matches the empirical findings in Table 2. More productive firms are larger on average, not only because they sell more of a given product, but also because they manufacture more products. The explanation for this positive correlation lies in managerial ability’s role as a component of productivity that is common across products. On the one hand, higher managerial ability raises a firm’s productivity in a given product, thereby lowering the firm’s price for that product, and increasing the quantity sold. On the other hand, higher managerial ability also lowers the zero-profit cutoff for expertise above which the firm finds it profitable to enter a product market, thereby expanding the range of products manufactured by the firm. Note that the correlation between firms’ intensive and extensive margins exists even though we have assumed that expertise draws are themselves uncorrelated across products. Assuming that expertise draws are positively correlated across products would only strengthen the correlation.

**Corollary 1** Multi-product firms’ endogenous decisions about product scope magnify inequality in the firm-size distribution: the percentage difference in revenue between two firms with managerial abilities $\varphi''$ and $\varphi'$ (where $\varphi'' > \varphi'$) is strictly greater than when all firms have the same exogenous product scope ($\lambda^*(\varphi) = \lambda^*(\varphi^*) = \bar{\lambda}$ for all $\varphi$).

**Proof.** See appendix. ■

The positive correlation between the intensive and extensive margin magnifies inequality in the firm-size production. Firms with higher managerial abilities are larger than those with lower managerial abilities, not only because they sell more of any given product, but also because they manufacture more products. This prediction of the model has implications for the large theoretical and empirical literature concerned with the firm-size distribution (see for example Axtell 2001, Rossi-Hansberg and Wright 2005, and the survey by Sutton 1997). The positive correlation between firms’ intensive and extensive margins in our model implies that a smaller variance in managerial ability is required to explain the observed dispersion in firm sizes than in a model where product scope is exogenous. A given increase in managerial ability leads to a larger increase in firm size than it would with exogenous product scope because of expansion along the extensive margin of the number of products.
Proposition 2  (a) Firm weighted average productivity $\tilde{\lambda}(\varphi)$ is monotonically increasing in managerial ability. (b) The percentage difference in firm weighted average productivity $\tilde{\lambda}(\varphi)$ is smaller than the percentage difference in firm managerial ability.

Proof. See appendix. ■

Whereas the inequality in firm sizes is magnified by endogenous product scope, the dispersion in average productivity is compressed. As managerial ability increases, a firm’s average productivity rises because it becomes more productive in all the products that it currently manufactures. At the same time, the increase in managerial ability leads the firm to expand along the extensive margin into new products, where it has less expertise and so is less productive. The addition of these lower expertise products reduces the firm’s weighted average productivity, which therefore rises less than proportionately with managerial ability.\(^{13}\) An implication of this finding is that the measurement of productivity for multi-product firms becomes problematic unless data on output and factor inputs is available by both product and firm. If product mix were held constant, the difference in productivity between a high and low managerial ability firm would be greater than when product scope is endogenous, because the high managerial ability firm expands along the extensive margin into products where it has less expertise and so is less productive.\(^{14}\)

The expansion of higher managerial ability firms along the extensive margin into lower expertise products has a variety of general equilibrium implications. The most basic of these implications arises from the expansion of labor demand at higher managerial ability firms, which bids up factor prices and so reduces the profitability of lower managerial ability firms. As a result, the zero-profit cutoff below which firms exit is higher when product scope is endogenous than when all firms manufacture the same exogenous range of products. A more subtle implication is for the economy’s response to changes in model parameters such as the sunk cost of entry. The potential to respond along a new margin – the number of goods produced – dampens responses along existing adjustment margins such as the range of productivities where firms produce.

Proposition 3 Endogenous adjustments along firms’ extensive margins dampen the response of the zero-profit cutoff productivity to changes in the sunk cost of entry.

Proof. See appendix. ■

Following a rise in the sunk entry cost, the expected value of entry must rise in order for the free entry condition to continue to be satisfied in equilibrium. When product scope is exogenous, all of the adjustment in the expected value of entry occurs through an expansion in the range of managerial abilities where firms produce (a fall in the zero-profit cutoff below which firms exit). In contrast, when product scope is endogenous, some of the adjustment in the expected value of entry occurs through an expansion in the range of products manufactured

\(^{13}\) As long as the cumulative distribution function for product expertise is continuous, the measure of products added as managerial ability rises is small relative to the measure of products already manufactured. Therefore, as shown formally in the proof of Proposition 2, the reduction in firm weighted average productivity from expanding into lower expertise products cannot outweigh the increase caused by enhanced productivity in existing products, which implies that firm weighted average productivity is monotonically increasing in managerial ability. Although the proposition focuses on the weighted average productivity of firms, a directly analogous result holds for the unweighted average productivity of firms.

\(^{14}\) For further discussion of the empirical problems in measuring productivity with multi-product firms, see De Loecker (2005) and Bernard, Redding and Schott (2006c).
by firms, thereby reducing the extent of adjustment in the zero-profit cutoff productivity that has to occur.

Intuitively, as the sunk entry cost rises, entry declines and product market competition is diminished. With less competition, firms are able to derive positive profits from products which were not previously viable, and so expand their product scope, which increases the expected value of entry. Less competition also enables lower managerial ability firms who were not previously viable to make positive profits, and so the zero-profit cutoff productive falls, which also increases the expected value of entry. But the increase in labor demand caused by the expansion of firms along the extensive margin into new products bids up wages and so reduces the profitability of lower managerial ability firms. Therefore, the zero-profit cutoff productivity falls by less than it would if product scope were exogenously fixed.

6. Open Economy

In this section, we examine the implications of opening to trade within a framework of two symmetric countries, while in a later section we introduce country asymmetries in the form of comparative advantage. We denote domestic market variables with a superscript $d$ and export market variables with a superscript $x$. In line with a large body of empirical evidence, we assume that trade is costly. There are fixed costs of operating across national borders, such as the costs of mastering customs procedures and building distribution networks. Therefore we assume that, to begin exporting, a firm must incur additional fixed headquarters costs of $f_{xh} > 0$ which raise total headquarters costs to $f_h + f_{xh}$. There are also fixed costs of entering export markets that are specific to individual products, such as costs of market research, advertising and conforming to foreign regulatory standards. Hence we assume that, to export a particular product, a firm must incur an additional product-specific fixed cost of $f_x > 0$. As more products are exported, total fixed exporting costs rise, but average fixed exporting costs fall because the headquarters costs of becoming an exporter are spread over a larger number of products. Finally we allow for variable costs of trade, such as transportation costs, which take the standard “iceberg” form where a fraction $\tau > 1$ of a variety must be shipped in order for one unit to arrive.\(^{15}\)

6.1. Consumption and Production

Profit maximization implies that equilibrium price of a product variety is again a constant mark-up over marginal cost, with export prices a constant multiple of domestic prices due to the variable costs of trade:

$$p_{ix}(\varphi, \lambda_i) = \tau p_{id}(\varphi, \lambda_i) = \frac{w}{\rho \varphi \lambda_i}. \tag{25}$$

Given firms’ pricing rule and symmetric countries, equilibrium product revenue in the export market is proportional to revenue in the domestic market: $r_{ix}(\varphi, \lambda_i) = \tau^{1-\sigma} r_{id}(\varphi, \lambda_i)$. The combined revenue from a product depends upon whether or not it is exported:

$$r(\varphi, \lambda_i) = \begin{cases} r_{id}(\varphi, \lambda_i) & \text{if the product is not exported} \\ (1 + \tau^{1-\sigma}) r_{id}(\varphi, \lambda_i) & \text{if the product is exported} \end{cases}. \tag{26}$$

\(^{15}\)For evidence on the magnitude of overall trade costs, see Anderson and van Wincoop (2004) and Hummels (2001). For evidence on the fixed costs of exporting, see Bernard and Jensen (2004) and Roberts and Tybout (1997).
Consumer love of variety and fixed production costs imply that no firm ever exports a product without also selling it in the domestic market. Therefore, we may separate a firm’s profit from a product into components earned from domestic sales, $\pi_{id} (\varphi, \lambda_i)$, and export sales, $\pi_{ix} (\varphi, \lambda_i)$, where we apportion the entire fixed production cost to domestic profits and the fixed exporting cost to export profits:

$$\pi_{id} (\varphi, \lambda_i) = \frac{r_{id} (\varphi, \lambda_i)}{\sigma} - f_p, \quad \pi_{ix} (\varphi, \lambda_i) = \frac{r_{ix} (\varphi, \lambda_i)}{\sigma} - f_x.$$  \hfill (27)

The combined profit from a product is therefore:

$$\pi_i (\varphi, \lambda_i) = \pi_{id} (\varphi, \lambda_i) + \max \{0, \pi_{ix} (\varphi, \lambda_i)\}.$$  \hfill (28)

6.2. Firm-Product Profitability

Given a firm’s managerial ability ($\varphi$), the zero-profit condition in equation (11) determines a cutoff for product expertise, such that the firm sells a product domestically if it draws an expertise equal to or greater than $\lambda^* (\varphi)$. Manipulating the zero-profit condition for expertise for firms with different managerial abilities $\varphi$ yields the same expressions for the equilibrium range of products manufactured as in equations (22) and (23). However, the zero-profit cutoffs for managerial ability ($\varphi^*$) and product expertise ($\lambda^* (\varphi)$) will differ between the closed and open economies and thus induce differences in equilibrium product scope.

Entry into export markets within products is determined in an analogous fashion. For each managerial ability $\varphi$, there is an exporting cutoff for product expertise, i.e. the firm exports the product if it draws an expertise equal to or greater than $\lambda^*_{ix} (\varphi)$:

$$r_{ix} (\varphi, \lambda^*_{ix} (\varphi)) = \sigma f_x.$$  \hfill (29)

Since products are identical, the exporting cutoff for expertise is the same across products: $\lambda^*_{ix} (\varphi) = \lambda^*_{ix} (\varphi)$. As expertise is independently and identically distributed across products, the fraction of products exported by a firm with managerial ability $\varphi$ is $[1 - Z (\lambda^*_{ix} (\varphi))]$.

Exporters vary in terms of their managerial ability $\varphi$, and we refer to the lowest level of managerial ability observed among exporters as the exporting cutoff managerial ability $\varphi^*_x$. Using equation (29) for two exporting firms with managerial abilities, $\varphi$ and $\varphi^*_x$, we obtain the following expression for the firms’ relative exporting cutoffs for expertise:

$$\lambda^*_{x} (\varphi) = \left( \frac{\varphi^*_x}{\varphi} \right) \lambda^*_{x} (\varphi^*_x).$$  \hfill (30)

The intuition underlying this relationship in the export market is directly analogous to the intuition underlying the similar relationship in the domestic market (equation (22)). As managerial ability $\varphi$ increases, firms become more productive in each product, and so have a lower exporting cutoff for product expertise. As the exporting cutoff for managerial ability $\varphi^*_x$ increases, the average productivity of the product varieties supplied to the export market by rival firms increases, which intensifies export market competition, and so raises the exporting cutoff for product expertise.

Since revenue in the export market is proportional to revenue in the domestic market, there is an equilibrium relationship between the exporting and zero-profit cutoffs for expertise.

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$^{16}$This is a convenient accounting device which simplifies the exposition. Rather than comparing revenue from exporting to the fixed cost of exporting, we could equivalently compare the sum of domestic and export revenue to the sum of fixed production and exporting costs.
Combining the expression for the relative revenue of a variety in the export and domestic markets, \( r_{ix} (\varphi, \lambda_i) = \tau^{1-\sigma} r_{id} (\varphi, \lambda_i) \), the zero-profit cutoff condition for expertise (11), and the exporting cutoff condition for expertise (29) yields the following relationship between the exporting and zero-profit cutoffs for expertise for those firms that export:

\[
\lambda_{x}^* (\varphi) = \tau \left( \frac{f_x}{f_p} \right)^{\frac{1}{1-\sigma}} \lambda^* (\varphi).
\]

(31)

There is selection into export markets within products if the exporting cutoff for expertise lies above the zero-profit cutoff: \( \lambda_{x}^* (\varphi) > \lambda^* (\varphi) \). In this case, for a given level of managerial ability \( \varphi \), a firm with a sufficiently high draw for expertise finds it profitable to export the product, while another firm with a lower draw for expertise only sells the product in the domestic market. Since there is strong empirical evidence supporting the partitioning of firms by export status, we focus on parameter values where self-selection into export markets within products occurs: \( \tau^{\sigma-1} f_x > f_p \) in equation (31) above.\(^{17}\)

6.3. Firm Profitability

We noted earlier that no firm ever exports a product without also selling it in the domestic market. For each product, we are therefore able to apportion fixed production costs \( (f_p) \) to profits from domestic sales and fixed exporting costs \( (f_x) \) to profits from export sales. Similarly, across the continuum of products, we are able to apportion fixed headquarters costs to firm profits from domestic sales and exporting headquarters costs \( (f_{xh}) \) to firm profits from export sales.\(^{18}\)

As in the closed economy, with a continuum of identical products of unit interval, firm profits from domestic sales equal average profits from domestic sales in an individual product market minus fixed headquarters costs. The requirement that firm profits net of fixed headquarters costs are greater than or equal to zero defines a zero-profit cutoff for managerial ability \( \varphi^* \), as in equations (14) and (15). Only firms who draw a managerial ability equal to or greater than \( \varphi^* \) enter, and the zero-profit cutoff for managerial ability varies between the closed and open economy, as established below. Following a similar line of reasoning, firm profits from export sales equal average profits from export sales in an individual product market minus exporting headquarters costs:

\[
\pi_x (\varphi) = \int_{\lambda_{x}^* (\varphi)}^{\infty} \left[ \frac{r_x (\varphi, \lambda)}{\sigma} - f_x \right] z (\lambda) d\lambda - f_{xh}.
\]

(32)

The lower the managerial ability of a firm, the higher \( \lambda_{x}^* (\varphi) \), and the lower the probability of drawing a product expertise sufficiently high to profitably export a product. Firms with lower managerial abilities have lower expected profits from an individual export market and export a smaller fraction of products \( [1 - Z (\lambda_{x}^* (\varphi))] \). For sufficiently low draws of managerial ability, the excess of revenue over fixed exporting costs in the small range of products that the firm could profitably sell abroad falls short of the exporting headquarters cost. Therefore, the following zero-profit condition defines an exporting cutoff for managerial ability \( \varphi_{x}^* \), such that only firms who draw a managerial ability equal to or greater than \( \varphi_{x}^* \) export:

\[
\pi_x (\varphi_{x}^*) = 0.
\]

\(^{17}\)For empirical evidence on selection into export markets, see Bernard and Jensen (1995, 1999, 2004), Clerides, Lach and Tybout (1998), and Roberts and Tybout (1997).

\(^{18}\)Following the earlier discussion, we could equivalently compare the sum of firm profits from domestic sales and export sales together to the sum of fixed headquarters costs and exporting headquarters costs.
When the exporting cutoff lies above the zero-profit cutoff for both firm-product expertise and firm managerial ability, namely \( \lambda_x^\star (\varphi) > \lambda^\star (\varphi) \) and \( \varphi_x^\star > \varphi^\star \), there is selection into export markets across and within firms. A firm who draws a managerial ability in between \( \varphi^\star \) and \( \varphi_x^\star \) only serves the domestic market for all the products that it manufactures. Even though such a firm draws an expertise above the exporting cutoff in a fraction \( [1 - Z (\lambda_x^\star (\varphi))] \) of products that it manufactures, the total export profits received across this fraction of products are insufficient to cover the exporting headquarters costs. A firm who draws a managerial ability above \( \varphi_x^\star \) exports, but manufactures a larger fraction of products than it exports, so that \( [1 - Z (\lambda_x^\star (\varphi))] < [1 - Z (\lambda^\star (\varphi))] \). For some of the firm’s products, the expertise draw is high enough to cover fixed production costs for the product, but too low given variable trade costs to cover fixed exporting costs for the product. Such products are supplied to the domestic market but not exported. The parameter values under which \( \varphi_x^\star > \varphi^\star \) are determined below and are closely related to those discussed above under which \( \lambda_x^\star (\varphi) > \lambda^\star (\varphi) \).

6.4. Free Entry

The expected value of entry is now equal to the sum of two terms: the \textit{ex ante} probability of successful entry times the average firm profits from the domestic market conditional on entry \((\bar{\pi_d})\), plus the \textit{ex ante} probability of successful entry times the probability of exporting times the average firm profits from the export market conditional on exporting \((\bar{\pi_x})\), all discounted by the probability of firm death. The free entry condition requires that the expected value of entry equals the sunk entry cost:

\[
v_e = \frac{[1 - G (\varphi^\star)] [\bar{\pi_d} + \chi \bar{\pi}_x]}{\delta} = f_e, \tag{34}\]

where the \textit{ex ante} probability of successful entry is \([1 - G (\varphi^\star)]\) and \( \chi = [1 - G (\varphi^\star)] / [1 - G (\varphi^\star)] \) denotes the probability of exporting. The expression for average firm profits in the domestic market takes expectations over possible values for managerial ability in equation (14); similarly, the expression for average firm profits in the export market takes expectations over possible values for managerial ability in equation (32).

6.5. Goods and Labor Markets

The steady-state equilibrium is characterized by a constant mass of firms entering each period, \( M_e \), a constant mass of firms producing, \( M \), a constant mass of firms producing in each product market, \( M_{px} \), as well as a constant mass of firms exporting in each product market, \( M_{px} \). The steady-state equilibrium is also characterized by stationary distributions of managerial ability and expertise in domestic and export markets, which are determined by the zero-profit cutoffs \( \varphi^\star \) and \( \lambda^\star (\varphi) \) and the exporting cutoffs \( \varphi_x^\star \) and \( \lambda_x^\star (\varphi) \), and are the same across products.

Using the equilibrium pricing rule (25) and the symmetry of the two countries, the aggregate price index for each product may be written as a function of the mass of firms manufacturing the product for domestic and export markets, as well as the prices of products with weighted averages of managerial ability and expertise in the domestic and export markets:

\[
P = \left[ M_p \left( \frac{1}{\rho \tilde{\varphi}} \right)^{1/\sigma} + \frac{M_{px}}{\rho \tilde{\varphi}_x} \right]^{1/\sigma} \tag{35}\]

where the weighted averages \( \tilde{\varphi} \) and \( \tilde{\varphi}_x \) depend on the \textit{ex post} distributions of managerial ability and expertise and are defined in the appendix.
Of the mass of firms with a particular managerial ability $\varphi$, a fraction $[1 - Z (\lambda^* (\varphi))]$ serve the domestic market in a product, while a fraction $[1 - Z (\lambda_x^* (\varphi))]$ serve the export market in a product. Therefore, the expression for the mass of firms supplying the domestic market in a product remains as in equation (19) for the closed economy, while the expression for the mass of firms supplying the export market in a product is analogous:

$$M_x = \left[ \int_{\varphi_x^*}^{\infty} [1 - Z (\lambda_x^* (\varphi))] \left( \frac{g(\varphi)}{1 - G (\varphi_x^*)} \right) d\varphi \right] M.$$

With a constant mass of firms producing in steady-state, the steady-state stability condition remains as before in equation (20). Finally, labor market clearing requires that the demand for labor in production for the domestic market, in production for the export market and in entry equals the economy’s aggregate supply of labor.

7. Open Economy Equilibrium

The open economy equilibrium is completely referenced by the sextuple $\{\varphi^*, \varphi_x^*, \lambda^* (\varphi^*), \lambda_x^* (\varphi_x^*), P, R\}$. All other endogenous variables can be written as functions of these six elements of the equilibrium vector.

7.1. Multi-Product Firms

The range of products manufactured by a firm in the open economy is governed by equations (22) and (23) as in the closed economy. We determine the range of products exported in a similar way. To determine the exporting cutoff for expertise for the lowest managerial ability exporter, $\lambda_{i,x}^* (\varphi_x^*)$, we combine the requirement that zero profits are made from export sales by a firm with managerial ability $\varphi_x^*$ in equation (33) with the expression for firm profits from export sales in equation (32). We substitute for product revenue from export sales using the relationship between relative variety revenues in equation (10) and the exporting cutoff condition for expertise in equation (29). Together these equations yield the following expression that implicitly defines a unique value of $\lambda_{i,x}^* (\varphi_x^*)$ as a function of model parameters alone:

$$\int_{\lambda_x^* (\varphi_x^*)}^{\infty} \left( \left( \frac{\lambda}{\lambda_x^* (\varphi_x^*)} \right)^{\sigma-1} - 1 \right) f_{x,z} (\lambda) d\lambda = f_{xh}, \quad \Rightarrow \quad \lambda_x^* (\varphi_x^*) = \lambda_x. \quad (37)$$

Once $\lambda_x^* (\varphi_x^*)$ is known, the exporting cutoff for expertise for an exporter with a managerial ability greater than $\varphi_x^*$ can be determined from equation (30) as a function of the firm’s managerial ability $\varphi$ and the exporting cutoff managerial ability $\varphi_x^*$: $\lambda_x^* (\varphi) = (\varphi_x^*/\varphi) \lambda_x$. Since the exporting cutoff for expertise determines the fraction of products that a firm exports (which equals $[1 - Z (\lambda_x^* (\varphi))]$), equations (30) and (37) together determine the equilibrium range of products exported for each value of managerial ability.

The comparative statics of the range of products exported parallel those of the range of products manufactured. An increase in the fixed exporting cost for a product ($f_x$) or the elasticity of substitution ($\sigma$) raises the exporting cutoff for expertise ($\lambda_x^* (\varphi)$) for firms of all managerial abilities, since a higher value for expertise is required to generate sufficient revenue to cover the fixed exporting cost. An increase in the exporting headquarters cost ($f_{hx}$) has the opposite effect, reducing the exporting cutoff for expertise ($\lambda_x^* (\varphi)$) for firms of all managerial abilities. The third comparative static is again a property of general equilibrium. As the exporting headquarters cost rises, the total ex post profits of exporting firms decline,
thereby reducing the expected value of entry, depressing entry, and diminishing product market competition. As product market competition declines, firms with lower expertise draws can generate sufficient profits to cover fixed exporting costs, and so the exporting cutoff for expertise falls.

The ranges of products exported and manufactured are linked in general equilibrium. From the exporting cutoff condition for expertise (29) for a firm with managerial ability \( \varphi^*_x \), the zero-profit cutoff condition for expertise (11) for a firm with managerial ability \( \varphi^* \), and the equation for relative variety revenues in (10), the exporting cutoff managerial ability \( \varphi^*_x \) and the zero-profit cutoff managerial ability \( \varphi^* \) are related as follows:

\[
\varphi^*_x = \Gamma \varphi^*, \quad \Gamma \equiv \tau \left( \frac{f_x}{\bar{f}_p} \right) \frac{1}{\bar{x}_x} \frac{\bar{\lambda}}{\lambda_x},
\]

where we have used \( \lambda_x^*(\varphi^*_x) = \bar{x}_x \) and \( \lambda^*(\varphi^*) = \bar{x} \).

The parameter values where \( \varphi^*_x > \varphi^* \) (some firms export while others only serve the domestic market) are closely related to those where \( \lambda_x^*(\varphi) > \lambda^*(\varphi) \) (exporters ship some products abroad while others are only supplied to the domestic market). From the discussion above, \( \lambda_x^*(\varphi) > \lambda^*(\varphi) \) whenever \( \tau \sigma^{-1} f_x > f_p \). From equation (38), \( \varphi^*_x > \varphi^* \) whenever \( \tau \sigma^{-1} f_x \lambda_x^* \sigma^{-1} > f_p \bar{\lambda} \sigma^{-1} \), where \( \lambda \) is determined by equation (23) and depends on fixed production costs and fixed headquarters costs, while \( \bar{\lambda}_x \) is determined by equation (37) and depends on fixed exporting costs and exporting headquarters costs. In the light of the overwhelming evidence of selection into export markets, and since the interior equilibrium is the most interesting one, we focus on parameter values where both sets of inequalities are satisfied, so that \( \varphi^*_x > \varphi^* \) and \( \lambda_x^*(\varphi) > \lambda^*(\varphi) \).

### 7.2. Free Entry

Average firm revenue and average firm profits in the domestic market can be re-written as decreasing functions of the zero-profit cutoff managerial ability \( \varphi^* \) and the zero-profit cutoff for product expertise for a firm with this managerial ability \( (\lambda^*(\varphi^*) = \bar{\lambda}) \), as in the closed economy: \( \bar{r}_d = \bar{r}_d(\varphi^*, \bar{\lambda}) \) and \( \bar{\pi}_d = \bar{\pi}_d(\varphi^*, \bar{\lambda}) \). Similarly, average firm revenue and average firm profits in the export market can be re-written as decreasing functions of the exporting cutoff managerial ability \( \varphi^*_x \) and the exporting cutoff for product expertise for a firm with this managerial ability \( (\lambda_x^*(\varphi^*_x) = \bar{x}_x) \): so that \( \bar{r}_x = \bar{r}_x(\varphi^*_x, \bar{\lambda}_x) \) and \( \bar{\pi}_x = \bar{\pi}_d(\varphi^*_x, \bar{\lambda}_x) \).

Substituting for average firm profits in the domestic and export markets in the free entry condition, the requirement that the expected value of entry equals the sunk entry cost becomes:

\[
v_e = \frac{1}{\delta} \int_{\varphi^*}^{\infty} \left[ \int_{\lambda^*(\varphi)}^{\infty} \left( \frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_p z(\lambda) d\lambda - f_h \right] g(\varphi) d\varphi \]

\[
+ \frac{1}{\delta} \int_{\varphi^*_x}^{\infty} \left[ \int_{\lambda_x^*(\varphi)}^{\infty} \left( \frac{\lambda}{\lambda_x^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_x z(\lambda) d\lambda - f_{xh} \right] g(\varphi) d\varphi = f_e,
\]

which is again monotonically decreasing in \( \varphi^* \), since \( \lambda^*(\varphi) = (\varphi^*/\varphi) \bar{\lambda} \), \( \lambda_x^*(\varphi) = (\varphi_x^*/\varphi) \bar{\lambda} \) and \( \varphi^*_x = \Gamma \varphi^* \). This equation implicitly defines a unique zero-profit cutoff for managerial ability as a function of model parameters alone.

As in the closed economy, a rise in the fixed production cost \( f_p \) leads to a rise in the zero-profit cutoff for managerial ability \( \varphi^* \), and an increase in either the fixed headquarters cost \( f_h \) or the sunk entry cost \( f_e \) leads to a decrease in the zero-profit cutoff for managerial ability.
A rise in the fixed exporting cost \((f_x)\) has similar effects to a rise in the fixed production cost and an increase in the exporting headquarters cost \((f_{xh})\) has analogous effects to an increase in the fixed headquarters cost \((f_h)\). The economic intuitions for these comparative statics are directly analogous to those for the closed economy discussed above.

### 7.3. Goods and Labor Markets

We have now solved for the zero-profit cutoff for expertise for a firm with managerial ability \(\varphi^* (\lambda^* (\varphi^*) = \lambda)\), the exporting cutoff for expertise for a firm with managerial ability \(\varphi_x^* (\lambda_x^* (\varphi^*_x) = \lambda_x)\), and the zero-profit managerial ability \(\varphi^*\) as a function of model parameters. These three variables in turn determine the exporting cutoff managerial ability \(\varphi^*_x\) in equation (38). The quadruple \(\{\varphi^*, \varphi_x^*, \lambda, \lambda_x\}\) pins down the zero-profit cutoff for expertise \(\lambda^* (\varphi)\) and hence product scope, as well as the exporting cutoff for expertise \(\lambda^*_x (\varphi)\) and hence export scope, for firms of all managerial abilities. Together these variables determine the ex post distributions of managerial ability and product expertise in both domestic and export markets.

We can therefore solve for average revenue and profits in the domestic market \((\bar{r}_d = \bar{r}_d (\varphi^*, \lambda))\) and \(\bar{\pi}_d = \bar{\pi}_d (\varphi^*, \lambda)\), average revenue and profit in the export market \((\bar{r}_x = \bar{r}_x (\varphi^*_x, \lambda_x))\) and \(\bar{\pi}_x = \bar{\pi}_x (\varphi^*_x, \lambda_x)\), as well as average variety prices in the domestic and export market in equation (35).

Combining the steady-state stability, free entry and labor market clearing conditions, it can again be shown that aggregate revenue equals the economy’s aggregate supply of labor \((R = L)\). The mass of firms producing a product, \(M_p\), and the mass of firms exporting a product, \(M_{px}\), are constant fractions of the mass of firms producing any product, \(M\). These fractions depend solely on the zero-profit cutoff managerial ability \(\varphi^*\), the zero-profit cutoffs for expertise \(\lambda^* (\varphi)\), the exporting cutoff managerial ability \(\varphi^*_x\) and the exporting cutoffs for expertise \(\lambda^*_x (\varphi)\) for which we have solved. The mass of firms producing any product equals aggregate revenue divided by average revenue, \(M = R/\bar{r}\), where we have determined both aggregate and average revenue above. The price index for each product in equation (35) follows immediately from the mass of firms producing each product, \(M_p\), and the mass of firms exporting each product, \(M_{px}\), as well as average variety prices in the domestic and export market for which we have already solved. This completes the characterization of the equilibrium sextuple \(\{\varphi^*, \varphi_x^*, \lambda^* (\varphi^*), \lambda_x^* (\varphi_x^*), P, R\}\).

### 8. Properties of the Open Economy Equilibrium

Multi-product firms’ decisions about endogenous product scope introduce a new adjustment margin along which the economy can respond to trade liberalization. The presence of this new adjustment margin yields a variety of novel implications. First and foremost, trade liberalization induces firms to drop marginal lower expertise products and to “focus on their core competencies” in higher expertise products. Since low expertise, and thus low productivity products are dropped, average firm productivity rises. Trade liberalization induces within-firm productivity growth through reallocations of resources inside the firm. Thus, in contrast to standard models of industry dynamics, where firm productivity is a constant and aggregate productivity growth is driven entirely by reallocations of resources across firms, our framework incorporates an endogenous response of firm productivity to trade liberalization. Our model therefore facilitates analysis of firm as well as industry dynamics following the opening of economies to trade.
Proposition 4 The opening of trade increases steady-state aggregate productivity through:

(a) firm-level productivity growth: multi-product firms drop low productivity products (focus on their “core competencies”), and therefore firm and aggregate weighted-average productivity rise

(b) between-firm reallocations of resources: low productivity firms exit and therefore aggregate weighted-average productivity rises.

Proof. See appendix.

The key to understanding these results is that the opening of trade has heterogeneous effects across firms depending on whether they are exporters as well as heterogeneous effects across products within firms depending on whether the product is exported. As output of higher expertise exported products rises, the resulting increase in labor demand bids up wages and reduces the profitability of lower expertise products that are only supplied to the domestic market. As profitability declines in these lower expertise products, previously viable products become unprofitable and are dropped. The zero-profit cutoff for expertise $\lambda^* (\phi)$ rises for firms of all managerial abilities, so that in equilibrium a firm with managerial ability $\phi$ manufactures a smaller range of products $[1 - Z (\lambda^* (\phi))]$. Thus, the opening of trade leads to a contraction in equilibrium product scope, as firms focus on their “core competencies” in a narrower range of higher expertise products.

Additional intuition for these findings can be gained from comparing the free entry conditions in the closed and open economies (equations (24) and (39)). The opening of trade increases the expected value of entry for a firm because of the positive ex ante probability of drawing a managerial ability high enough to export. As a result, there is increased entry, which enhances product market competition and raises the zero-profit cutoff managerial ability $\phi^*$ below which firms exit. This rise in $\phi^*$ elevates the average productivity of the varieties manufactured by competing firms, strengthens product market competition, and so induces surviving firms to drop lower-expertise products. The product expertise cutoff $\lambda^* (\phi)$ of a firm with a particular managerial ability $\phi$ in equations (22) and (23) increases as $\phi^*$ rises, which implies that the firm manufactures a smaller fraction of products $[1 - Z (\lambda^* (\phi))]$. Figure 2 illustrates the impact of the opening of trade as a rightward shift in the schedule $[1 - Z (\lambda^* (\phi))]$, where $\phi^*A$ and $\phi^*CT$ in the figure denote the zero-profit cutoffs for managerial ability in the autarkic and costly trade economies, respectively. Although the product expertise cutoff for a firm with managerial ability $\phi^*$ remains unchanged at $\lambda^* (\phi^*) = \bar{\lambda}$, the value of $\phi^*$ for which the product expertise cutoff is equal to $\bar{\lambda}$ has increased.

The focusing on core competencies that follows trade liberalization leads to a change in the composition of firm output that generates firm-level productivity growth. Marginal low expertise products are dropped; output of all surviving products for the domestic market contracts; and entry into exporting generates new output for the export market of higher expertise products. Each of these responses shifts the composition of firm output towards higher expertise products. Therefore, even though firm managerial ability and product expertise are themselves parameters, the model generates firm-level productivity growth due to reallocations of resources across products within firms. This new source of aggregate productivity gains, which is driven by multi-product firms’ decisions about endogenous product scope, is excluded by construction from the standard model of industry dynamics where firms implicitly manufacture a single product.

While we have developed these results in the context of opening a closed economy to trade, similar results hold for reductions in trade costs once international trade occurs, as a result for
example of improvements in transportation technology or multilateral trade liberalization. The proof is directly analogous to the proof of Proposition 4 and involves differentiating the free entry condition with respect to trade costs. Trade liberalization again generates within-firm productivity growth and leads to between-firm reallocations of resources from lower to higher managerial ability firms.\(^{19}\)

The prediction that trade liberalization generates within-firm productivity growth is supported by an extensive body of empirical research.\(^{20}\) In an influential study, Pavcnik (2002) finds that within-plant productivity growth accounts for one third (6.6 percentage points) of the 19 percent increase in aggregate productivity in the Chilean manufacturing sector following the trade liberalization reform of the late 1970s and early 1980s. In our model, the causal mechanism that gives rise to within-firm productivity growth following trade liberalization is the concentration on core competencies. This underlying mechanism is also supported by empirical evidence. Using census data on Canadian plants, Baldwin and Gu (2006) develop a measure of product diversification that captures the extent to which plant output is concentrated in the largest products or diversified across many smaller products. Following the Canada-US Free Trade Agreement (CUSFTA) and the North American Free Trade Agreement (NAFTA), there is a sharp decline in product diversification as plant output becomes increasingly concentrated towards the largest products. Similarly, Liu (2006) uses U.S. Compustat data to measure firms’ core and peripheral products, where a firm’s core product accounts for the largest proportion of its sales. The paper presents evidence that a rise in import competition leads to an increase in the share of firm sales accounted for by the core product as firms increasingly focus on their core competences.

The second main implication of our new extensive margin relates to the distinction between domestic and export markets. While firms contract the range of products that they manufacture and supply to the domestic market as trade costs fall, exporting firms expand the range of products that they supply to the export market, so that exports account for an increasing fraction of firm sales. Thus, there is a positive correlation between firms’ intensive and extensive margins in both the domestic and export market. In the domestic market the quantity of each product sold and the range of products offered decline as trade costs fall, while in the export market the amount of each product exported and the range of products shipped abroad increase.

**Proposition 5**  
Reductions in variable trade costs \( (\tau) \) lead to:  
\[(a) \] exit from the domestic market by low managerial ability firms  
\[(b) \] contraction in the domestic market along both the extensive margin (the number of products manufactured for the domestic market) and the intensive margin (domestic sales of each product)  
\[(c) \] entry into the export market by firms who previously only served the domestic market  
\[(d) \] expansion in the export market along both the extensive margin (the number of products exported) and the intensive margin (export sales of each product).

**Proof.** See appendix.  

\(^{19}\)Throughout the following, we focus on reductions in variable trade costs, but reductions in the fixed costs of exporting have similar effects as long as there remains selection into export markets: \( \lambda^*_x (\phi) > \lambda^* (\phi) \) and \( \phi^*_x > \phi^* \).

The intensive and extensive margin adjustments in domestic and export markets imply that our framework features a richer and more realistic range of export behavior than in the standard model of industry dynamics with single-product firms. There is selection into export markets across firms (exporters and non-exporters), within products (some firms export a product while other firms only supply the product to the domestic market) and also within firms (some products are exported by a firm while other products are only sold domestically). This richer range of export behavior enables our model to better account for a number of features of the micro data on the trade exposure of firms and plants. In particular, the model is able to explain the positive correlation between exports per product and the number of products exported in Table 2 above. We are also able to account for empirical findings that exports comprise a small share of the output of many exporters, while other firms export a large share of their output.\footnote{For empirical evidence on the distribution of firm export shares, see Bernard and Jensen (1995), Bernard et al. (2003) and Brooks (2006).} In our model, low managerial ability exporters export a small fraction of their products, and so exports account for a low share of firm output, while higher managerial ability exporters export a larger share of their products.\footnote{Another complementary explanation for variation in export shares involves single-product firms and fixed exporting costs that are specific to individual export markets, so that only higher productivity firms find it profitable to serve destinations with higher fixed exporting costs (see for example Eaton, Kortum and Kramarz 2005). While this alternative explanation accounts for variation in export shares, it does not explain the positive correlation between exports per product and the number of products exported by a firm.}

There are other general equilibrium implications of firms’ extensive margin adjustments. On the one hand, reductions in trade costs lead to a larger increase in the expected value of entry when product scope is endogenous than when it is exogenous, which implies a larger increase in the zero-profit cutoff for managerial ability. Intuitively, when the range of products exported expands with reductions in trade costs, the ex post profits of exporters rise from both products already exported and newly-exported products. On the other hand, some of the adjustment to reductions in trade costs occurs through a contraction of the range of products manufactured when product scope is endogenous, which implies a smaller increase in the zero-profit cutoff for managerial ability. As the expected value of entry rises and increased entry occurs, the resulting increase in product market competition leads to both a contraction in product range and a rise in the zero-profit cutoff for managerial ability. In contrast, when product scope is exogenous, all of the adjustment must occur through a rise in the zero-profit cutoff for managerial ability.

One final prediction of the model relates to the firm-size distribution. The positive correlation between the intensive and extensive margins in our model of endogenous product scope enhances the size inequality between exporting and non-exporting firms relative to a framework where product scope is exogenous. High managerial ability exporting firms are larger than low managerial ability non-exporting firms, not only because they sell more of a given range of products in the domestic market, but also because they manufacture a larger range of products, and because the range of products that they export increases with managerial ability.

**Proposition 6** Multi-product firms decisions about endogenous product scope magnify the inequality in firm size between high managerial ability exporters and low managerial ability domestic producers relative to when firms manufacture and export exogenous ranges of products \((\lambda^*(\varphi) = \lambda^*(\varphi^*) = \bar{\lambda} \text{ and } \lambda^*_x(\varphi) = \lambda^*_x(\varphi^*_x) = \bar{\lambda}_x \text{ for all } \varphi).\)

**Proof.** See appendix. \(\blacksquare\)
Therefore, endogenous product scope magnifies the impact of trade liberalization on the firm-size distribution. As trade costs fall, higher managerial ability exporting firms become larger relative to lower managerial ability firms that only serve the domestic market, as a result of intensive and extensive margin adjustments in the domestic and export markets. Within the export market, endogenous product scope magnifies the inequality in the distribution of export shipments. As managerial ability rises, firm export volumes increase, both because of greater exports per product and a larger range of products exported. Therefore, the skewness observed in firm-level data on export shipments (Bernard, Jensen and Schott 2005) can be explained with a smaller variance in managerial ability, mirroring our earlier result for the domestic market in the closed economy.

9. Multi-Product Firms and Comparative Advantage

The framework developed in the preceding sections endogenizes product scope across heterogeneous firms in general equilibrium, while remaining highly tractable and consistent with the standard models of intra-industry trade. As a result, our framework can be extended in a variety of directions and applied to a number issues in international trade to see how existing intuitions are altered by firms’ extensive margin decisions. In this section, we develop an extension that addresses a particularly influential issue in the trade literature: factor endowments as a source of comparative advantage and trade. We embed our framework of multi-product firms within the model of comparative advantage and heterogeneous firms of Bernard, Redding and Schott (2006a). We examine how the effects of trade liberalization are influenced by an interaction between firms’ extensive margins and comparative advantage.

The model so far can be viewed as capturing a single industry (e.g. apparel), where there are many products within the industry (e.g. shirts and skirts), and where firms supply differentiated varieties of products (e.g. J. Crew and Gap). In this section, we generalize the analysis by introducing comparative advantage and disadvantage industries that each have this structure. The two industries enter an upper tier of the representative consumer’s utility that takes the Cobb-Douglas form:

\[
\Upsilon = U_1^{\alpha_1} U_2^{\alpha_2}, \quad \alpha_1 + \alpha_2 = 1, \quad \alpha_1 = \alpha, \quad (40)
\]

where \(U_j\) is an index for industry \(j\) that is defined over the consumption \(C_i\) of products \(i\) within the industry (as in equation (4)); \(C_i\) is itself an index for each product \(i\) that is defined over the consumption \(c_i(\omega)\) of a continuum of varieties \(\omega\) (as in equation (5)). Thus, in the extended model, “industries” constitute an upper tier of utility, “products” form an intermediate tier and “varieties” occupy a lower tier. We assume for simplicity that the two industries have the same elasticity of substitution across products within industries (\(\kappa\)) and across varieties within products (\(\sigma\)).

The two industries differ in the intensity with which they use two factors of production: skilled and unskilled labor. To enter an industry, a firm from the competitive fringe must incur a sunk entry cost of \(f_{ej}(w_S)^{\beta_j}(w_L)^{1-\beta_j}\), where \(w_S\) denotes the skilled wage, \(w_L\) corresponds to the unskilled wage and \(\beta_j\) parameterizes industry factor intensity. After the sunk entry cost is paid, the firm draws its managerial ability and values of product expertise for the industry as above. The distributions of managerial ability and product expertise are identically and independently distributed across industries, so that information about productivity within an industry can only be obtained by incurring the sunk entry cost for that industry. The distributions of managerial ability and product expertise are also identically and inde-
pendently distributed across countries, which ensures consistency with the Heckscher-Ohlin model’s assumption of common technologies across countries.

The technology for production has the same factor intensity as for entry.\(^{23}\) While the factor intensity of production varies across industries, all products within an industry are modelled symmetrically and therefore have the same factor intensity.\(^{24}\) To manufacture a product, a firm must incur a fixed and variable cost of production as above. The variable cost depends upon the firm’s productivity in manufacturing the product, which is again determined by both the firm’s managerial ability and its expertise in that product. Fixed and variable costs use the two factors of production with the same proportions, so that the total cost of production for a firm in industry \(j\) is as follows:

\[
\Gamma_j = \left( f_{hj} + \int_0^1 I_{ji} \left[ f_{pj} + \frac{q_j (\varphi_j I_{ji})}{\varphi_j I_{ji}} \right] dt \right) (w_S)^{\beta_j} (w_L)^{1-\beta_j}, \quad 1 > \beta_1 > \beta_2 > 0, \tag{41}
\]

where \(I_{ji}\) is an indicator variable which equals one if a firm in industry \(j\) manufactures product \(i\) and zero otherwise. The fixed costs of becoming an exporter and of exporting a particular product are modelled analogously, and use skilled and unskilled labor with the same factor intensity as production and entry.

We assume that industry 1 is skill-intensive relative to industry 2 and that the home country is skill-abundant relative to the foreign country. The skilled wage in the home country is chosen as the numeraire. In order to determine general equilibrium, we follow a very similar approach as in previous sections. The range of products manufactured and exported by a firm with a particular managerial ability can be determined using analogous expressions to those above. Similarly, the zero-profit cutoff and exporting cutoff for managerial ability can be determined following the same line of reasoning as previously. Entry, production and exporting costs all have the same factor intensity, and therefore terms in factor prices cancel from the relevant expressions. There is however an important general equilibrium interaction between comparative advantage and firms’ product scope and entry decisions. In contrast to the previous sections, the two countries are no longer symmetric. Comparative advantage and trade costs generate cross-country differences in industry price indices, which generate greater export opportunities in comparative advantage industries than comparative disadvantage industries, as in Bernard, Redding and Schott (2006a).

The relative price indices for the two industries vary across countries because of the combination of comparative-advantage-based specialization and trade costs. Specialization leads to a larger mass of domestic firms relative to foreign firms in a country’s comparative advantage industry than in its comparative disadvantage industry. Variable trade costs introduce a wedge between domestic and export prices for a variety, while fixed costs to becoming an exporter imply that not all firms export, and fixed costs to exporting individual products imply that only a subset of the products manufactured by an exporter are shipped abroad. Combining specialization and trade costs, the comparative advantage industry has a greater mass of lower-priced domestic varieties relative to the mass of higher-price foreign varieties than the comparative disadvantage industry. Therefore, the price index in the domestic market is lower relative to the price index in the export market in the comparative advantage industry. Hence, the degree of competition in the domestic market is higher relative to the degree of competition

\(^{23}\) Allowing factor intensity differences between entry and production introduces additional interactions with comparative advantage as discussed in Bernard, Redding and Schott (2006a).

\(^{24}\) The symmetry of products within industries is clearly a simplification, but as in the previous sections of the paper is useful for the law of large numbers results that determine the fraction of products manufactured by a firm with a particular managerial ability.
in the export market in the comparative advantage industry. These differences in the degree of competition in turn imply that export demand is greater relative to domestic demand in the comparative advantage industry than in the comparative disadvantage industry.

**Proposition 7** Other things equal, the opening of trade leads to:

(a) greater firm-level productivity growth in the comparative advantage industry than the comparative disadvantage industry (\(\Delta \lambda_1^H (\phi) > \Delta \lambda_2^H (\phi)\) and \(\Delta \lambda_2^F (\phi) > \Delta \lambda_1^F (\phi)\)),

(b) a larger increase in the zero-profit cutoff for managerial ability below which firms exit in the comparative advantage industry than the comparative disadvantage industry (\(\Delta \varphi_1^*H > \Delta \varphi_2^*H\) and \(\Delta \varphi_2^*F > \Delta \varphi_1^*F\)),

(c) a larger increase in weighted-average productivity in the comparative advantage industry than in the comparative disadvantage industry (\(\Delta \tilde{\varphi}_1^*H > \Delta \tilde{\varphi}_2^*H\) and \(\Delta \tilde{\varphi}_2^*F > \Delta \tilde{\varphi}_1^*F\)).

**Proof.** See Appendix.

Following the opening of trade, countries specialize according to comparative advantage, which leads to a rise in the mass of firms in the comparative advantage industry relative to the comparative disadvantage industry. While the comparative advantage industry expands and the comparative disadvantage industry contracts, the shedding of low productivity products (focusing on core competencies) is greater in the comparative advantage industry. Therefore, the opening of trade leads to greater within-firm productivity growth in the comparative advantage industry. In addition to the greater focusing on core competencies in the comparative advantage industry, there is also a larger rise in the zero-profit cutoff for managerial ability below which firms exit. Hence, the opening of trade causes a larger increase in average industry productivity in the comparative advantage industry, not only because of stronger within-firm productivity growth, but also because of greater between-firm reallocations of resources.

The key to understanding these results is the greater export opportunities in comparative advantage industries. Following the opening of trade, there is a larger increase in labor demand at exported products in the comparative advantage industry than in the comparative disadvantage industry. This larger increase in labor demand leads to a rise in the relative price of the abundant factor, because this factor is used intensively in the comparative advantage industry. As the relative price of the abundant factor rises, the difference in factor intensity between the two sectors implies that production costs in the comparative advantage industry increase relative to production costs in the comparative disadvantage industry. This change in relative production costs reduces the profits derived from products that are supplied only to the domestic market in the comparative advantage industry relative to the comparative disadvantage industry. The larger reduction in domestic market profits induces greater shedding of low productivity products (more focusing on core competencies) and hence greater within-firm productivity growth in the comparative advantage industry. Similarly, the larger reduction in domestic market profits leads to a greater rise in the zero-profit cutoff for managerial ability and hence greater between-firm reallocations of resources in the comparative advantage industry than in the comparative disadvantage industry.

Therefore, this extension of our baseline model of multi-product firms features firm-level responses to trade liberalization that vary with comparative advantage. The measured productivity of a firm with a particular managerial ability depends on comparative advantage, since this shapes the endogenous range of products that the firm chooses to manufacture. Multi-product firms’ endogenous product scope decisions have general equilibrium consequences. The greater within-firm productivity growth and between-firm reallocations of resources in the
comparative advantage industry magnify Heckscher-Ohlin based comparative advantage and so act as an additional source of welfare gains from trade.

10. Conclusions

Although multi-product firms dominate world production and trade, they have played a peripheral role in theoretical analyses of international trade. This paper develops a general equilibrium model of multi-product firms and analyzes their behavior during trade liberalization. Firm productivity in a given product is modeled as a combination of firm-level “managerial ability” and firm-product-level “product expertise”, both of which are stochastic and unknown prior to the firm’s payment of a sunk cost of entry. Firm productivity is positively correlated across products, which in equilibrium induces a positive correlation between a firm’s intensive (output per product) and extensive (number of products) margins.

Multi-product firms’ decisions about endogenous product scope introduce a new adjustment margin along which the economy can respond to trade liberalization. This novel dimension of responses creates firm-level productivity growth following trade liberalization, as multi-product firms drop low productivity products in order to focus on their core competencies in a narrower range of higher productivity products. This source of aggregate productivity gains is excluded by construction from models of single-product firms, and is consistent with a large body of empirical evidence of firm-level productivity growth following trade liberalization, as well as with more recent findings that deeper trade integration leads firms to reduce product diversification and shed peripheral products.

The positive correlation between firms’ intensive and extensive margins magnifies inequality in the firm-size distribution, while at the same time compressing the dispersion in average productivity across firms, as higher managerial ability firms expand along the extensive margin into lower expertise products. There is selection into export markets across firms (exporters and non-exporters), within products (some firms export a product while other firms only supply the product to the domestic market) and also within firms (some products are exported by a firm while other products are only sold domestically). As trade costs fall, there is contraction in the domestic market and expansion in the export market along both the intensive and extensive margins, magnifying the impact of trade liberalization on the firm-size distribution.

Our framework endogenizes product scope in a general equilibrium model of industry dynamics, featuring ongoing firm entry and exit and heterogeneity both across and within firms. Nonetheless, the analysis remains tractable and the framework lends itself to a number of extensions, including the introduction of Heckscher-Ohlin comparative advantage. Since export opportunities are more attractive in comparative advantage industries than comparative disadvantage industries, the opening of trade leads to greater focusing on core competencies and stronger within-firm productivity growth in comparative advantage industries.

Taken together, our findings emphasize the role of a new firm-level adjustment margin - the extensive margin of the number of products - in shaping both firm and industry dynamics and in influencing the general equilibrium implications of trade liberalization.
References


A Data Appendix

The empirical results employ two data sources. For results on firm output, output per product and product count, we use the data from Bernard, Redding and Schott (2006b). Those data are derived from the U.S. Census of Manufactures of the Longitudinal Research Database (LRD) maintained by the U.S. Census Bureau. Manufacturing Censuses are conducted every five years and we examine data from 1972 to 1997. The sampling unit for each Census is a manufacturing “establishment”, or plant, and the sampling frame in each Census year includes information on the mix of products produced by the plant. Very small manufacturing plants (referred to as Administrative Records) are excluded from the analysis unless otherwise noted because data on their mix of products are unavailable. Because product-mix decisions are made at the level of the firm, we aggregate the LRD to that level for our analysis.

Our definition of “product” is based upon 1987 Standard Industrial Classification (SIC) categories, which segment manufacturing output generally according to its end use. We refer to five-digit SIC categories as products or goods. In the LRD, aggregate manufacturing contains 1848 products. For each firm in each Census year, we record the set of products in which the firm produces. We also observe firms’ total and product-level output. There are an average of 141,561 surviving firms in each Census year for which such extensive-margin adjustments can be observed. For further details about the construction of the dataset, see Bernard, Redding and Schott (2006b).

For export value, exports per product and the count of exported products we use the Linked/Longitudinal Firm Trade Transaction Database (LFTTD) from Bernard, Jensen and Schott (2005) which links individual trade transactions to firms in the United States. This dataset has two components. The first, foreign trade data assembled by the U.S. Census Bureau and the U.S. Customs Bureau, captures all U.S. international trade transactions between 1993 and 2000 inclusive. For each flow of goods across a U.S. border, this dataset records the product classification and the value. Products in the LFTTD are tracked according to ten-digit Harmonized System (HS) categories, which break exported goods into 8572 products. The second component of the LFTTD is the Longitudinal Business Database (LBD) of the U.S. Census Bureau, which records annual employment and survival information for most U.S.

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25 The description here draws upon that in Bernard, Redding and Schott (2006b).

26 The SIC classification scheme was revised substantially in 1977 and again in 1987. Industry identifiers from Censuses prior to 1987 have been concorded to the 1987 scheme. Our results are not sensitive to this concordance: we get substantially similar results if we focus exclusively on the 1987 to 1997 portion of the LRD.
establishments.\textsuperscript{27} Employment information for each establishment is collected in March of every year and we aggregate the establishment data up to the level of the firm. Matching the annual information in the LBD to the transaction-level trade data yields the LFTTD. For further details about the construction of the dataset, see Bernard, Jensen and Schott (2005).

\section*{B Theory Appendix}

\textbf{B1. Weighted Average Productivity}

Aggregate weighted-average productivity is:

\[
\bar{\varphi} \equiv \left[ \frac{1}{1 - G(\varphi^*)} \int_{\varphi^*}^{\infty} \tilde{\lambda}(\varphi) g(\varphi) \, d\varphi \right]^{\frac{1}{\sigma - 1}},
\]

(42)

where \(\tilde{\lambda}(\varphi)\) denotes weighted-average productivity for a firm of managerial ability \(\varphi\):

\[
\tilde{\lambda}(\varphi) = \frac{1}{1 - Z(\lambda^*(\varphi))} \int_{\lambda^*(\varphi)}^{\infty} (\varphi \lambda)^{\sigma - 1} z(\lambda) \, d\lambda.
\]

(43)

Similarly, aggregate weighted-average productivity in the export market is:

\[
\bar{\varphi}_x \equiv \left[ \frac{1}{1 - G(\varphi^*_x)} \int_{\varphi^*_x}^{\infty} \tilde{\lambda}_x(\varphi) g(\varphi) \, d\varphi \right]^{\frac{1}{\sigma - 1}},
\]

(44)

where \(\tilde{\lambda}_x(\varphi)\) denotes weighted-average productivity in the export market for a firm of managerial ability \(\varphi\):

\[
\tilde{\lambda}_x(\varphi) = \frac{1}{1 - Z(\lambda^*_x(\varphi))} \int_{\lambda^*_x(\varphi)}^{\infty} (\varphi \lambda)^{\sigma - 1} z(\lambda).
\]

(45)

\textbf{B2. Closed Economy Average Firm Revenue and Profit}

Combining equations (17), (10), (11), (22) and (23), average revenue and profits can be re-written as:

\[
\bar{r} = \int_{\varphi^*}^{\infty} \left[ \int_{\lambda^*(\varphi)}^{\infty} \left( \frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma - 1} f_p z(\lambda) \, d\lambda \right] \left( \frac{g(\varphi)}{1 - G(\varphi^*)} \right) \, d\varphi,
\]

\[
\bar{\pi} = \int_{\varphi^*}^{\infty} \left[ \int_{\lambda^*(\varphi)}^{\infty} \left( \frac{\lambda}{\lambda^*(\varphi)} \right)^{\sigma - 1} f_p z(\lambda) \, d\lambda - f_h \right] \left( \frac{g(\varphi)}{1 - G(\varphi^*)} \right) \, d\varphi.
\]

(46)

where \(\lambda^*(\varphi) = (\varphi^*/\varphi) \tilde{\lambda} \).

\textsuperscript{27}This dataset excludes the U.S. Postal Service and firms in agriculture, forestry and fishing, railroads, education, public administration and several smaller sectors. See Jarmin and Miranda (2002) for an extensive discussion of the LBD and its construction.
B3. Open Economy Average Firm Revenue and Profit

Combining equations (17), (10), (11), (22) and (23), the expression for average revenue and profits in the domestic market are the same as those in equation (46). Combining equations (32), (10), (29), (30) and (37), average revenue and profits in the export market can be rewritten as:

\[
\bar{r}_x = \int_{\phi_x^-}^{\infty} \left[ \int_{\lambda_x^* (\phi')}^{\infty} \left( \frac{\lambda}{\lambda_x^* (\phi')} \right)^{\sigma-1} \sigma f_{x\bar{z}} (\lambda) d\lambda \right] \left( \frac{g (\phi)}{1 - G (\phi_x^* \phi')} \right) d\phi
\]

\[
\tilde{\pi}_x = \int_{\phi_x^-}^{\infty} \left[ \int_{\lambda_x^* (\phi')}^{\infty} \left( \frac{\lambda}{\lambda_x^* (\phi')} \right)^{\sigma-1} \left( \lambda_x^* (\phi) \right) - 1 \right] f_{x\bar{z}} (\lambda) d\lambda - f_{xh} \left( \frac{g (\phi)}{1 - G (\phi_x^* \phi')} \right) d\phi.
\]

where \( \lambda_x^* (\phi) = (\phi_x^*/\phi) \lambda_x \).

B4. Proof of Proposition 1

Proof. The relative revenue of two firms with managerial productivities \( \phi'' \) and \( \phi' \) and product productivities \( \lambda'' \) and \( \lambda' \) within a product market is: \( IM \equiv r (\phi'', \lambda'') / r (\phi', \lambda') = (\phi''/\phi')^{\sigma-1} (\lambda''/\lambda')^{\sigma-1} \), where \( \lambda \) is independently distributed across firms and products. The relative fraction of products manufactured by the two firms is: \( EM \equiv [1 - Z (\lambda^* (\phi''))] / [1 - Z (\lambda^* (\phi'))] \).

Note that \( \lambda^* (\phi) \) in equations (22) and (23) is monotonically decreasing in \( \phi \), while \( Z (\lambda^* (\phi)) \) is a cumulative distribution function which is monotonically increasing in its argument. It follows that \( IM \) and \( EM \) are both monotonically increasing in \( (\phi''/\phi') \) and are positively correlated.

B5. Proof of Corollary 1

Proof. Consider two firms with managerial abilities \( \phi'' \) and \( \phi' \), where \( \phi'' > \phi' \). Combining the equation for firm revenue in (13), the relationship between variety revenues in (10), the zero-profit cutoff for product expertise in (11), and the relationship between product zero-profit cutoffs in (22) yields the following expression for the relative revenues of the two firms:

\[
r (\phi'') = \left( \frac{\phi''}{\phi'} \right)^{\sigma-1} r (\phi') + \int_{\lambda^* (\phi')}^{\lambda^* (\phi'')} \left( \frac{\lambda}{\lambda^* (\phi')} \right)^{\sigma-1} \sigma f_{\bar{p}z} (\lambda) d\lambda,
\]

which is strictly greater than the relative revenue of the two firms with a common exogenous product scope: \( r (\phi'') = (\phi''/\phi')^{\sigma-1} r (\phi') \).
B6. Proof of Proposition 2

Proof. (a) Differentiating firm weighted average productivity \( \tilde{\lambda}(\varphi) \) in equation (43), we obtain:

\[
\frac{d\tilde{\lambda}(\varphi)}{d\varphi} = \left( \frac{z(\lambda^*(\varphi))}{1 - Z(\lambda^*(\varphi))} \right) \frac{d\lambda^*(\varphi)}{d\varphi} \left( \frac{1}{1 - Z(\lambda^*(\varphi))} \right) \int_{\lambda^*(\varphi)}^{\infty} (\varphi \lambda)^{\sigma - 1} z(\lambda) \, d\lambda
\]

Term A < 0

\[+ \left( \frac{1}{1 - Z(\lambda^*(\varphi))} \right) \int_{\lambda^*(\varphi)}^{\infty} (\sigma - 1)(\lambda)^{\sigma - 1} (\varphi)^{\sigma - 2} z(\lambda) \, d\lambda \]

Term B > 0

\[- \left( \frac{z(\lambda^*(\varphi))}{1 - Z(\lambda^*(\varphi))} \right) \frac{d\lambda^*(\varphi)}{d\varphi} (\varphi \lambda^*(\varphi))^{\sigma - 1} > 0, \]

Term C > 0

where \( d\lambda^*(\varphi) / d\varphi < 0 \). Since \( z(\lambda) \) is a continuous distribution function, \( z(\lambda^*(\varphi)) / [1 - Z(\lambda^*(\varphi))] \) is of measure zero, and so the derivative \( d\tilde{\lambda}(\varphi) / d\varphi > 0 \).

(b) Consider two firms with managerial abilities \( \varphi'' \) and \( \varphi' \), where \( \varphi'' > \varphi' \), and so \( \lambda^*(\varphi'') < \lambda^*(\varphi') \). From the definition of firm weighted-average productivity in equation (43):

\[
\tilde{\lambda}(\varphi'') = \left( \frac{1 - Z(\lambda^*(\varphi'))}{1 - Z(\lambda^*(\varphi''))} \right) \int_{\lambda^*(\varphi'')}^{\lambda^*(\varphi')} (\lambda^*(\varphi'))^{\sigma - 1} z(\lambda) \, d\lambda + \left( \frac{Z(\lambda^*(\varphi')) - Z(\lambda^*(\varphi''))}{1 - Z(\lambda^*(\varphi''))} \right) \frac{\hat{\lambda}_0}{\lambda^*(\varphi')} \left( \frac{\varphi''}{\varphi'} \right) \tilde{\lambda}(\varphi'),
\]

(49)

where \( \hat{\lambda}_0 \equiv \frac{1}{Z(\lambda^*(\varphi')) - Z(\lambda^*(\varphi''))} \int_{\lambda^*(\varphi'')}^{\lambda^*(\varphi')} (\lambda^*(\varphi'))^{\sigma - 1} z(\lambda) \, d\lambda \) < \( \tilde{\lambda}(\varphi') \),

and where the sum of the terms in square parenthesis in equation (49) is strictly less than one, since Terms A and B sum to one, and Term C is strictly less than one. □

B7. Proof of Proposition 3

Proof. Consider an increase in the sunk entry cost: \( \Delta f_e > 0 \). Denote the value of variables before and after the change in the sunk entry cost by the subscripts 1 and 2. With endogenous product scope: \( \varphi_1^* > \varphi_2^* \) and \( \lambda_1^* (\varphi) > \lambda_2^* (\varphi) \). Taking differences in the free entry condition (24) before and after the parameter change, we obtain:

\[
\Delta v_e = \frac{1}{\delta} \left[ \int_{\varphi_1^*}^{\varphi_1^*} \left( \int_{\lambda_1^*(\varphi)}^{\infty} \left( \frac{\lambda}{\lambda_1^* (\varphi)} \right)^{\sigma - 1} - \left( \frac{\lambda}{\lambda_2^* (\varphi)} \right)^{\sigma - 1} \right) f_p z(\lambda) \, d\lambda - f_h \right] g(\varphi) \, d\varphi
\]

Term A > 0

\[+ \frac{1}{\delta} \left[ \int_{\varphi_1^*}^{\varphi_1^*} \left( \int_{\lambda_1^*(\varphi)}^{\lambda_2^*(\varphi)} \left( \frac{\lambda}{\lambda_2^*(\varphi)} \right)^{\sigma - 1} - 1 \right) f_p z(\lambda) \, d\lambda - f_h \right] g(\varphi) \, d\varphi
\]

Term B > 0

\[+ \frac{1}{\delta} \left[ \int_{\varphi_1^*}^{\varphi_2^*} \left( \int_{\lambda_2^*(\varphi)}^{\infty} \left( \frac{\lambda}{\lambda_2^*(\varphi)} \right)^{\sigma - 1} - 1 \right) f_p z(\lambda) \, d\lambda - f_h \right] g(\varphi) \, d\varphi = \Delta f_e.
\]

Term C > 0
With endogenous product scope: \( \varphi_1^* > \varphi_2^* \) and \( \lambda^* (\varphi) = \lambda_1^* (\varphi) \) remains unchanged. Taking differences in the free entry condition before and after the parameter change, we obtain:

\[
\Delta v_e = \frac{1}{\tau} \int_{\varphi_2}^{\varphi_1} \left[ \left( \frac{\lambda}{\lambda_1^*(\varphi)} \right)^{\sigma-1} - 1 \right] f_p z (\lambda) d\lambda - f_h \int_{\varphi_2}^{\varphi_1} g(\varphi) d\varphi = \Delta f_e.
\]

where Term D < Term C, since \( \lambda_1^* (\varphi) > \lambda_2^* (\varphi) \).

For both endogenous and exogenous product scope, we require: \( \Delta v_e = \Delta f_e \). For a given pair of values of \( \varphi_1^* \) and \( \varphi_2^* \), note that Term A > 0, Term B > 0 and Term C > Term D > 0. Therefore, since \( \Delta v_e \) is monotonically increasing in \( \Delta \varphi^* = \varphi_1^* - \varphi_2^* > 0 \), the change in the zero-profit cutoff productivity \( \Delta \varphi^* \) must be smaller when product scope is endogenous than when it is exogenous.

**B8. Proof of Proposition 4**

**Proof.** (a) The expected value of entry for the open economy in equation (39) equals that for the closed economy in equation (24) plus an additional positive term. Since the expected value of entry \( v_e \) is monotonically decreasing in \( \varphi^* \), the equilibrium value of \( \varphi^* \) must be higher in the open economy than in the closed economy in order to equate \( v_e \) with the unchanged sunk entry cost. The product expertise cutoff \( \lambda^* (\varphi) = (\varphi^*/\varphi) \lambda \) is monotonically increasing in \( \varphi^* \) and therefore rises following the opening of trade. Firm weighted average productivity in equation (43) is monotonically increasing in \( \lambda^* (\varphi) \) and so also rises, which contributes towards an increase in aggregate weighted average productivity \( \bar{\varphi} \) in equation (42).

(b) From the proof of Proposition 2, firms with higher managerial abilities \( \varphi \) have higher weighted average productivities \( \lambda (\varphi) \). Therefore, the increase in \( \varphi \) raises weighted average productivity \( \bar{\varphi} \) in equation (42), not only because each surviving firm has a higher weighted average productivity \( \bar{\lambda} (\varphi) \), but also because of the selection of firms with higher managerial abilities.

**B9. Proof of Proposition 5**

**Proof.** (a) From the expression for the expected value of entry \( v_e \) in equation (39): \( dv_e/d\tau < 0 \). Therefore, since (39) is monotonically decreasing in \( \varphi^* \), \( d\varphi^*/d\tau < 0 \).

(b) First, we consider the extensive margin in the domestic market. From equations (22) and (23): \( d\lambda^* (\varphi)/d\varphi^* > 0 \). Since the fraction of products manufactured \( [1 - Z (\lambda^* (\varphi))] \) is decreasing in \( \lambda^* (\varphi) \), the rise in \( \lambda^* (\varphi) \) following a reduction in \( \tau \) diminishes the range of products manufactured for the domestic market.

Second, we consider the intensive margin in the domestic market. The revenue received in the domestic market from a product with productivity \( \lambda \) at a firm with managerial ability \( \varphi \) can be written as the following decreasing function of \( \lambda^* (\varphi) \): \( r (\varphi, \lambda) = (\lambda/\lambda^* (\varphi)) \sigma f_p \). Therefore, \( dr (\varphi, \lambda)/d\tau > 0 \).

(c) Differentiating the expression for equilibrium \( \varphi^*_x \) in equation (38) with respect to \( \tau \):

\[
\left( \frac{d\varphi^*_x}{d\tau} \right) \frac{\tau}{\varphi^*_x} = 1 + \left( \frac{d\varphi^*_x}{d\tau} \right) \frac{\tau}{\varphi^*_x},
\]

where, from (a), \( d\varphi^*/d\tau < 0 \).

Define \( \Omega = v_e - f_e \). By the implicit function theorem, \( d\varphi^*/d\tau = -(d\Omega/d\tau)/(d\Omega/d\varphi^*) \).
Applying the implicit function theorem and using $d \lambda^*_x (\varphi) / d \tau = \lambda^*_x (\varphi) / \tau$, $d \lambda^*_x (\varphi) / d \varphi^* = \lambda^*_x (\varphi) / \varphi^*$, $d \varphi^*_x / d \tau = \varphi^*_x / \tau$ and $d \varphi^*_x / d \varphi^* = \varphi^*_x / \varphi^*$, it can be shown that $(d \varphi^* / d \tau) / (\tau / \varphi^*) > -1$. Therefore, from equation (50), $d \varphi^*_x / d \tau > 0$.

(d) First, we consider the extensive margin in the export market. From equations (31), (22) and (23):

$$\left( \frac{d \lambda^*_x (\varphi)}{d \tau} \right) \left( \frac{\tau}{\lambda^*_x (\varphi)} \right) = 1 + \left( \frac{d \varphi^*_x \tau}{d \tau \varphi^*} \right) > 0,$$

where we have used the result, from (c), that $(d \varphi^* / d \tau) / (\tau / \varphi^*) > -1$. Since the fraction of products exported $[1 - Z (\lambda^*_x (\varphi))]$ is decreasing in $\lambda^*_x (\varphi)$, the decline in $\lambda^*_x (\varphi)$ following a reduction in $\tau$ expands the range of products exported.

Second, we consider the intensive margin in the export market. The revenue received in the export market from a product with productivity $\varphi$ can be written as the following decreasing function of $\lambda^*_x (\varphi)$: $r_x (\varphi, \lambda) = (\lambda / \lambda^*_x (\varphi)) \sigma f_x$. Therefore, $dr_x (\varphi, \lambda) / d \tau < 0$. 

**B10. Proof of Proposition 6**

**Proof.** Consider an exporting firm of managerial ability $\varphi''$ and a non-exporting firm of managerial ability $\varphi'$, where $\varphi'' > \varphi'$. When $\lambda^* (\varphi) = \lambda^* (\varphi^*) = \lambda$ and $\lambda^*_x (\varphi) = \lambda^*_x (\varphi^*_x) = \lambda_x$, the relative revenue of the two firms is:

$$r (\varphi'') = \left( \frac{\varphi''}{\varphi'} \right)^{\sigma - 1} r (\varphi') + \int^\infty_{\lambda_x} \left( \frac{\varphi''}{\varphi'} \right)^{\sigma - 1} \left( \frac{\lambda}{\lambda_x} \right)^{\sigma - 1} \sigma f_x.$$

When the ranges of products manufactured and exported are endogenous, the relative revenues of the two firms may be determined by combining the equation for firm revenue in (13), the relationship between variety revenues in (10), the expression for the zero-profit product cutoff in (11), the relationship between the zero-profit product cutoffs in (22), the equation for the exporting product cutoff in (29), and the relationship between exporting product cutoffs in (30):

$$r (\varphi'') = \left( \frac{\varphi''}{\varphi'} \right)^{\sigma - 1} r (\varphi') + \int^\infty_{\lambda_x} \left( \frac{\varphi''}{\varphi'} \right)^{\sigma - 1} \left( \frac{\lambda}{\lambda_x} \right)^{\sigma - 1} \sigma f_x + \int^\lambda_{\lambda^* (\varphi')} \left( \frac{\lambda}{\lambda^* (\varphi')} \right)^{\sigma - 1} \sigma f_{p^x} (\lambda) \lambda_x + \int^\lambda_{\lambda^* (\varphi''')} \left( \frac{\lambda}{\lambda^* (\varphi'')} \right)^{\sigma - 1} \sigma f_{p^x} \left( \frac{\varphi'''}{\varphi'} \right) \lambda_x,$$

which is strictly greater than the expression above. 

**B11. Proof of Proposition 7**

**Proof.** The introduction of comparative advantage implies that countries are no longer symmetric. Therefore, the relative revenue from a product $i$ in the domestic and export markets in industry $j$ depends on price indices and aggregate revenue in the two countries: $r_{jix} (\varphi_j, \lambda_{ji}) = \tau_{j - i}^{-\sigma} \left( P^F_{ji} / P^H_{ji} \right)^{\sigma - 1} (P^F / P^H) r_{jix} (\varphi_j, \lambda_{ji})$. In equilibrium, the price indices are the same for all products within an industry due to symmetry: $P^F_{ji} = P^F_{j}$ and $P^H_{ji} = P^H_{j}$ for all $i$.

The equations determining the equilibrium range of products manufactured as a function of the zero-profit cutoff managerial ability (equations (22) and (23)) and the equations determining
the equilibrium range of products exported as a function of the exporting cutoff managerial ability (equations (30) and (37)) remain exactly as in previous sections. These relationships only compare the relative revenue of products within a particular market within an individual industry, and are therefore unchanged by the introduction of country asymmetries. The additional terms in factor prices due to the introduction of skilled and unskilled labor cancel from the left and right-hand sides of equations (23) and (37).

The free entry condition also takes the same form as previously (equation (39)), since average profits in the domestic and export market are evaluated separately relative to the lowest expertise product supplied by a firm, and terms in factor prices again cancel from the left and right-hand sides of the equation.

However, the introduction of country asymmetries changes the relationship between the exporting cutoff managerial ability \( \phi_{xj}^* \) and the zero-profit cutoff managerial ability \( \phi_j^* \), which instead of equation (38) is given by the following expression for the home country:

\[
\phi_{xj}^* = \Gamma_j^H \phi_j^*, \quad \Gamma_j^H \equiv \tau_j \left( \frac{f_{ej}}{f_{pj}} \frac{R_j^H}{R_j^F} \right)^{\frac{1}{\Gamma_j^H}} \left( \frac{P_j^H}{P_j^F} \right) \left( \frac{\lambda_j}{\lambda_{xj}} \right),
\]

where an analogous expression holds for the foreign country. Since \( \lambda_{xj}^* (\varphi_j) = \left( \phi_{xj}^*/\phi_j^* \right) \lambda_{xj} \), the change in the relationship between \( \phi_{xj}^* \) and \( \phi_j^* \) affects the exporting cutoffs for product expertise \( \lambda_{xj}^* (\varphi_j) \) and hence average profits from the export market in the free entry condition (39).

Comparing the free entry conditions in the open and closed economies (equations (39) and (24)), the expected value of entry in the open economy is equal to the value for the closed economy plus an additional positive term which captures the expected profits from the export market. Since \( \lambda_{xj}^* (\varphi_j) = \left( \phi_{xj}^*/\phi_j^* \right) \lambda_{xj} \) and \( \phi_{xj}^* = \Gamma_j^H \phi_j^* \), this additional positive term is larger, the smaller the value of \( \Gamma_j \). Dividing equation (51) for the two industries:

\[
\frac{\phi_{xj}^H / \phi_j^H}{\phi_{xj}^F / \phi_j^F} = \frac{\Gamma_j^H}{\Gamma_j^F} = \frac{\tau_1}{\tau_2} \frac{\left( f_{e1}/f_{pj} \right)}{\left( f_{e2}/f_{pj} \right)} \frac{\lambda_{1j}}{\lambda_{2j}} \lambda_{xj} \left( \frac{P_{j}^H}{P_{j}^F} \right) \left( \frac{\lambda_{xj}}{\lambda_{xj}} \right).
\]

The remainder of the proof follows the same structure as the proof of Proposition 4 in Bernard, Redding and Schott (2006b) and we present an abbreviated version here. The price index for the skill-intensive industry relative to the labor-intensive industry is lower in the skill-abundant country than the labor-abundant country: \( P_1^H/P_2^H < P_1^F/P_2^F \). Therefore, in the absence of other differences in parameters across industries except factor intensity (common values of \( \tau_i, f_i, P_i, \) and \( f_e \) across industries): \( \Gamma_i^H < \Gamma_i^F \) and similarly \( \Gamma_j^H < \Gamma_j^F \). Hence, the additional positive term in the free entry condition capturing expected profits in the export market is larger in the comparative advantage industry than the comparative disadvantage industry.

Noting that \( \lambda_j^*(\varphi) = \left( \phi_j^*/\phi_j^* \right) \lambda_j \), \( \lambda_{xj}^* (\varphi) = \left( \phi_{xj}^*/\phi_j^* \right) \lambda_{xj} \), and \( \phi_{xj}^H = \Gamma_j^H \phi_j^* \), the expected value of entry in the free entry condition (39) is monotonically decreasing in \( \phi_j^* \). Therefore, the comparative advantage industry’s larger increase in the expected value of entry following the opening of trade requires a larger rise in the zero-profit cutoff for managerial ability \( \phi_j^* \) in order to restore equality between the expected value of entry and the unchanged sunk entry cost: \( \Delta \phi_{j1}^* > \Delta \phi_{j2}^* \) and \( \Delta \phi_{j1}^* > \Delta \phi_{j2}^* \).

Since \( \lambda_j^*(\varphi) = \left( \phi_j^*/\phi_j^* \right) \lambda_j \), the comparative advantage industry’s larger rise in the zero-profit cutoff for managerial ability \( \phi_j^* \) implies a greater focusing on core competencies in the comparative advantage industry than in the comparative disadvantage industry: \( \Delta \lambda_{1j}^* (\varphi) > \Delta \lambda_{2j}^* (\varphi) \).
and \( \Delta \lambda_2^F (\varphi) > \Delta \lambda_1^F (\varphi) \).

From the definition of weighted average productivity in equations (42) and (43), the comparative advantage industry’s larger rise in the zero-profit cutoff for managerial ability and greater focusing on core competencies implies a larger increase in weighted average productivity in the comparative advantage industry than in the comparative disadvantage industry: \( \Delta \tilde{\varphi}_1^H > \Delta \tilde{\varphi}_2^H \) and \( \Delta \tilde{\varphi}_2^F > \Delta \tilde{\varphi}_1^F \).
Multi-Product Firms and Trade Liberalization

Figure 1: Product Scope and Managerial Ability

Figure 2: Trade Liberalization and Product Scope