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## Interpreting tests of the convergence hypothesis

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### Abstract

This paper provides a framework for understanding the cross-section and time series approaches which have been used to test the convergence hypothesis. First, we present two definitions of convergence which capture the implications of the neo-classical growth model for the relationship between current and future cross-country output differences. Second, we identify how the cross-section and time series approaches relate to these definitions. Cross-section tests are shown to be associated with a weaker notion of convergence than time series tests. Third, we show how these alternative approaches make different assumptions on whether the data are well characterized by a limiting distribution. As a result, the choice of an appropriate testing framework is shown to depend on both the specific null and alternative hypotheses under consideration as well as on the initial conditions characterizing the data being studied.

*Key words:* Growth models; Unit roots; Time series tests; Cross-section tests

*JEL classification:* O41; C31; C32

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### 1. Introduction

The neoclassical growth model, originating with Solow (1956), has profoundly affected the way in which economists conceptualize long-run interrelationships between macroeconomies. By ascribing economic growth to the joint impact of

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exogenous technical change and capital deepening on an economy with concave short-run production opportunities, the neoclassical model makes very strong predictions concerning the behavior of economies over time. In particular, given a microeconomic specification of technologies and preferences, per capita output in an economy will converge to the same level regardless of initial capital endowments. In comparing different economies, this means that differences in per capita output for economies with identical technologies and preferences will be transitory.

Starting with Romer (1986) and Lucas (1988), a body of theoretical research has challenged the strong cross-country implications of the neoclassical model. 'New growth' theorists have pointed to the failure of per capita output to equalize across first and third world economies as well as the failure of growth rates in less developed countries to exceed those of advanced industrialized countries as evidence that there is little observable tendency for poorer economies to catch up to richer ones. In terms of theory, these authors have argued that a fundamental factor in growth is the presence of nonconvexities in production which can create a nondiminishing relationship between an economy's initial conditions and its output level over arbitrarily long horizons. Authors such as Azariadis and Drazen (1990) and Durlauf (1993) have specifically shown how production complementarities can interact with market incompleteness to generate multiple equilibria in long-term output paths, which implies that similarly specified economies need not converge.

The striking differences in the empirical implications of the neoclassical and new growth perspectives have led to a literature which has formally tested the convergence hypothesis. Empirical tests of convergence fall into two categories. The first class of tests studies the cross-section correlation between initial per capita output levels and subsequent growth rates for a group of countries. A negative correlation is taken as evidence of convergence as it implies that, on average, countries with low per capita initial incomes are growing faster than those with high initial per capita incomes.<sup>1</sup>

A second set of tests has examined the long-run behavior of differences in per capita output across countries. These tests interpret convergence to mean that these differences are always transitory in the sense that long-run forecasts of the difference between any pair of countries converges to zero as the forecast horizon grows. Convergence, according to this approach, has the strong implication that output differences between two economies cannot contain unit roots or time trends and the weak implication that output levels in two economies must be cointegrated.

These different testing frameworks have tended to give contrary results when applied to output series. Cross-section tests generally reject the no convergence

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<sup>1</sup> Other work has explored whether the cross-section variance of per capita output for a set of countries has decreased over time. This approach may be shown to suffer from similar problems to those we identify for cross-section regression tests.

null in the cases of advanced industrialized economies (Baumol, 1986; Dowrick and Nguyen, 1989) and US regions (Barro and Sala-i-Martin, 1991, 1992), as well as in large international cross-sections after controlling for variables such as population growth or savings rates (Barro, 1991; Mankiw, Romer, and Weil, 1992). On the other hand, time series tests have generally accepted the no convergence null for a range of data sets, as shown by Quah (1992), Bernard (1992), and Bernard and Durlauf (1995).

The purpose of this paper is to explore the properties of various tests of the convergence hypothesis. Our analysis contributes to the convergence literature in three respects. First, we propose two definitions of convergence which directly focus on the transience or permanence of contemporary output differences. Both types of convergence are implications of the neoclassical growth model. Second, we relate these definitions to the cross-section and time series tests which have been employed in the empirical growth literature. This analysis indicates how time series tests are based on a stricter notion of convergence than cross-section tests. Further, cross-section tests turn out to be unable to distinguish between the neoclassical growth model and certain new growth alternatives. Third, we show how the different testing approaches also make different assumptions concerning the statistical properties of the data. While cross-section tests assume that the data under analysis are generated by economies far from a steady state, time series tests assume that the data possess well-defined population moments in either levels or first differences. Inferences from the time series approach may be invalid when based on data which are far from a limiting distribution.

Together, these results illustrate how the cross-section and time series approaches to convergence make different assumptions both about what one means by convergence and about the properties of the economies under study, and therefore how tests within the two frameworks can lead to very different conclusions concerning cross-country output relationships.

Section 2 of this paper reviews the dynamics of neoclassical growth models. Section 3 links the neoclassical model to two notions of convergence. Section 4 describes the different convergence tests in the literature and characterizes their relationships to the convergence definitions. Section 5 gives a summary and conclusions.

## 2. Review of neoclassical growth model

Following Solow (1956), a typical neoclassical stochastic growth model can be written as follows.<sup>2</sup> Output  $Y_t$  obeys a production function of the form

$$Y_t = A_t F(K_{t-1}, H_{t-1}, L_{t-1}, \xi_t), \quad (1)$$

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<sup>2</sup> We focus on the Solow model versus a model which endogenizes rates for ease of exposition. As the new growth theory differs from the old in terms of assumptions about the production function rather than in terms of preferences, no loss of generality occurs.

where  $A_t$  denotes the level of productivity,  $K_t$  and  $H_t$  denote physical and human capital respectively,  $L_t$  denotes labor, and  $\xi_t$  is a productivity shock.  $A_t$  can embody both deterministic and stochastic technical change; examples include  $\ln A_t = \rho t$  or  $\ln A_t = \rho t + \zeta_t$ , where  $\zeta_t$  contains a unit root. While  $\xi_t$  is assumed to be unconditionally stationary in theoretical analyses, the necessary condition for empirical work is that for a given  $K$ ,  $H$ , and  $L$ , the limiting sample moments of  $F(K, H, L, \xi_t)$  be identical across different realizations of the economy.

Physical and human capital obey the laws of motion

$$K_t = (1 - \delta_K)K_{t-1} + s_K Y_t, \quad (2)$$

$$H_t = (1 - \delta_H)H_{t-1} + s_H Y_t. \quad (3)$$

Here  $s_K$  and  $s_H$  denote savings rates and  $\delta_K$  and  $\delta_H$  denote depreciation rates. Labor grows at a constant rate  $n$ ,

$$L_t = (1 + n)^t L_0. \quad (4)$$

Finally, some restrictions are placed on the function  $F(\cdot, \cdot, \cdot)$ .  $F(\cdot, \cdot, \cdot)$  is assumed to exhibit nonincreasing returns to scale. Further, the function obeys the Inada-type conditions

$$\frac{\partial F(0, H, L, \xi)}{\partial K} = \frac{\partial F(K, 0, L, \xi)}{\partial H} = \infty, \quad (5)$$

$$\frac{\partial F(\infty, H, L, \xi)}{\partial K} = \frac{\partial F(K, \infty, L, \xi)}{\partial H} = 0. \quad (6)$$

Under these conditions, the long-run behavior of the economy will be independent of initial conditions in the following sense. Consider  $A_t^{-1} L_t^{-1} Y_t$ , the level of output normalized by the levels of productivity and labor. The limiting behavior of this stochastic process is independent of the initial conditions in the economy.

*Proposition 1. Convergence in the neoclassical growth model. For any economy obeying the neoclassical growth model that was defined by Eqs. (1)–(6),  $\lim_{t \rightarrow \infty} \text{Prob}(A_t^{-1} L_t^{-1} Y_t \mid K_0, H_0, L_0)$  is independent of  $K_0, H_0,$  and  $L_0$ .*

Intuitively, the concavity of  $F(\cdot, \cdot, \cdot)$  in the capital stocks means that capital-poor economies will grow sufficiently faster than capital-rich ones to offset differences in initial conditions. Although the concavity assumptions on the production function are sufficient rather than necessary, it is straightforward to construct examples of models where the relaxation of either requirement causes Proposition 1 to break down. For example, Jones and Manuelli (1990) show how output levels across economies can fail to equalize when the marginal product of capital fails to converge to zero even as the capital-labor ratio becomes

unbounded – which is inconsistent with (6). Romer's (1986) model of capital complementarities is a case where the production function exhibits increasing returns, allowing output differences between economies to become unbounded.

### 3. Defining convergence

In this section, we propose two definitions of convergence which capture some of the implications of the neoclassical growth model for the permanence of contemporaneous output differences. These definitions characterize convergence between a pair of economies  $i$  and  $j$ ; convergence between members of a set of  $I$  economies may be defined analogously by requiring that every pair within the set exhibits convergence. Throughout  $\mathfrak{F}_t$  denotes all information available at  $t$ .

Our first definition considers the behavior of the output difference between two economies over a fixed time interval and equates convergence with the tendency of the difference to narrow.

*Definition 1. Convergence as catching up. Countries  $i$  and  $j$  converge between dates  $t$  and  $t + T$  if the (log) per capita output disparity at  $t$  is expected to decrease in value. If  $y_{i,t} > y_{j,t}$ ,*

$$E(y_{i,t+T} - y_{j,t+T} \mid \mathfrak{F}_t) < y_{i,t} - y_{j,t}.^3 \quad (7)$$

Our second definition asks whether the long-run forecasts of output differences tend to zero as the forecasting horizon increases. This definition is violated if history matters, i.e., the effects of a shock on output differences persist into the indefinite future.

*Definition 2. Convergence as equality of long-term forecasts at a fixed time. Countries  $i$  and  $j$  converge if the long-term forecasts of (log) per capita output for both countries are equal at a fixed time  $t$ ,*

$$\lim_{k \rightarrow \infty} E(y_{i,t+k} - y_{j,t+k} \mid \mathfrak{F}_t) = 0.^4 \quad (8)$$

<sup>3</sup> We express definitions in terms of logarithm of per capita output between economies, denoted as  $y_{i,t}$ , as the empirical literature has generally focused on logs rather than levels.

<sup>4</sup> Notice that if this conditional expectation is taken with respect to the linear space generated by current and lagged  $y_{i,t} - y_{j,t}$  rather than a general  $\mathfrak{F}_t$ , as is conventionally done in empirical work, then this definition is equivalent to requiring that  $y_{i,t} - y_{j,t}$  is a linearly regular process. Also, this definition implies that convergence does not hold if  $y_{i,t} - y_{j,t}$  does not converge to a limiting stochastic process. For example, if  $y_{i,t} - y_{j,t}$  equals 1 in even periods and  $-1$  in odd periods, two countries will fail to converge in the sense of Definition 2, although the sample mean of the differences is equal to zero. This problem has no impact on either the conduct or interpretation of empirical work so long as any stochastic process whose population moments coincide with the limiting samples moments of  $y_{i,t} - y_{j,t}$  obeys Definition 2.

The relationship between these definitions is quite straightforward. In fact, it is easy to show that the definitions can be ordered in terms of the range of restrictions placed on the behavior of output differences.

*Proposition 2. Relationships between definitions. Definition 2  $\Rightarrow$  Definition 1 (for some  $T$ ).*

From the perspective of empirical work, both definitions are useful as they each represent implications of the neoclassical growth model.

*Proposition 3. Relationship between the neoclassical growth model and convergence definitions. Any pair of economies which obey Eqs. (1)–(6) and possess identical savings rates, population growth rates, production functions, and probability distributions of shocks will exhibit convergence in the senses of both Definition 1 and Definition 2.*

## 4. Convergence tests

### 4.1. Cross-section tests

A first set of convergence tests, used by Baumol (1986), DeLong (1988), Dowrick and Nguyen (1989), Barro (1991), and Barro and Sala-i-Martin (1991, 1992), among others, examines how an economy's average growth comoves with initial income. Defining the average growth rate  $g_{i,T} = T^{-1}(y_{i,T} - y_{i,0})$  for each of  $I$  economies, these authors have examined

$$g_{i,T} = \alpha + \beta y_{i,0} + \varepsilon_{i,T}, \quad (9)$$

where  $T$  is a fixed horizon and  $E(\varepsilon_{i,T} | \mathfrak{F}_0) = 0$ .<sup>5</sup> Empirical work using this regression has equated convergence with a negative value of  $\beta$ , treating  $\beta \geq 0$  as the no convergence null hypothesis.

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<sup>5</sup> In some formulations of cross-section tests, Eq. (9) is modified to include a set of control variables  $X_i$ ,

$$g_{i,T} = \alpha + \beta y_{i,0} + \Pi X_i + \varepsilon_{i,T}. \quad (9')$$

Here a negative  $\beta$  means that convergence holds conditional on some set of exogenous factors. When  $X_i$  includes the savings and population growth rates for country  $i$ , Mankiw, Romer, and Weil (1992) show that this equation can represent the law of motion describing economies which obey the Solow growth model with constant returns to scale in the aggregate production function, and is thus compatible with versions of the model described in Section 2. We ignore the presence of control variables in the subsequent discussion without any loss of generality.

This requirement may be rewritten as a constraint on the mean of output differences between two time series. Observing that  $g_{i,T} = T^{-1} \sum_{t=1}^T \Delta y_{i,t}$ , where  $\Delta y_{i,t} = y_{i,t} - y_{i,t-1}$ , (9) implies that

$$T^{-1} \sum_{i=1}^I \Delta y_{i,t} - T^{-1} \sum_{j=1}^I \Delta y_{j,t} = \beta(y_{i,0} - y_{j,0}) + \varepsilon_{i,T} - \varepsilon_{j,T}. \quad (10)$$

If  $y_{i,0} - y_{j,0}$  is positive, then the requirement that  $\beta$  is negative implies that the expected value of  $T^{-1} \sum_{i=1}^I \Delta y_{i,t} - T^{-1} \sum_{j=1}^I \Delta y_{j,t}$  is negative. From the perspective of bivariate comparisons, the cross-section  $\beta$  tests consequently examine whether the average change in the per capita output of an initially poorer country exceeds that of an initially richer country.

Equivalently, letting  $\bar{y}_{i,0} = I^{-1} \sum_{i=1}^I y_{i,0}$  and  $\bar{g}_{i,T} = I^{-1} \sum_{i=1}^I g_{i,T}$ , recall that the ordinary least squares estimator  $\hat{\beta}$  can be written as

$$\hat{\beta} = \sum_{i=1}^I \phi_i \psi_i, \quad (11)$$

where

$$\phi_i = \frac{(y_{i,0} - \bar{y}_{i,0})^2}{\sum_{i=1}^I (y_{i,0} - \bar{y}_{i,0})^2}, \quad (12)$$

$$\psi_i = \frac{(g_{i,T} - \bar{g}_{i,T})}{(y_{i,0} - \bar{y}_{i,0})}. \quad (13)$$

In other words,  $\hat{\beta}$  equals a weighted average of the ratio of differences of growth rates from the sample means to differences of initial incomes from the sample mean. Cross-section tests therefore require that a weighted average of countries with above average initial incomes grow at a slower rate than the mean growth for the cross-section. In equating convergence with the neoclassical model, the testable restriction of the model as analyzed in cross-section tests requires that the first moments of the stochastic process governing growth rates differ for initially rich and poor economies.

This derivation shows how the cross-section tests may be interpreted with respect to Definition 1. Suppose that  $\hat{\beta} < 0$ . Since  $\hat{\beta}$  is a weighted average of  $\psi_i$ 's, a negative  $\hat{\beta}$  means that the output differences between some pairs of countries have declined over the sample. Hence, for the information set consisting exclusively of a constant, some pairs of countries are converging in the sense of Definition 1. However, the cross-section tests cannot identify groupings of countries which are converging. This class of tests is thus ill-designed to analyze data where some countries are converging and others are not.

Further, this test does not provide evidence on whether economies converge in the sense of Definition 2. To see this, consider a cross-section law of motion for growth where economies converge to one of  $n$  long-run steady states. Durlauf and Johnson (1995) show how a class of endogenous growth models produces the law of motion<sup>6</sup>

$$g_{i,T} = \alpha + \beta(y_{i,0} - \gamma_i) + \varepsilon_{i,T}, \quad (14)$$

where  $\beta < 0$  and  $\gamma_i = \mu_n$  if economy  $i$  is converging to equilibrium  $n$ . Here, the law of motion for each economy is the same except for a fixed displacement determined by the economy's long-run equilibrium. If the various economies are distributed across  $N$  long-run steady states, then convergence as characterized by Definition 2 does not hold, although countries associated with the same equilibrium are converging.

Suppose that Eq. (9) is estimated in order to test for convergence. From the perspective of Eq. (14), Eq. (9) is a misspecified regression where the regressor  $y_{i,0} - \gamma_i$  has been replaced by  $y_{i,0}$ ; equivalently,  $\gamma_i$  is an omitted regressor. However, the biased OLS estimate  $\hat{\beta}$  from Eq. (9) may still be negative. To see this, suppose that  $I$  is large so that we can replace sample moments with population moments. Observe that the  $\hat{\beta}$  from estimating Eq. (9) will follow

$$\hat{\beta} = \beta \left( 1 - \frac{\text{cov}(\gamma_i, y_{i,0})}{\text{var}(y_{i,0})} \right). \quad (15)$$

To identify the direction of this bias, we can rewrite this limit as

$$\beta \left( 1 - \frac{\text{cov}(\gamma_i, y_{i,0})}{\text{var}(y_{i,0})} \right) = -\beta \frac{\text{cov}(y_{i,0}, \gamma_i - y_{i,0})}{\text{var}(y_{i,0})}. \quad (16)$$

The sign of  $\text{cov}(y_{i,0}, \gamma_i - y_{i,0})/\text{var}(y_{i,0})$  is ambiguous and depends upon the distribution of initial incomes around the different equilibria. The distribution of initial incomes for a cross-section of economies is not part of the set of restrictions imposed on data by any class of growth models. When this covariance is negative, the estimated coefficient  $\hat{\beta}$  will also be negative, which

<sup>6</sup> Specifically, Durlauf and Johnson (1995) show that an equation of the form

$$g_{i,T} = \alpha + \beta(y_{i,0} - \gamma_i) + \Pi X_i + \varepsilon_{i,T}, \quad (14')$$

where  $X_i$  includes the savings and population growth rates for country  $i$ , is the implied law of motion for economies obeying a version of the Solow growth model where the aggregate production function exhibits increasing returns to scale over some range of capital levels. The economies studied by Azariadis and Drazen (1990) fall into this class. Such economies can converge to different long-run equilibria depending on initial conditions. The economies analyzed by Durlauf (1993) will possess similar laws of motion except that the regression coefficients  $\alpha$ ,  $\beta$ , and  $\Pi$  will depend on the long-run steady state to which an economy converges.



means the econometrician will erroneously conclude that all countries in the cross-section are converging when countries are converging to different steady states.

To illustrate how a negative  $\hat{\beta}$  can emerge from some distribution of initial conditions, consider the following argument. Observe that for fixed savings and population growth rates, the level of  $\gamma_i$  differs from a country's long-run equilibrium  $y_{i,\infty}$ , by a constant and a positive scale factor. Suppose that there are only two long-run equilibria and that all low-equilibrium economies have the same initial income and that all high-equilibrium countries have the same initial income. Under these conditions, a negative  $\hat{\beta}$  can occur if economies converging to low-output equilibria start far below their steady states, whereas economies converging to high-output equilibria start at values near their steady states.<sup>7</sup>

The relationship between the cross-section tests and the convergence definitions is summarized in Proposition 4.

*Proposition 4. Convergence definitions and cross-section tests. A negative coefficient  $\hat{\beta}$  in the cross-section growth regression (9) or (9') is compatible with a class of structural models which violate Definition 2 of convergence.*

Proposition 4 calls into question the utility of cross-section tests in adjudicating theoretical disputes in the growth literature. From the perspective of new growth models, the relevant notion of convergence would appear to be Definition 2 since this definition equates convergence with the independence of limiting behavior from initial conditions, whose failure is a hallmark of that class of models.

These results are complementary of those derived in Quah (1993a). Quah shows how it is possible for a negative cross-section relationship between initial income and growth to be compatible with a stable cross-section variance in output levels. Intuitively, this can hold because shocks to country-specific growth rates can offset the effect of this negative coefficient.<sup>8</sup> Notice however that in Quah's model, countries are converging in the sense of Definition 2 and so by that definition, the interpretation given to a negative  $\beta$  is correct for the model he studies. Conversely, our analysis shows how a negative  $\beta$  can occur

<sup>7</sup> Durlauf and Johnson (1994) show that if one modifies the cross-section tests to allow (14) to be the alternative to (9), convergence is rejected for the countries in the Summers–Heston (1988) data set.

<sup>8</sup> This argument assumes that the initial income series used on the right-hand side is the same as that used in constructing the left-hand side growth rates in the cross-section regression. When the initial income series used to construct the growth rate series differs from the one used as a right-hand side variable, as might occur if the dates of the series differ, Quah further shows that there is no a priori restriction on the sign of  $\beta$ .

due to a divergence between the transition and steady state properties for a cross-section of economies where Definition 2 is violated.

#### 4.2. Time series tests

A second approach, employed by Quah (1992), Bernard (1992), and Bernard and Durlauf (1995), has relied on the time series properties of output series. Time series approaches to convergence check for the compatibility of  $y_{i,t} - y_{j,t}$  with a time-invariant Wold representation of the form

$$y_{i,t} - y_{j,t} = \kappa_{i,j} + \sum_{r=0}^{\infty} \pi_{i,j,r} \varepsilon_{i,j,t-r} \quad (17)$$

such that the  $\kappa_{i,j} = 0$  and  $\pi_{i,j,r}$  is square-summable. As stated in Proposition 5, the presence of either a deterministic or unit-root component in  $y_{i,t} - y_{j,t}$  is a violation of Definition 2, as either component implies that forecasts of output differences do not converge to zero in expected value as the forecast horizon becomes arbitrarily long.

*Proposition 5. Relationship between convergence definitions and time series tests. If  $y_{i,t} - y_{j,t}$  contains either a nonzero mean or a unit root, then Definition 2 of convergence is violated.*

Notice that if  $y_{i,t} - y_{j,t}$  is a zero-mean stationary process, then Eq. (17) immediately implies that  $T^{-1} \sum_{t=1}^T \Delta y_{i,t} - T^{-1} \sum_{t=1}^T \Delta y_{j,t}$  is also a zero-mean stationary process.

#### 4.3. Relationships between tests and transition versus steady state behavior

Despite their links to related convergence definitions, cross-section and time series tests place different implications on the data. At one level, if the two types of tests are applied to the same data set, they must necessarily be inconsistent. This holds because, as a comparison of (10) and (17) shows, cross-section tests require that the first difference of cross-economy output differences possesses a nonzero mean, whereas time series tests require the same series to possess a zero mean.

*Proposition 6. Incompatibility of cross-section and time series tests. Under cross-section tests, convergence requires that the expected value of  $T^{-1} \sum_{t=1}^T \Delta y_{i,t} - T^{-1} \sum_{t=1}^T \Delta y_{j,t}$  is negative if  $y_{i,0} - y_{j,0}$  is positive, whereas under time series tests, convergence requires that the expected value of  $T^{-1} \sum_{t=1}^T \Delta y_{i,t} - T^{-1} \sum_{t=1}^T \Delta y_{j,t}$  is zero regardless of  $y_{i,0} - y_{j,0}$ . Thus, there*

*exists a set of values for the sample moments  $\Delta y_{i,t}$  and  $\Delta y_{j,t}$  which implies convergence under one test yet implies no convergence under the other.*

In order to understand this dissimilarity in the data restrictions of the two classes of tests, it is necessary to observe that the two approaches to convergence have fundamentally different views concerning the properties of the data under question. In cross-section tests, one assumes that the data are in transition towards a limiting distribution and convergence is interpreted as meaning that initial output differences between economies dissipate over a fixed time period. In time series tests, one assumes that the data are generated by economies near their limiting distributions and convergence is interpreted to mean that initial conditions have no (statistically significant) effect on the expected value of output differences. Consequently, a given approach is appropriate depending upon whether one regards the data as better characterized by transition or steady state dynamics.

Further, if the data are taken from economies which are far from their steady states, then the sample moments of the data might inaccurately approximate the limiting population moments. For example, per capita output for an economy at a unique steady state will strictly exceed output of an economy converging to the same steady state from below. Hence the mean of the first economy will be strictly greater than the second, which violates Definition 2 when sample means are used to proxy for asymptotic means. In such a situation, the null of no convergence may be erroneously accepted when time series tests are used. In other words, time series tests may have poor power properties when applied to data from economies in transitions.<sup>9</sup>

As a result, cross-section tests appear to more naturally apply to transition data whereas time series tests appear to more naturally apply to data whose sample moments well approximate the properties of the limiting distribution of the economies under study. The empirical relevance of this distinction is indicated in Quah (1993b), where the instability of mean growth rates over different subsamples for countries in the Summers–Heston (1988) data set illustrates how for this large cross-section, the data fail to possess the invariant moments necessary for time series analysis.

## 5. Conclusions

The new growth theory has led to many empirical efforts to identify whether per capita output is equalizing across economies with equivalent micro-economic structures. The empirical growth literature has produced diverse

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<sup>9</sup> Unit root tests take no convergence as the null. On the other hand, a test of convergence based on the null hypothesis that the mean of this process is zero would take convergence as the null. We thank an associate editor for this observation.

definitions and testing procedures as well as conflicting conclusions with respect to the convergence hypothesis. This paper has attempted to provide a framework for interpreting this body of research.

We have shown how identically specified economies with a common concave technology will provide restrictions on the properties of output differences between the economies which can be given precise statistical formulations. The cross-section and time series tests which have been proposed by various authors to test for convergence can be related to these formulations. Cross-section tests turn out to place much weaker restrictions on the behavior of growth across countries than the associated time series tests. As a result, the cross-section tests can reject a no convergence null hypothesis for data generated by economies with different long-run steady states, such as those considered by Romer (1986). Time series tests do not spuriously reject the no convergence null for data generated by multiple long-run equilibria. However, the time series approach requires that the economies under analysis are near their long-run equilibria since the tests assume that the sample moments of the data accurately approximate the limiting moments for the data under analysis. The tests may therefore be invalid if the data are largely driven by transition dynamics.

Overall, our results suggest that neither testing framework is likely to yield unambiguous conclusions with respect to competing models of growth. At a minimum, our work shows that the bulk of cross-section evidence on convergence which has thus far appeared can be construed as consistent with some versions of the new growth theory. Additionally, the time series results accepting the no convergence null may be due to transitional dynamics in the data. An important advance over both approaches, though, would lie in the integration of the transition information in the cross-section approach with the steady state information in the time series approach to create a more general empirical methodology. One possibility is to follow the approach in Quah (1993a,b) and estimate a general Markov transition function for the data and then infer the limiting distribution of the cross-section.

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